Dr Oliver Mathematics
GCSE Mathematics
2005 June Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Expand and simplify (2)

$$(x+7)(x-4).$$

Solution

$\times$	x	$+7$
x	x^2	$+7x$
-4	$-4x$	-28

$$(x+7)(x-4) = \underline{\underline{x^2+3x-28}}.$$

(b) Expand (2)

$$y(y^3+2y).$$

Solution

$$y(y^3+2y) = \underline{\underline{y^4+2y^2}}.$$

(c) Factorise (2)

$$p^2+6p.$$

Solution

$$p^2+6p = \underline{\underline{p(p+6)}}.$$

(d) Factorise completely (2)

$$6x^2 - 9xy.$$

Solution

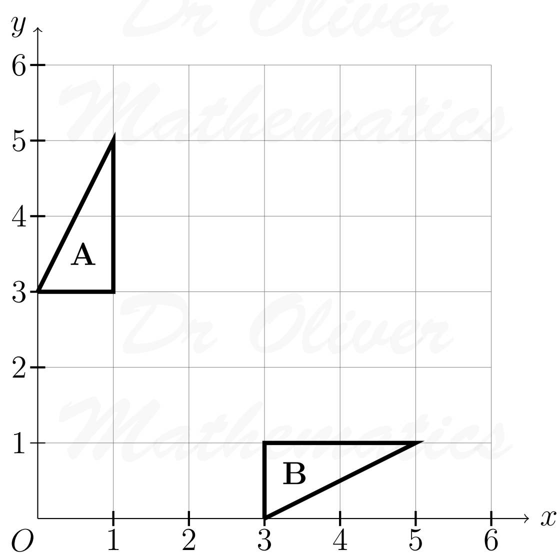
$$6x^2 - 9xy = \underline{\underline{3x(2x - 3y)}}.$$

2. Janie wants to collect information about the amount of sleep the students in her class get. (2)
Design a suitable question she could use.

Solution

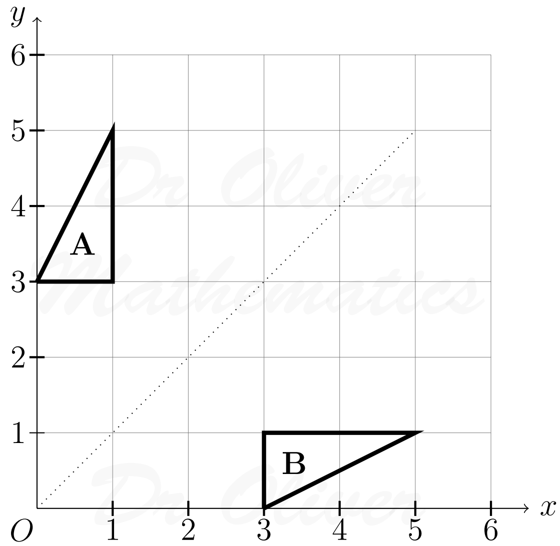
A suitable question with a time frame, e.g., "How long did you sleep last night/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, $0 \leqslant t$ hours < 5, $5 \leqslant t$ hours < 7, $7 \leqslant t$ hours < 8, t hours $\geqslant 8$.

3. Triangle **A** and triangle **B** have been drawn on the grid. (2)



Describe fully the single transformation which will map triangle **A** onto triangle **B**.

Solution



Reflection in the line $y = x$.

4. (a) Solve (3)

$$5 - 3x = 2(x + 1).$$

Solution

$$\begin{aligned} 5 - 3x = 2(x+1) &\Rightarrow 5 - 3x = 2x + 2 \\ &\Rightarrow 3 = 5x \\ &\Rightarrow x = \tfrac{3}{5}. \end{aligned}$$

(b) $-3 \leqslant y < 3$. (2)
y is an integer.
Write down all the possible values of y.

Solution

$-3, -2, -1, 0, 1, 2$.

