

Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2013 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 52.

You must write down all the stages in your working.

1. A snail crawls 3 kilometres in 16 days.

(4)

What is the average speed of the snail in metres per second?

Give your answer **in scientific notation correct to 2 significant figures**.

Solution

$$\begin{aligned} \text{Average speed} &= \frac{3 \text{ km}}{16 \text{ days}} \\ &= \frac{3000 \text{ m}}{384 \text{ hours}} \\ &= \frac{3000 \text{ m}}{23040 \text{ mins}} \\ &= \frac{3000 \text{ m}}{1382400 \text{ s}} \\ &= 0.002170138 \text{ (FCD)} \\ &= \underline{\underline{2.2 \times 10^{-3} \text{ ms}^{-1} \text{ (2 sf)}}} \end{aligned}$$

2. Solve the equation

(4)

$$2x^2 + 7x - 3 = 0.$$

Give your answers **correct to 1 decimal place**.

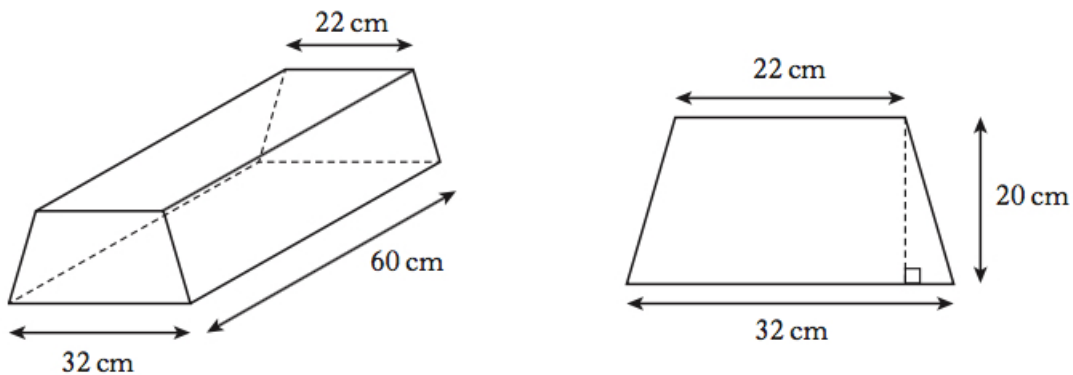
Solution

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Mathematics*

$a = 2$, $b = 7$, and $c = -3$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times (-3)}}{2 \times 2} \\&= \frac{-7 \pm \sqrt{73}}{4} \\&= -3.886\,000\,936, 0.386\,000\,936\,3 \text{ (FCD)} \\&= \underline{\underline{-3.9, 0.4}} \text{ (1 dp).}\end{aligned}$$

3. A concrete block is in the shape of a prism.



The cross-section of the prism is a trapezium with dimensions as shown.

(a) Calculate the area of the cross-section.

(3)

Solution

$$\begin{aligned}\text{CSA} &= \frac{1}{2} \times (22 + 32) \times 20 \\&= \underline{\underline{540 \text{ cm}^2}}.\end{aligned}$$

(b) Calculate the volume of the concrete block.

(1)

Solution

$$\begin{aligned} \text{Volume} &= 540 \times 60 \\ &= \underline{\underline{32\,400 \text{ cm}^3}}. \end{aligned}$$

4. Last year, 1 296 learner drivers from “Topflight” school of motoring passed their driving test. (3)
 This was 72% of those who sat their driving test from Topflight.
 How many failed their driving test?

