Dr Oliver Mathematics
Mathematics: National Qualifications N5
2018 Paper 1: Non-Calculator
1 hour 15 minutes

The total number of marks available is 46.
You must write down all the stages in your working.

1. Evaluate (2)

$$2\tfrac{1}{3} + \tfrac{4}{5}.$$

Solution

$$\begin{aligned} 2\tfrac{1}{3} + \tfrac{4}{5} &= 2 + \tfrac{5}{15} + \tfrac{12}{15} \\ &= 2 + \tfrac{17}{15} \\ &= \underline{\underline{3\tfrac{2}{15}}}. \end{aligned}$$

2. Expand and simplify (3)

$$(3x+1)(x-1) + 2(x^2-5).$$

Solution

$\times$	$3x$	$+1$
x	$3x^2$	$+x$
-1	$-3x$	-1

$$\begin{aligned} (3x+1)(x-1) + 2(x^2-5) &= (3x^2 - 2x - 1) + (2x^2 - 10) \\ &= \underline{\underline{5x^2 - 2x - 11}}. \end{aligned}$$

3. Solve, algebraically, the system of equations (3)

$$4x + 5y = -3$$
$$6x - 2y = 5.$$

Solution

$$4x + 5y = -3 \quad (1)$$
$$6x - 2y = 5 \quad (2)$$

Do $3 \times (1)$ and $2 \times (2)$:

$$12x + 15y = -9 \quad (3)$$
$$12x - 4y = 10 \quad (4)$$

Subract $(3) - (4)$:

$$19y = -19 \Rightarrow \underline{\underline{y = -1}}$$
$$\Rightarrow 4x - 5 = -3$$
$$\Rightarrow 4x = 2$$
$$\Rightarrow \underline{\underline{x = \tfrac{1}{2}}}.$$

4. Two vectors are given by (2)

$$\mathbf{u} = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} \text{ and } \mathbf{u} + \mathbf{v} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}.$$

Find vector $\mathbf{v}$.
Express your answer in component form.

Solution

$$\begin{aligned}\mathbf{v} &= (\mathbf{u}+\mathbf{v}) - \mathbf{u} \\ &= \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 5 \\ -9 \\ 2 \end{pmatrix}}}.\end{aligned}$$

5. Solve (2)

$$x^2 - 11x + 24 = 0.$$

Solution

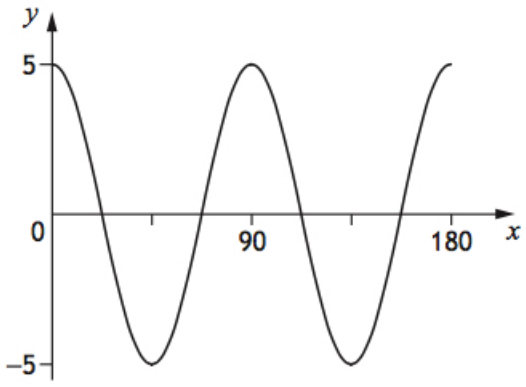
$$\left.\begin{array}{ll} \text{add to:} & -11 \\ \text{multiply to:} & +24 \end{array}\right\} -8, -3$$

$$\begin{aligned} x^2 - 11x + 24 = 0 &\Rightarrow (x-8)(x-3) = 0 \\ &\Rightarrow x - 3 = 0 \text{ or } x - 8 = 0 \\ &\Rightarrow \underline{\underline{x = 3 \text{ or } x = 8}}. \end{aligned}$$

6. Part of the graph of (2)

$$y = a\cos bx^\circ$$

is shown in the diagram.

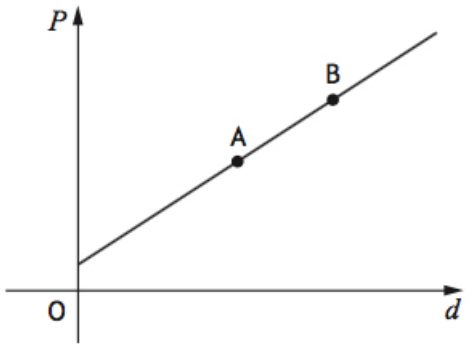


State the values of a and b.

Solution

$\underline{\underline{a = 5}}$ (because of the amplitude) and $\underline{\underline{b = 4}}$ (because there 'four' the graph in there).

7. The cost of a journey with Tom's Taxis depends on the distance travelled.
The graph below shows the cost, P pounds, of a journey with Tom's Taxis against the distance travelled, d miles.



