

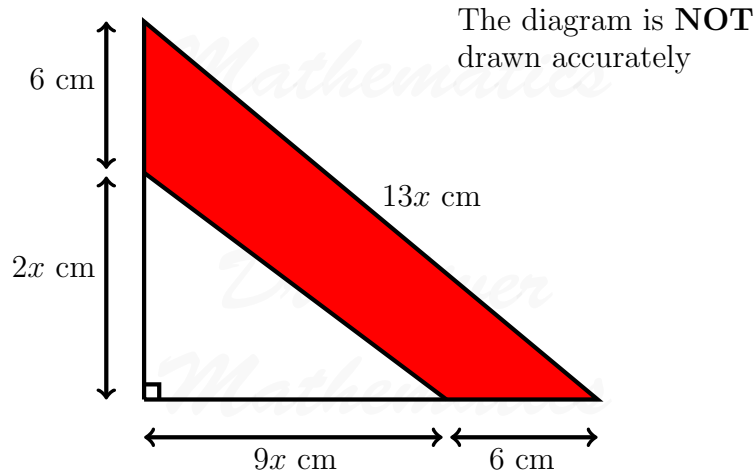
Dr Oliver Mathematics

Worked Examples

Area 10

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- Find the area of the red triangle.



Solution

We begin by using Pythagoras' Theorem:

$\times$	$2x$	$+6$
$2x$	$4x^2$	$+12x$
$+6$	$+12x$	$+36$

and

$\times$	$9x$	$+6$
$9x$	$81x^2$	$+54x$
$+6$	$+54x$	$+36$

Now,

$$\begin{aligned}(13x)^2 &= (2x + 6)^2 + (9x + 6)^2 \\ \Rightarrow 169x^2 &= (4x^2 + 24x + 36) + (81x^2 + 108x + 36) \\ \Rightarrow 169x^2 &= 85x^2 + 132x + 72 \\ \Rightarrow 84x^2 - 132x - 72 &= 0\end{aligned}$$

take a factor of 12 out:

$$\Rightarrow 12(7x^2 - 11x - 6) = 0$$

we factorise:

$$\left. \begin{array}{l} \text{add to:} \quad \quad \quad -11 \\ \text{multiply to:} \quad (+7) \times (-6) = -42 \end{array} \right\} -14, +3$$

$$\begin{aligned}\Rightarrow 12[7x^2 - 14x + 3x - 6] &= 0 \\ \Rightarrow 12[7x(x - 2) + 3(x - 2)] &= 0 \\ \Rightarrow 12(7x + 3)(x - 2) &= 0 \\ \Rightarrow 7x + 3 = 0 \quad x - 2 = 0 \\ \Rightarrow x = -\frac{3}{7} \quad x = 2;\end{aligned}$$

clearly, $x \neq -\frac{3}{7}$ (why?) so $x = 2$.

Finally,

$$\begin{aligned}\text{area of the red triangle} &= \text{big triangle} - \text{small triangle} \\ &= \left(\frac{1}{2} \times 10 \times 24\right) - \left(\frac{1}{2} \times 4 \times 18\right) \\ &= 120 - 36 \\ &= \underline{\underline{84 \text{ cm}^2}}.\end{aligned}$$