Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2009 Paper 2: Calculator
1 hour 20 minutes

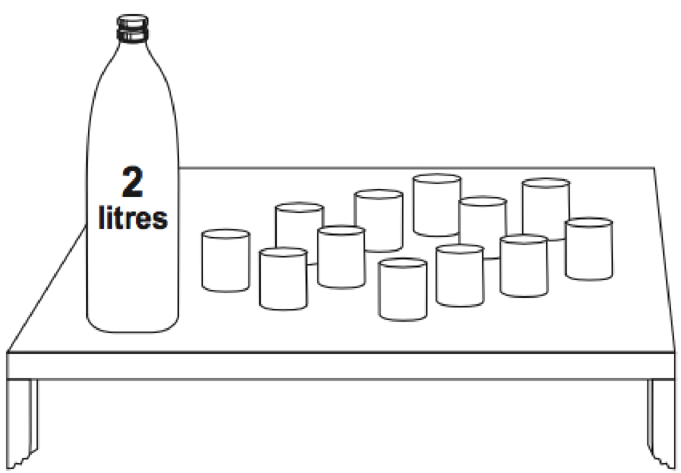
The total number of marks available is 52.
You must write down all the stages in your working.

1. One atom of gold weighs 3.27×10^{-22} grams. (3)
   How many atoms will there be in one kilogram of gold?
   Give your answer **in scientific notation correct to 2 significant figures**.

Solution

$$\begin{aligned}\text{One kilogram of gold} &= \frac{1}{3.27 \times 10^{-22} \text{ g}} \\ &= \frac{1}{3.27 \times 10^{-25} \text{ kg}} \\ &= 3.058\,103\,976 \times 10^{24} \text{ (FCD)} \\ &= \underline{\underline{3.1 \times 10^{24} \text{ atoms (2 sf)}}}.\end{aligned}$$

2. Lemonade is to be poured from a 2 litre bottle into glasses. (4)
   Each glass is in the shape of a cylinder of radius 3 centimetres and height 8 centimetres.



How many full glasses can be poured from the bottle?

Solution

$$\begin{aligned}\text{Full glasses} &= \frac{2\ \text{l}}{\pi \times 3^2 \times 8\ \text{cm}^3} \\ &= \frac{2\,000\ \text{cm}^3}{\pi \times 3^2 \times 8\ \text{cm}^3} \\ &= 8.841\,941\,283\ \text{(FCD)};\end{aligned}$$

hence, he can pour 8 glasses.

3. Solve the quadratic equation (4)

$$x^2 - 4x - 6 = 0.$$

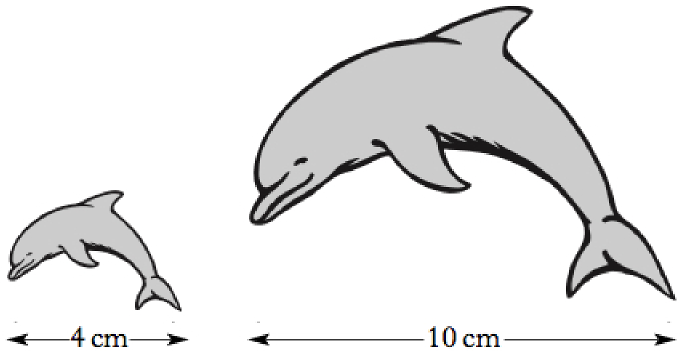
Give your answers **correct to 1 decimal place**.

Solution

$a = 1$, $b = -4$, and $c = -6$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-6)}}{2 \times 1} \\ &= \frac{4 \pm \sqrt{40}}{2} \\ &= -1.162\,277\,66, 5.162\,277\,66\ \text{(FCD)} \\ &= -1.2, 5.2\ \text{(1 dp)}.\end{aligned}$$

4. Two fridge magnets are mathematically similar. (3)
One magnet is 4 centimetres long and the other is 10 centimetres long.



The area of the smaller magnet is 18 square centimetres.
Calculate the area of the larger magnet.