5. (a) Work out the value of (2)

$$\frac{2}{3} \times \frac{3}{4}.$$

Give your answer as a fraction in its simplest form.

Solution

$$\frac{2}{3} \times \frac{3}{4} = \frac{1}{1} \times \frac{1}{2}$$
$$= \underline{\underline{\frac{1}{2}}}.$$

(b) Work out the value of (3)

$$1\tfrac{2}{3} + 2\tfrac{3}{4}.$$

Give your answer as a fraction in its simplest form.

Solution

$$1\tfrac{2}{3} + 2\tfrac{3}{4} = 3 + \frac{8}{12} + \frac{9}{12}$$
$$= 3 + \frac{17}{12}$$
$$= \underline{\underline{4\tfrac{5}{12}}}.$$

6. (a) Write as a power of 5 (2)

(i) $5^4 \times 5^2$,

Solution

$$5^4 \times 5^2 = \underline{\underline{5^6}}.$$

(ii) $5^9 \div 5^6$.

Solution

$$5^9 \div 5^6 = \underline{\underline{5^3}}.$$

(b) (3)

$$2^x \times 2^y = 2^{10}$$

and

$$2^x \div 2^y = 2^4.$$

Work out the value of x and the value of y.

Solution

The first line is

$$x + y = 10$$

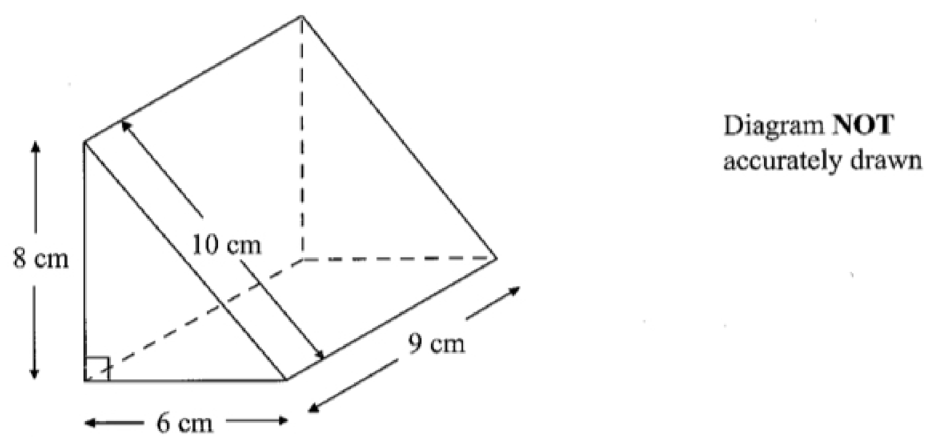
and the second line is

$$x - y = 4.$$

Add:

$$2x = 14 \Rightarrow \underline{\underline{x = 7}}$$
$$\Rightarrow \underline{\underline{y = 3}}.$$

7. Work out the surface area of the triangular prism. (4)



State the units with your answer.

Solution

$$\begin{aligned}\text{Area} &= (2 \times \tfrac{1}{2} \times 6 \times 8) + (9 \times 10) + (9 \times 8) + (9 \times 6) \\ &= 48 + 90 + 72 + 54 \\ &= \underline{\underline{264 \text{ cm}^2}}.\end{aligned}$$

8. The table shows some expressions. a, b, c, and d represent lengths. π and 3 are numbers which have no dimensions. (3)

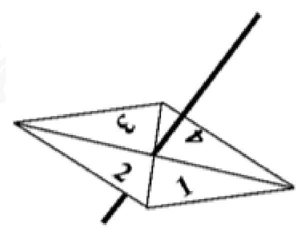
$3a^2$	$\dfrac{\pi ab^3}{3d}$	πbc	$ac + bd$	$\pi(a + b)$	$3(c + d)^3$	$3\pi bc^2$

Tick (✓) the boxes underneath the three expressions which could represent volumes.