Point A represents a journey of 8 miles which costs £14.
Point B represents a journey of 12 miles which costs £20.

(a) Find the equation of the line in terms of P and d. (3)
Give the equation in its simplest form.

Solution

$$\text{Gradient} = \frac{20-14}{12-8} = \frac{6}{4} = \tfrac{3}{2}$$

and the equation of the line is

$$P - 14 = \tfrac{3}{2}(d-8) \Rightarrow P - 14 = \tfrac{3}{2}d - 12 \Rightarrow \underline{\underline{P = \tfrac{3}{2}d + 2}}.$$

(b) Calculate the cost of a journey of 5 miles. (1)

Solution

$$\begin{aligned} P &= (\tfrac{3}{2} \times 5) + 2 \\ &= \underline{\underline{£9.50}}. \end{aligned}$$

8. Determine the nature of the roots of the function (2)

$$\mathrm{f}(x) = 2x^2 + 4x + 5.$$

Solution

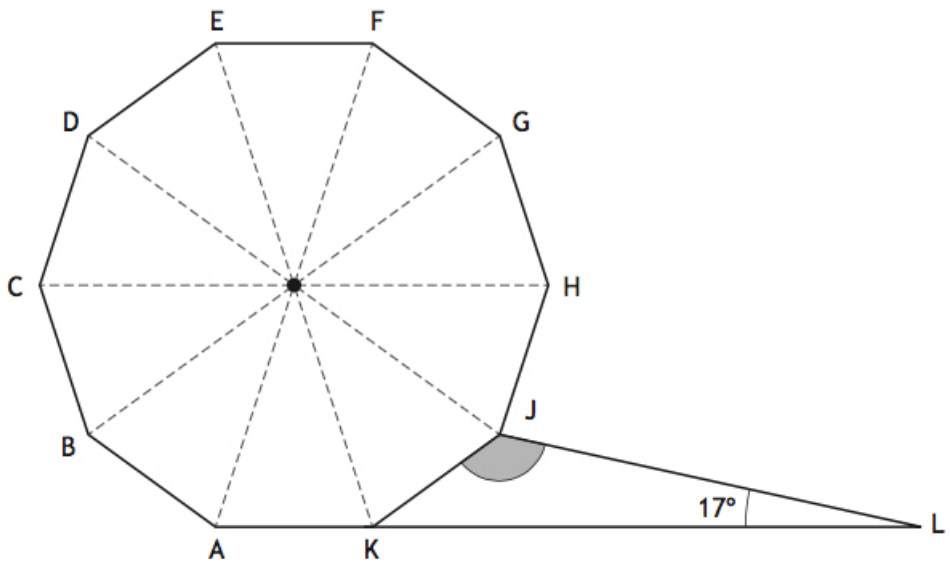
$a = 2$, $b = 4$, and $c = 5$:

$$\begin{aligned} b^2 - 4ac &= 4^2 - 4 \times 2 \times 5 \\ &= 16 - 40 \\ &= -24 \\ &< 0; \end{aligned}$$

hence, there are <u>no real roots</u>.

9. In the diagram shown below, $ABCDEFGHJK$ is a regular decagon. (2)

- Angle KLJ is 17°.
- AKL is a straight line.



Calculate the size of shaded angle KJL.

Solution

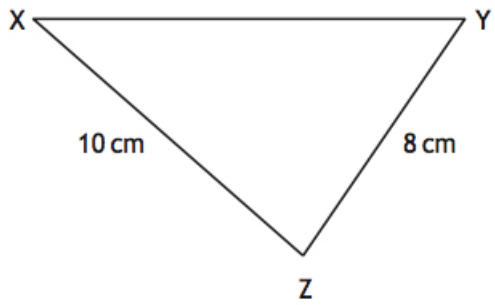
$$\begin{aligned}\angle KOJ = \frac{360}{10} &\Rightarrow \angle KOJ = 36^\circ \\ &\Rightarrow \angle JKO = \tfrac{1}{2}(180 - 36) \\ &\Rightarrow \angle JKO = \tfrac{1}{2}(144) \\ &\Rightarrow \angle JKO = 72^\circ.\end{aligned}$$

Now,

$$\begin{aligned}\angle AKJ = \angle KJL + \angle KLJ &\Rightarrow 2 \times 72 = \angle KJL + 17 \\ &\Rightarrow \angle KJL = 144 - 17 \\ &\Rightarrow \underline{\underline{\angle KJL = 127^\circ}}.\end{aligned}$$

10. In triangle XYZ: (2)

- $XZ = 10$ centimetres,
- $YZ = 8$ centimetres, and
- $\cos Z = \frac{1}{8}$.



