Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 November Paper 1 Variant 3: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Given that (3)

$$\frac{(6x^{\frac{3}{2}}y^{\frac{4}{5}})^4}{2x^{\frac{1}{2}}y^{-1}} = ax^p y^q,$$

find the values of the constants a, p, and q.

2. Express in the form (4)

$$\sqrt{\frac{1-\cos^2\theta}{4\sec^2\theta - 4}}$$

in the form

$$k\cos\theta,$$

where k is a constant to be found.

3. (a) Given that (2)

$$\mathbf{A} = \begin{pmatrix} 4 & 3 \\ -8 & -2 \end{pmatrix},$$

find $\mathbf{A}^{-1}$.

(b) Hence, find the matrix $\mathbf{M}$ such that (3)

$$\begin{pmatrix} 4 & 3 \\ -8 & -2 \end{pmatrix}\mathbf{M} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}.$$

4. (a) Sets A and B are such that (2)
   - $n(A) = 11$,
   - $n(B) = 13$, and
   - $n(A \cup B) = 18$.

   Find $n(A \cap B)$.

   (b) Sets $\mathscr{E}$, X, and Y are such that
   - $\mathscr{E} = \{\theta : 0 \leqslant \theta \leqslant 2\pi\}$,

- $X = \{\theta : \sin\theta = -0.5\}$, and
- $Y = \left\{\theta : \sec^2\theta = \frac{4}{3}\right\}$.

(i) Find, in terms of π , the elements of the set X. (1)

(ii) Find, in terms of π , the elements of the set Y. (2)

(iii) Use set notation to describe the relationship between the sets X and Y. (1)

5. It is given that

$$\log_{10} p^3 q = 10a \text{ and } \log_{10}\left(\frac{p}{q^2}\right) = a.$$

(a) Find, in terms of a, expressions for $\log_{10} p$ and $\log_{10} q$. (5)

(b) Find the value of $\log_p q$. (1)

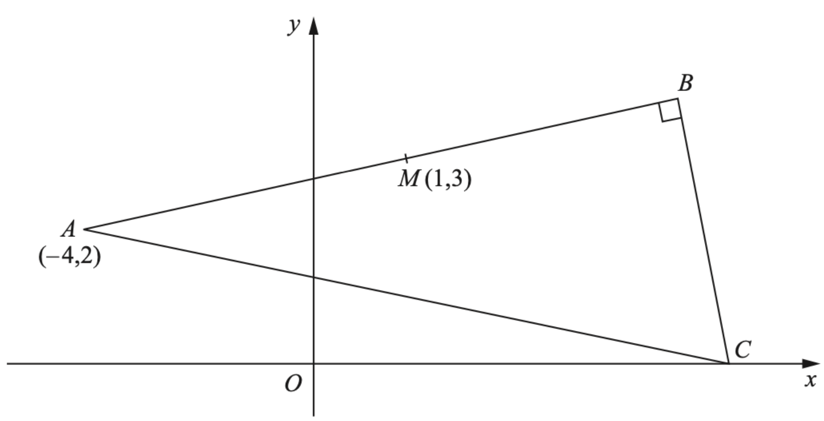
6. A curve has equation

$$y = 6\cos(\tfrac{1}{2}x) + 4\sin(\tfrac{1}{2}x), \text{ for } 0 < x < 2\pi \text{ radians.}$$

(a) Find the x-coordinate of the stationary point on the curve. (5)

(b) Determine the nature of this stationary point. (2)

7. The figure shows a right-angled triangle ABC, where the point A has coordinates $(-4, 2)$, the angle B is 90° and the point C lies on the x-axis. (7)



The point $M(1, 3)$ is the midpoint of AB.

Find the area of the triangle ABC.

8. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that

$$\mathbf{a} = \begin{pmatrix} 3+m \\ 5-2n \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 4-2n \\ 10+3m \end{pmatrix}.$$

(a) Given that (4)

$$3\mathbf{a} + \mathbf{b} = \begin{pmatrix} 1+n \\ -5 \end{pmatrix},$$

find the value of m and of n.

(b) Show that the magnitude of $\mathbf{b}$ is $k\sqrt{5}$, where k is an integer to be found. (2)

(c) Find the unit vector in the direction of $\mathbf{b}$. (1)

9. The function f is defined, for $0° \leqslant x \leqslant 360°$, by

$$\mathrm{f}(x) = 2\sin 3x - 1.$$

(a) State the amplitude and period of f. (2)

(b) State the maximum value of f and the corresponding values of x. (3)

(c) Sketch the graph of f. (2)

10. (a) Differentiate (2)

$$\tan(3x+2)$$

with respect to x.

(b) Differentiate (3)

$$(\sqrt{x}+1)^{\frac{2}{3}}$$

with respect to x.

(c) Differentiate (3)

$$\frac{\ln(x^3-1)}{2x+3}$$

with respect to x.

11. A particle moves in a straight line so that, t s after leaving a fixed point O, its velocity v ms^{-1} is given by

$$v = 3\mathrm{e}^{2t} + 4t.$$

(a) Find the initial velocity of the particle. (1)

(b) Find the initial acceleration of the particle. (3)

(c) Find the distance travelled by the particle in the third second. (4)

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12. A function f is such that

$$\mathrm{f}(x) = \ln(5x-10), \text{ for } x > 2.$$

(a) State the range of f. (1)

(b) Find $f^{-1}(x)$. (3)

(c) State the range of f^{-1}. (1)

(d) Solve (2)

$$f(x) = 0.$$

A function g is such that

$$g(x) = 2x - \ln 2, \text{ for } x \in \mathbb{R}.$$

(e) Solve (5)

$$g\,f(x) = f(x^2).$$

OR

13. A function f is such that

$$f(x) = 4e^{-x} + 2, \text{ for } x \in \mathbb{R}.$$

(a) State the range of f. (1)

(b) Solve (2)

$$f(x) = 26.$$

(c) Find $f^{-1}(x)$. (3)

(d) State the domain of f^{-1}. (1)

A function g is such that

$$g(x) = 2e^{x} - 4, \text{ for } x \in \mathbb{R}.$$

(e) Using the substitution (5)

$$t = e^{x}$$

or otherwise, solve

$$g(x) = f(x).$$