Solution

$3a^2$	$\dfrac{\pi ab^3}{3d}$	πbc	$ac + bd$	$\pi(a + b)$	$3(c + d)^3$	$3\pi bc^2$
	✓				✓	✓

9. Here is a 4-sided spinner.



The sides of the spinner are labelled 1, 2, 3, and 4.
The spinner is biased.
The probability that the spinner will land on each of the numbers 2 and 3 is given in the table. The probability that the spinner will land on 1 is equal to the probability that it will land on 4.

Number	1	2	3	4
Probability	x	0.3	0.2	x

(a) Work out the value of x. (2)

Solution

$$x + 0.3 + 0.2 + x = 1 \Rightarrow 2x = 0.5$$
$$\Rightarrow \underline{\underline{x = 0.25}}.$$

Sarah is going to spin the spinner 200 times.

(b) Work out an estimate for the number of times it will land on 2. (2)

Solution

$$200 \times 0.3 = \underline{\underline{60 \text{ times}}}.$$

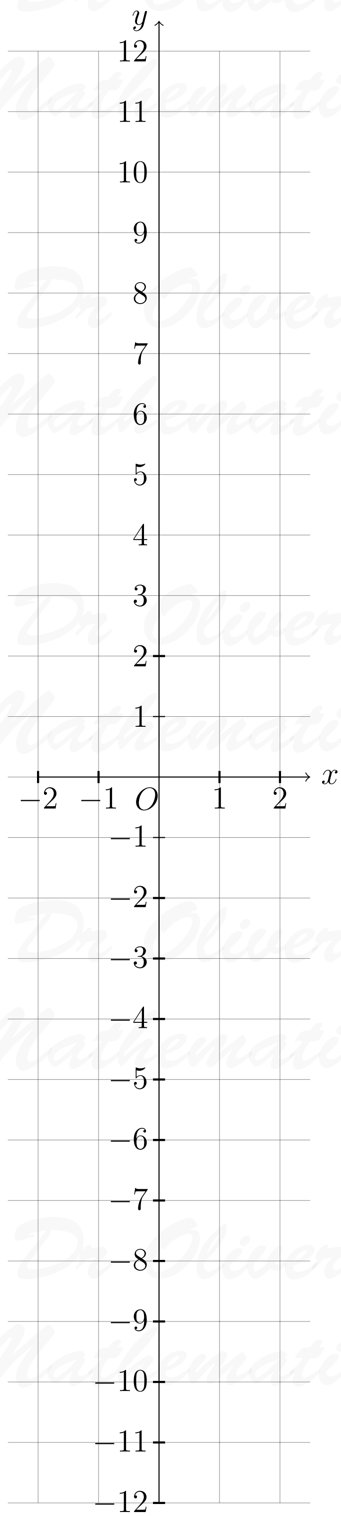
10. (a) Complete this table of values for $y = x^3 + x - 2$. (3)