Calculate the length of XY.

Solution

$$\begin{aligned} XY &= \sqrt{XZ^2 + YZ^2 - 2 \cdot XZ \cdot YZ \cdot \cos XZY} \\ &= \sqrt{10^2 + 8^2 - 2 \cdot 10 \cdot 8 \cdot \tfrac{1}{8}} \\ &= \sqrt{100 + 64 - 20} \\ &= \sqrt{144} \\ &= \underline{\underline{12 \text{ cm}}}. \end{aligned}$$

11. Express (2)

$$\frac{9}{\sqrt{6}}$$

with a rational denominator.
Give your answer in its simplest form.

Solution

$$\begin{aligned} \frac{9}{\sqrt{6}} &= \frac{9}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{9\sqrt{6}}{6} \\ &= \underline{\underline{\frac{3\sqrt{6}}{2}}}. \end{aligned}$$

12. Given that (1)

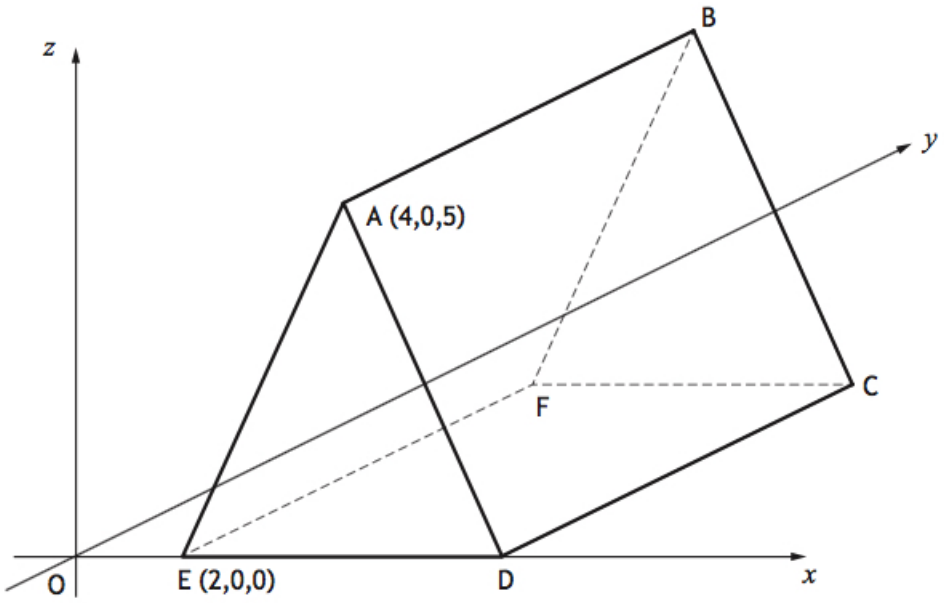
$$\cos 60^\circ = 0.5,$$

state the value of $\cos 240^\circ$.

Solution

$$\cos 60^\circ = 0.5 \Rightarrow \cos 300^\circ = 0.5$$
$$\Rightarrow \underline{\underline{\cos 240^\circ = -0.5}}.$$

13. The diagram shows a triangular prism, $ABCDEF$, relative to the coordinate axes. (2)



- $AD = AE$.
- $DC = 8$ units.
- Edges EF, DC, and AB are parallel to the y-axis.

Write down the coordinates of B and C.

Solution

$\underline{\underline{B(4, 8, 5)}}$ and $\underline{\underline{C(6, 8, 0)}}$.

14. 3] Change the subject of the formula

$$y = g\sqrt{x} + h$$

to x.

Solution

$$\begin{aligned} y = g\sqrt{x} + h &\Rightarrow g\sqrt{x} = y - h \\ &\Rightarrow \sqrt{x} = \frac{y-h}{g} \\ &\Rightarrow \underline{\underline{x = \left(\frac{y-h}{g}\right)^2}}. \end{aligned}$$