Solution

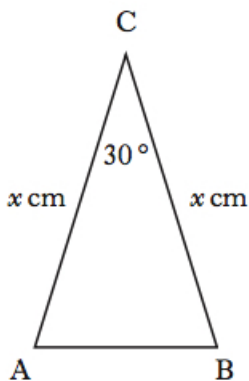
Let x be those that failed their driving test. Now,

$$100 - 72 = 28\%$$

failed their driving test so

$$\begin{aligned} 28 : 72 = x : 1\,296 &\Rightarrow \frac{x}{1\,296} = \frac{28}{72} \\ &\Rightarrow x = \frac{28 \times 1\,296}{72} \\ &\Rightarrow \underline{\underline{x = 504}}. \end{aligned}$$

5. ABC is an isosceles triangle with angle $ACB = 30^\circ$. (3)
 $AC = BC = x$ centimetres.

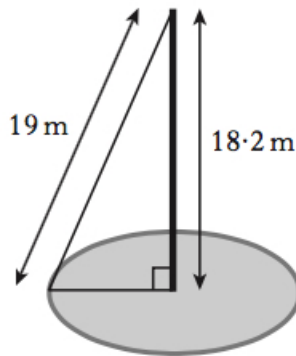


The area of triangle ABC is 9 square centimetres.
 Calculate the value of x .

Solution

$$\begin{aligned}\frac{1}{2} \times x^2 \times \sin 30^\circ &= 9 \Rightarrow x^2 = 36 \\ &\Rightarrow \underline{\underline{x = 6}}.\end{aligned}$$

6. A mobile phone mast, 18.2 metres high, stands vertically in the centre of a circle. (3)
It is supported by a wire rope, 19 metres long, attached to the ground at a point on the circumference of the circle, as shown.



Calculate the circumference of the circle.

Solution

Let x m be the adjacent side. Then

$$\begin{aligned}x &= \sqrt{19^2 - 18.2^2} \\ &= \frac{2}{5}\sqrt{186}\end{aligned}$$

and the

$$\begin{aligned}\text{circumference of the circle} &= 2 \times \pi \times \frac{2}{5}\sqrt{186} \\ &= 34.276\,489\,14 \text{ (FCD)} \\ &= \underline{\underline{34.3 \text{ m (3 sf)}}}.\end{aligned}$$

7. Jack weighs 94 kilograms. (4)
On the 1st of January, he starts a diet which is designed to reduce his weight by 7% per

month.

During which month should he achieve his target weight of 73 kilograms?

Show all your working.

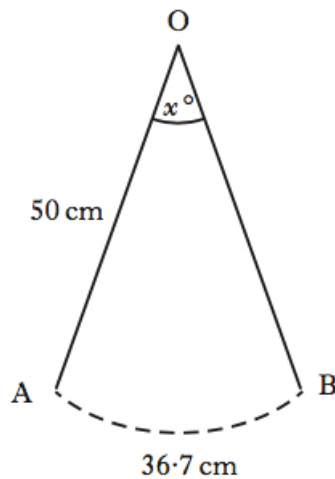
Solution

Month	Weight
Jan	$93 \times 0.93 = 86.49$
Feb	$93 \times 0.93^2 = 80.4357$
Mar	$93 \times 0.93^3 = 74.80\dots$
Apr	$93 \times 0.93^4 = 69.56\dots$

Hence, he will achieve his target weight in April.

8. As the pendulum of a clock swings, its tip moves through an arc of a circle.

(3)



The length of the pendulum is 50 centimetres.

The length of the arc is 36.7 centimetres.

Calculate x° , the angle through which the pendulum swings.

Solution

$$2 \times \pi \times 50 \times \frac{x}{360} = 36.7 \Rightarrow x = \frac{36.7 \times 360}{100\pi}$$

$$\Rightarrow x = 42.055\ 102\ 16 \text{ (FCD)}$$

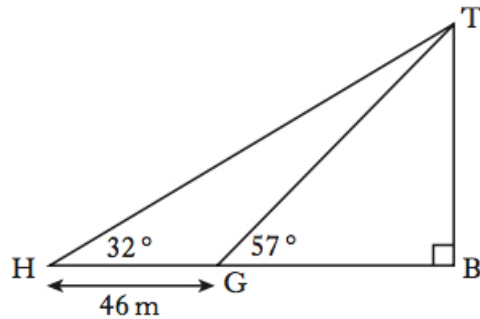
$$\Rightarrow \underline{\underline{x = 42.1 \text{ (1 dp)}}}$$

9. In triangle THB :

- angle $TBH = 90^\circ$,
- angle $THB = 32^\circ$.