Solution

The length scale ratio (LSF) is

$$\frac{10}{4} = \frac{5}{2}$$

and the area scale ratio (ASF) is

$$\left(\frac{5}{2}\right)^2 = \frac{25}{4}.$$

Hence, the area is

$$18 \times \frac{25}{4} = \underline{\underline{112.5 \text{ cm}^2}}.$$

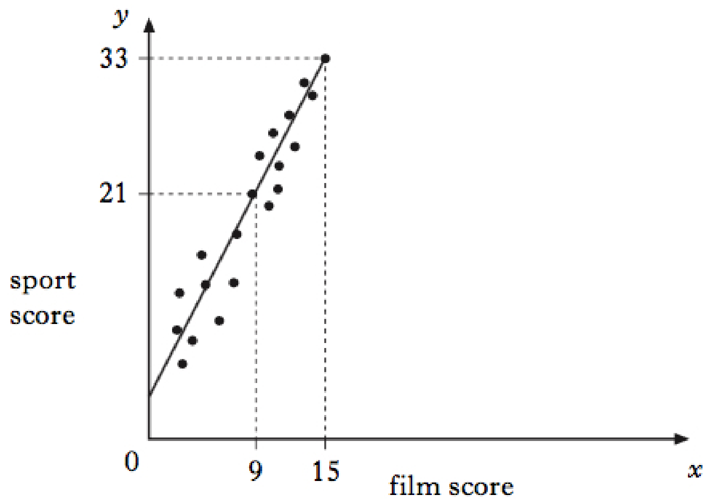
5. Tom looked at the cost of 10 different flights to New York. (2)
He calculated that the mean cost was £360 and the standard deviation was £74.
A tax of £12 is then added to each flight
Write down the new mean and standard deviation.

Solution

Mean: $360 + 12 = \underline{\underline{£372}}$.

Standard deviation: Exactly the same: $\underline{\underline{£74}}$.

6. Teams in a quiz answer questions on film and sport.
This scatter graph shows the scores of some of the teams.



A line of best fit is drawn as shown above.

(a) Find the equation of this straight line. (4)

Solution

Let f and s be the film and sport respectively. Then,

$$\text{gradient} = \frac{33-21}{15-9} = \frac{12}{6} = 2$$

and the equation is

$$s - 33 = 2(f - 15) \Rightarrow s - 33 = 2f - 30 \Rightarrow \underline{\underline{s = 2f + 3}}.$$

(b) Use this equation to estimate the sport score for a team with a film score of 20. (2)

Solution

$$s = 2 \times 20 + 3 = \underline{\underline{43}}.$$

7. The air temperature, t° Celsius, varies inversely as the square of the distance, d metres, from a furnace.

(a) Write down a formula connecting t and d. (2)

Solution

$$t \propto \frac{1}{d^2} \Rightarrow \underline{\underline{t = \frac{k}{d^2}}}$$

for some constant k.

At a distance of 2 metres from the furnace, the air temperature is 50°C.

(b) Calculate the air temperature at a distance of 5 metres from the furnace. (3)

Solution

$$50 = \frac{k}{2^2} \Rightarrow k = 200$$

and so

$$t = \frac{200}{d^2}.$$

Finally,

$$\begin{aligned} t &= \frac{200}{5^2} \\ &= \underline{\underline{8^\circ\text{C}}}. \end{aligned}$$

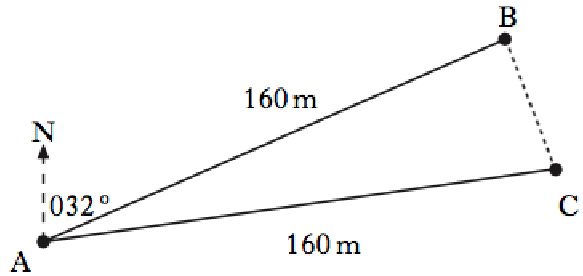
8. A company makes large bags of crisps which contain 90 grams of fat. (4)
The company aims to reduce the fat content of the crisps by 50%.
They decide to reduce the fat content by 20% each year.
Will they have achieved their aim by the end of the 3rd year?
Justify your answer.

Solution

$$\begin{aligned} \text{Fat} &= 90 \times (1.0.2)^3 \\ &= 46.08; \end{aligned}$$

since one-half of 90 is 45, <u>no</u>, they will not achieve their aim.

9. Jane is taking part in an orienteering competition.



She should have run 160 metres from A to B on a bearing of 032°.
However, she actually ran 160 metres from A to C on a bearing of 052°.

(a) Write down the size of angle BAC. (1)

Solution
$\angle BAC = 52 - 32 = \underline{\underline{20^\circ}}$.

(b) Calculate the length of BC. (3)

Solution

$$\begin{aligned} BC &= \sqrt{AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos BAC} \\ &= \sqrt{160^2 + 160^2 - 2 \times 160 \times 160 \times \cos 20^\circ} \\ &= 55.567\,416\,85 \text{ (FCD)} \\ &= \underline{\underline{55.6 \text{ m (3 sf)}}}. \end{aligned}$$

(c) What is the bearing from C to B? (2)

Solution

$$\angle ACB = \angle ABC = \tfrac{1}{2}(180 - 20) = 80^\circ$$

and the

$$\text{bearing} = 180 + 80 + 52 = \underline{\underline{312^\circ}}.$$

10. The weight, W kilograms, of a giraffe is related to its age, M months, by the formula (4)

$$W = \tfrac{1}{4}(M^2 - 4M + 272).$$

At what age will a giraffe weigh 83 kilograms?