15. Remove the brackets and simplify (2)

$$\left(\tfrac{2}{3}p^4\right)^2.$$

Solution

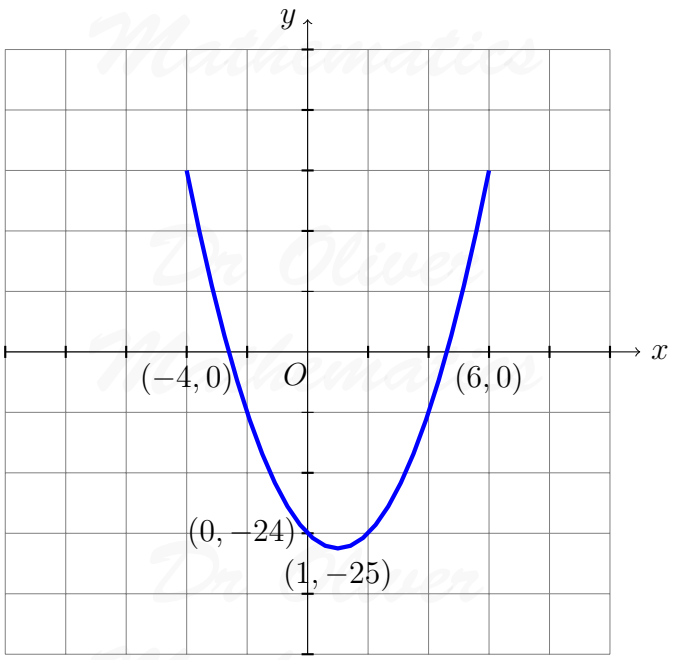
$$\left(\tfrac{2}{3}p^4\right)^2 = \underline{\underline{\tfrac{4}{9}p^8}}.$$

16. Sketch the graph of (3)

$$y = (x-6)(x+4).$$

On your sketch, show clearly the points of intersection with the x-axis and the y-axis, and the coordinates of the turning point.

Solution

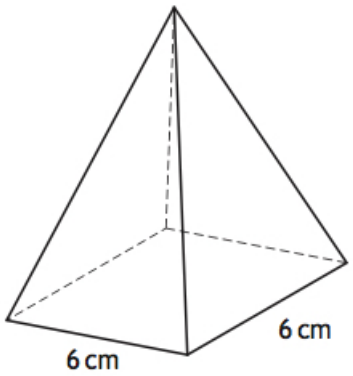


The points of intersection with the x-axis are $(-4, 0)$ and $(6, 0)$.

The point of intersection with the y-axis is $(0, -24)$.

The turning point is $(1, -25)$.

17. A square based pyramid is shown in the diagram below. (3)



The square base has length 6 centimetres.
The volume is 138 cubic centimetres.
Calculate the height of the pyramid.

Solution

$$\tfrac{1}{3} \times 6 \times 6 \times \text{height} = 138 \Rightarrow \text{height} = \frac{23}{2}$$
$$\Rightarrow \underline{\underline{\text{height} = 11\tfrac{1}{2} \text{ cm}}}.$$

18. Express (2)

$$\sin x^\circ \cos x^\circ \tan x^\circ$$

in its simplest form.
Show your working.

Solution

$$\sin x^\circ \cos x^\circ \tan x^\circ = \sin x^\circ \cos x^\circ \left(\frac{\sin x^\circ}{\cos x^\circ}\right)$$
$$= \underline{\underline{\sin^2 x^\circ}}.$$

19. (a) (i) Express (2)

$$x^2 - 6x - 81$$

in the form

$$(x - p)^2 + q.$$

Solution

$$x^2 - 6x - 81 = (x^2 - 6x + 9) - 81 - 9$$
$$= \underline{\underline{(x - 3)^2 - 90}};$$

hence, $\underline{\underline{p = 3}}$ and $\underline{\underline{q = -90}}$.

(ii) Hence state the equation of the axis of symmetry of the graph of (1)

$$y = x^2 - 6x - 81.$$

Solution

$\underline{\underline{x = 3}}$.

The roots of the equation

$$x^2 - 6x - 81 = 0$$

can be expressed in the form $x = d \pm d\sqrt{e}$.

(b) Find, algebraically, the values of d and e. (4)

Solution

$$\begin{aligned} x^2 - 6x - 81 = 0 &\Rightarrow (x-3)^2 - 90 = 0 \\ &\Rightarrow (x-3)^2 = 90 \\ &\Rightarrow x - 3 = \pm\sqrt{90} \\ &\Rightarrow x = 3 \pm \sqrt{9 \times 10} \\ &\Rightarrow x = 3 \pm (\sqrt{9} \times \sqrt{10}) \\ &\Rightarrow \underline{\underline{x = 3 \pm 3\sqrt{10}}}; \end{aligned}$$

hence, $\underline{\underline{d = 3}}$ and $\underline{\underline{e = 10}}$.

Dr Oliver Mathematics