G is a point on HB :

- angle $TGB = 57^\circ$,
- $GH = 46$ metres.



Calculate the length of TB .

Solution

Well,

$$TB = GB \tan 57^\circ$$

and

$$TB = (46 + GB) \tan 32^\circ.$$

Now,

$$\begin{aligned}GB \tan 57^\circ &= (46 + GB) \tan 32^\circ \Rightarrow GB \tan 57^\circ = 46 \tan 32^\circ + GB \tan 32^\circ \\&\Rightarrow GB \tan 57^\circ - GB \tan 32^\circ = 46 \tan 32^\circ \\&\Rightarrow GB(\tan 57^\circ - \tan 32^\circ) = 46 \tan 32^\circ \\&\Rightarrow GB = \frac{46 \tan 32^\circ}{\tan 57^\circ - \tan 32^\circ} \\&\Rightarrow TB = \frac{46 \tan 32^\circ \cdot \tan 57^\circ}{\tan 57^\circ - \tan 32^\circ} \\&\Rightarrow TB = 48.3738532 \text{ (FCD)} \\&\Rightarrow \underline{\underline{TB = 48.4 \text{ (1 dp)}}}.\end{aligned}$$

10. A function is given by the formula,

$$f(x) = 4 \times 2^x.$$

(a) Evaluate $f(3)$.

(2)

Solution

$$f(3) = 4 \times 2^3 = \underline{\underline{32}}.$$

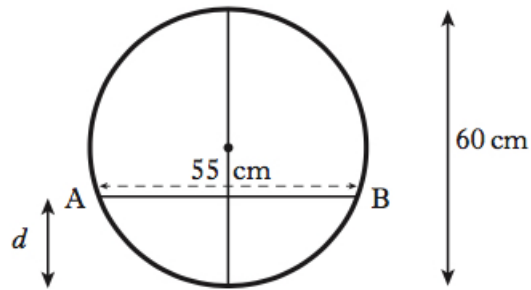
(b) Given that $f(m) = 4$, find the value of m .

(2)

Solution

$$\begin{aligned}f(m) = 4 &\Rightarrow 4 \times 2^m = 4 \\&\Rightarrow 2^m = 1 \\&\Rightarrow \underline{\underline{m = 0}}.\end{aligned}$$

11. Water flows through a horizontal pipe of diameter 60 centimetres.
The surface width, AB , of the water is 55 centimetres.



- (a) Calculate the depth, d , of the water in the pipe. (4)

Solution

$$x = \sqrt{30^2 - 27.5^2}$$

$$= \frac{5}{2}\sqrt{23}$$

and

$$d = 30 - \frac{5}{2}\sqrt{23}$$

$$= 18.010\ 421\ 19 \text{ (FCD)}$$

$$= \underline{\underline{18.0 \text{ cm (1 dp)}}}.$$

- (b) What other depth of water would give the same surface width? (1)

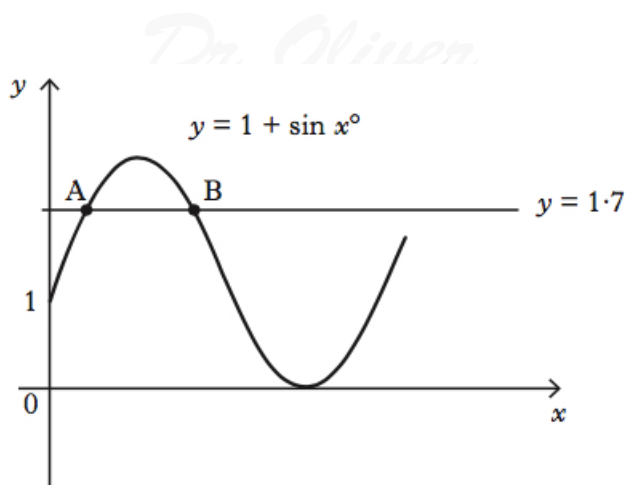
Solution

$$\text{Depth} = 30 + 11.989\dots = \underline{\underline{42.0 \text{ cm (1 dp)}}}.$$

12. Part of the graph of (4)

$$y = 1 + \sin x^\circ$$

is shown in the diagram below.



