

Dr Oliver Mathematics
Mathematics: Higher
2015 Paper 1: Non-Calculator
1 hour 10 minutes

The total number of marks available is 60.

You must write down all the stages in your working.

1. Vectors

$$\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k} \text{ and } \mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$$

are perpendicular.

Determine the value of t .

2. Find the equation of the tangent to the curve

$$y = 2x^3 + 3$$

at the point where $x = -2$.

3. Show that $(x + 3)$ is a factor of

$$x^3 - 3x^2 - 10x + 24$$

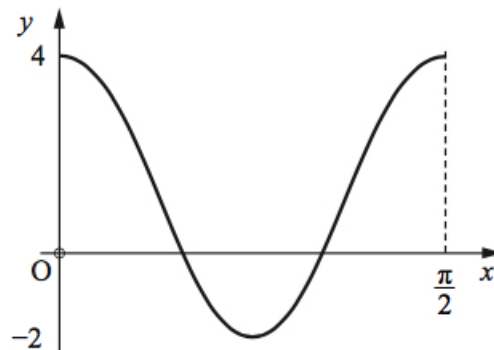
and hence factorise

$$x^3 - 3x^2 - 10x + 24$$

fully.

4. The diagram shows part of the graph of the function

$$y = p \cos qx + r.$$



Write down the values of p , q , and r .

5. A function g is defined on \mathbb{R} , the set of real numbers, by

$$g(x) = 6 - 2x.$$

(a) Determine an expression for $g^{-1}(x)$. (2)

(b) Write down an expression for $g(g^{-1}(x))$. (1)

6. Evaluate (3)

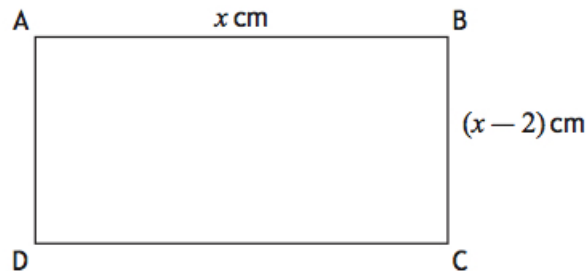
$$\log_6 12 + \frac{1}{3} \log_6 27.$$

7. A function f is defined on a suitable domain by (4)

$$f(x) = \sqrt{x} \left(3x - \frac{2}{x\sqrt{x}} \right).$$

Find $f'(4)$.

8. $ABCD$ is a rectangle with sides of lengths x centimetres and $(x - 2)$ centimetres, as shown. (4)



If the area of $ABCD$ is less than 15 cm^2 , determine the range of possible values of x .

9. A , B , and C are points such that AB is parallel to the line with equation (3)

$$y + \sqrt{3}x$$

and BC makes an angle of 150° with the positive direction of the x -axis.

Are the points A , B , and C collinear?

10. Given that

$$\tan 2x = \frac{3}{4}, \quad 0 < x < \frac{1}{4}\pi,$$

find the exact value of

(a) $\cos 2x$, (1)

(b) $\cos x$. (2)

11. $T(-2, -5)$ lies on the circumference of the circle with equation

$$(x + 8)^2 + (y + 2)^2 = 45.$$

(a) Find the equation of the tangent to the circle passing through T . (4)

This tangent is also a tangent to a parabola with equation

$$y = -2x^2 + px + 1 - p,$$

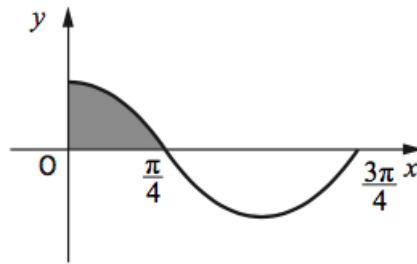
where $p > 3$.

(b) Determine the value of p . (6)

12. The diagram shows part of the graph of (2)

$$y = a \cos bx.$$

The shaded area is $\frac{1}{2}$ unit².



What is the value of

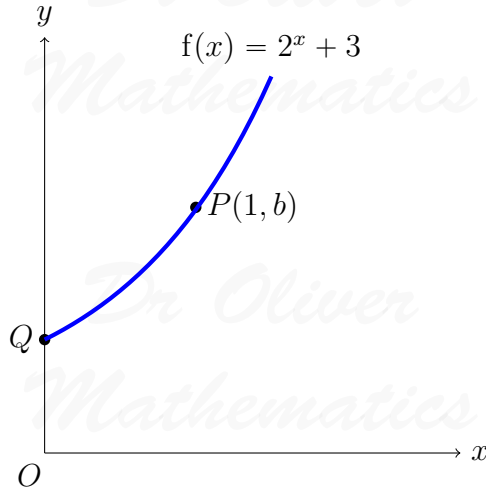
$$\int_0^{\frac{3}{4}\pi} (a \cos bx) dx?$$

13. The function

$$f(x) = 2^x + 3$$

is defined on \mathbb{R} , the set of real numbers.

The graph with equation $y = f(x)$ passes through the point $P(1, b)$ and cuts the y -axis at Q as shown in the diagram.



- (a) What is the value of b ? (1)
- (b) (i) Copy the above diagram. (1)
 On the same diagram, sketch the graph with equation $y = f^{-1}(x)$.
 (ii) Write down the coordinates of the images of P and Q . (3)

$R(3, 11)$ also lies on the graph with equation $y = f(x)$.

- (c) Find the coordinates of the image of R on the graph with equation $y = 4 - f(x + 1)$. (2)

14. The circle with equation (2)

$$x^2 + y^2 - 12x - 10y + k = 0$$

meets the coordinate axes at exactly three points.

What is the value of k ?

15. The rate of change of the temperature, $T^\circ\text{C}$ of a mug of coffee is given by (6)

$$\frac{dT}{dt} = \frac{1}{25}t - k, \quad 0 \leq t \leq 50,$$

where

- t is the elapsed time, in minutes, after the coffee is poured into the mug,
- k is a constant,
- initially, the temperature of the coffee is 100°C , and
- 10 minutes later the temperature has fallen to 82°C .

Express T in terms of t .