Solution

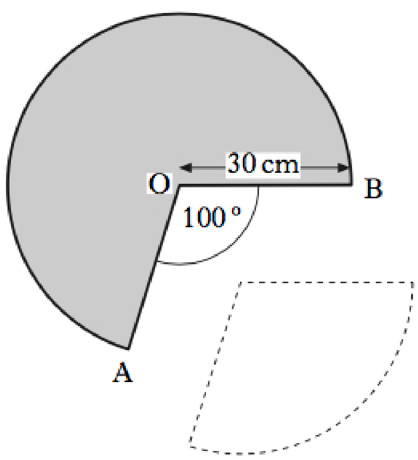
$$\frac{1}{4}(M^2 - 4M + 272) = 83 \Rightarrow M^2 - 4M + 272 = 332$$
$$\Rightarrow M^2 - 4M - 60 = 0$$

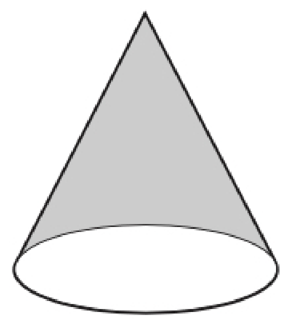
$$\left.\begin{array}{ll} \text{add to:} & -4 \\ \text{multiply to:} & -60 \end{array}\right\} -10,\ +6$$

$$\Rightarrow (M + 6)(M - 10) = 0$$
$$\Rightarrow M + 6 = 0 \text{ or } M - 10 = 0$$
$$\Rightarrow M = -6 \text{ or } M = 10;$$

hence, $M \neq -6$, $\underline{\underline{M = 10 \text{ months}}}$.

11. A cone is formed from a paper circle with a sector removed as shown.





The radius of the paper circle is 30 cm.
Angle $AOBis100^\circ$.

(a) Calculate the area of paper used to make the cone. (3)

Solution

$$\begin{aligned}\text{Area} &= \frac{(360-100)}{360} \times \pi \times 30^2 \\ &= 2\,042.035\,225 \text{ (FCD)} \\ &= \underline{\underline{2\,040 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

(b) Calculate the circumference of the base of the cone. (3)

Solution

$$\begin{aligned}\text{Circumference} &= \frac{(360-100)}{360} \times 2 \times \pi \times 30 \\ &= 136.135\,681\,7 \text{ (FCD)} \\ &= \underline{\underline{136 \text{ cm (3 sf)}}}.\end{aligned}$$

12. The nth term, T_n of the sequence 1, 3, 6, 10, ... is given by the formula

$$T_n = \tfrac{1}{2}n(n+1), n \geqslant 1.$$

For example,

$$\begin{aligned}\text{1st term}: &\quad T_1 = \tfrac{1}{2} \times 1 \times (1+1) = 1, \\ \text{2nd term}: &\quad T_2 = \tfrac{1}{2} \times 2 \times (2+1) = 3, \\ \text{3rd term}: &\quad T_3 = \tfrac{1}{2} \times 3 \times (3+1) = 6.\end{aligned}$$

(a) Calculate the 20th term, T_{20}. (1)

Solution

$$\begin{aligned}T_{20} &= \tfrac{1}{2} \times 20 \times (20+1) \\ &= \underline{\underline{210}}.\end{aligned}$$

(b) Show that (2)

$$T_{n+1} = \tfrac{1}{2}(n^2 + 3n + 2).$$

Solution

$$T_{n+1} = \tfrac{1}{2}(n+1)(n+2)$$

$\times$	n	$+1$
n	n^2	$+n$
$+2$	$+2n$	$+2$

$$= \underline{\underline{\tfrac{1}{2}(n^2+3n+2)}},$$

as required.

(c) Show that $T_n + T_{n+1}$ is a square number. (2)

Solution

$$\begin{aligned} T_n + T_{n+1} &= \tfrac{1}{2}n(n+1) + \tfrac{1}{2}(n+1)(n+2) \\ &= \tfrac{1}{2}(n+1)[n+(n+2)] \\ &= \tfrac{1}{2}(n+1)[2n+2] \\ &= \tfrac{1}{2}(n+1) \times 2(n+1) \\ &= \underline{\underline{(n+1)^2}} \end{aligned}$$

and it is a square number