The line $y = 1.7$ is drawn. It cuts the graph of $y = 1 + \sin x^\circ$ at A and B as shown.

Calculate the x -coordinates of A and B .

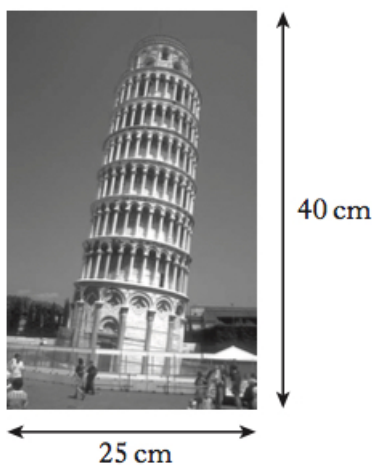
Solution

$$1 + \sin x^\circ = 1.7 \Rightarrow \sin x^\circ = 0.7$$

$$\Rightarrow x = 44.427\,004, 135.572\,996 \text{ (FCD);}$$

hence, A and B have x -coordinates of 44.4 (1 dp) and 135.6 (1 dp) respectively.

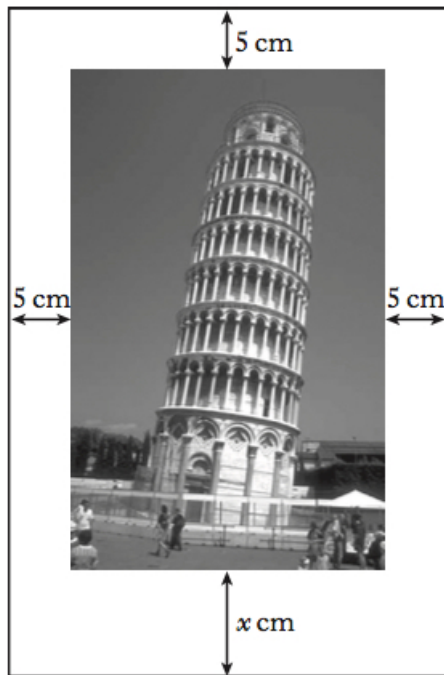
13. Asim has a poster which is 25 centimetres wide and 40 centimetres high. (4)



He decides to place it on a white card. The card and the poster are mathematically similar.



The border is 5 centimetres wide on three sides and x centimetres wide on the fourth side as shown.



Calculate the value of x .

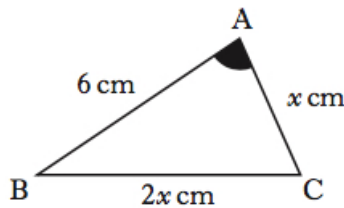
Solution

$$\begin{aligned}25 : 40 &= (25 + 5 + 5) : (40 + 5 + x) \Rightarrow 25 : 40 = 35 : (45 + x) \\ &\Rightarrow \frac{40}{25} = \frac{45 + x}{35} \\ &\Rightarrow 45 + x = \frac{40 \times 35}{25} \\ &\Rightarrow 45 + x = 56 \\ &\Rightarrow \underline{\underline{x = 11}}.\end{aligned}$$

14. In triangle ABC :

(3)

- $\cos A = 0.5$,
- $AB = 6$ centimetres,
- $BC = 2x$ centimetres,
- $AC = x$ centimetres.



Show that

$$x^2 + 2x - 12 = 0.$$

Solution

$$\begin{aligned}BC^2 &= AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos BAC \\ \Rightarrow (2x)^2 &= 6^2 + x^2 - 2 \cdot 6 \cdot x \cdot 0.5 \\ \Rightarrow 4x^2 &= 36 + x^2 - 6x \\ \Rightarrow 3x^2 + 6x - 36 &= 0 \\ \Rightarrow 3(x^2 + 2x - 12) &= 0 \\ \Rightarrow \underline{\underline{x^2 + 2x - 12 = 0}},\end{aligned}$$

as required.

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