

Dr Oliver Mathematics

Worked Examples

Radius of a Circle 3

From: LoveMath, 25 April 2023

1. A rectangle is inscribed a quarter-circle is drawn as follows: the circle is tangent to the bottom and left sides of the square and passes through its top-right corner, as shown in Figure 1.

The distance between the two vertices that lie on the quarter-circle are 1 cm and 2 cm respectively.

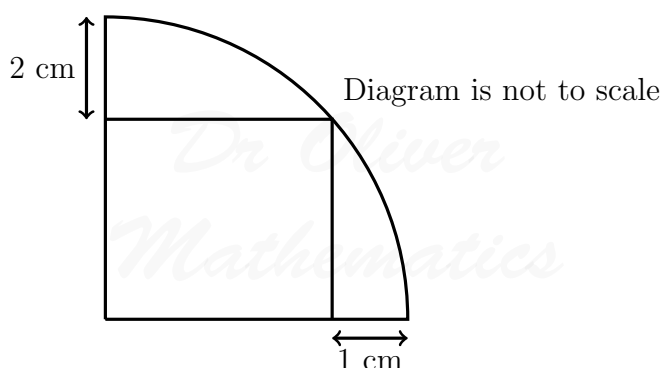


Figure 1: a rectangle and a quarter circle

Find the length of the radius of the circle.

Solution

Let r cm be the radius of the circle. We add in the new dimensions:

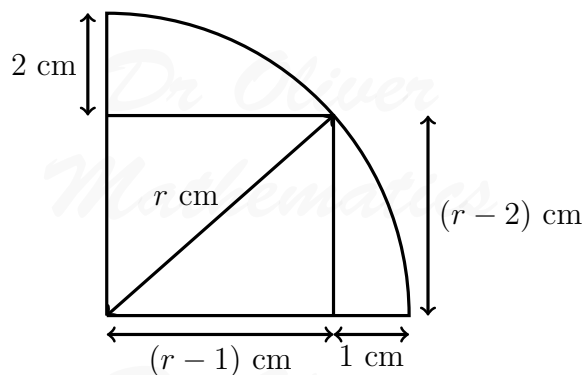


Figure 2: with the dimensions added on

Pythagoras' Theorem:

$$\begin{array}{r|rr} \times & r & -1 \\ \hline r & r^2 & -r \\ -1 & -r & +1 \\ \hline \end{array}$$

$$\begin{array}{r|rr} \times & r & -2 \\ \hline r & r^2 & -2r \\ -2 & -2r & +4 \\ \hline \end{array}$$

Now,

$$\begin{aligned} (r-1)^2 + (r-2)^2 &= r^2 \Rightarrow (r^2 - 2r + 1) + (r^2 - 4r + 4) = r^2 \\ &\Rightarrow 2r^2 - 6r + 5 = r^2 \\ &\Rightarrow r^2 - 6r + 5 = 0 \end{aligned}$$

$$\begin{array}{l} \text{add to:} \quad -6 \\ \text{multiply to:} \quad +5 \end{array} \left. \vphantom{\begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array}} \right\} -5, -1$$

$$\begin{aligned} &\Rightarrow (r-5)(r-1) = 0 \\ &\Rightarrow r-5 = 0 \text{ or } r-1 = 0 \\ &\Rightarrow r = 5 \text{ or } r = 1. \end{aligned}$$

So, two solutions, right? Well, it cannot be $r = 1$, as have a radius that is greater than 2 (why?). Hence, we need the other solution:

$$\underline{\underline{r = 5 \text{ cm.}}}$$