x	-2	-1	0	1	2
y	-12			0	

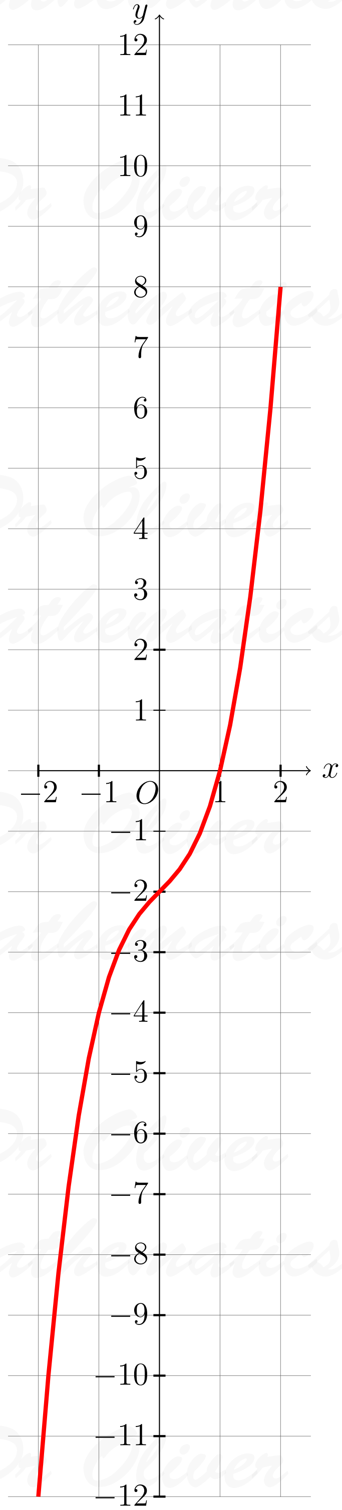
Solution

x	-2	-1	0	1	2
y	-12	$\underline{\underline{-4}}$	$\underline{\underline{-2}}$	0	$\underline{\underline{8}}$

(b) Draw the graph of $y = x^3 + x - 2$. (2)


y
x
O
12
11
10
9
8
7
6
5
4
3
2
1
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
−12
−2
−1
1
2

Solution



11. The number 40 can be written as $2^m \times n$, where m and n are prime numbers. (2)
Find the value of m and the value of n.

Solution

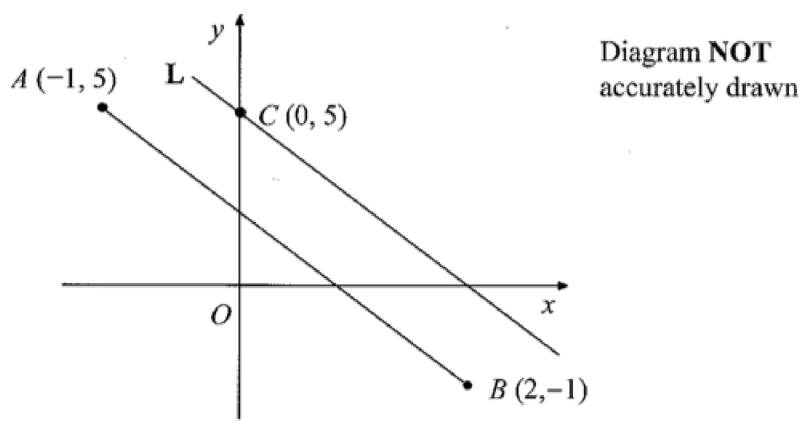
	40
2	20
2	10
2	5
5	1

So

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$$

which means $\underline{\underline{m = 3}}$ and $\underline{\underline{n = 5}}$.

12. The diagram shows three points $A(-1, 5)$, $B(2, -1)$, and $C(0, 5)$. (4)



The line **L** is parallel to AB and passes through C.
Find the equation of the line **L**.

Solution

The gradient of AB is

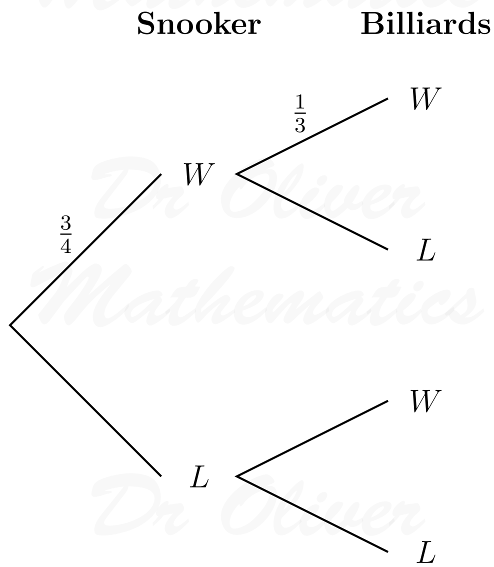
$$\frac{5-(-1)}{-1-2} = \frac{6}{-3} = -2$$

and the equation of **L** is

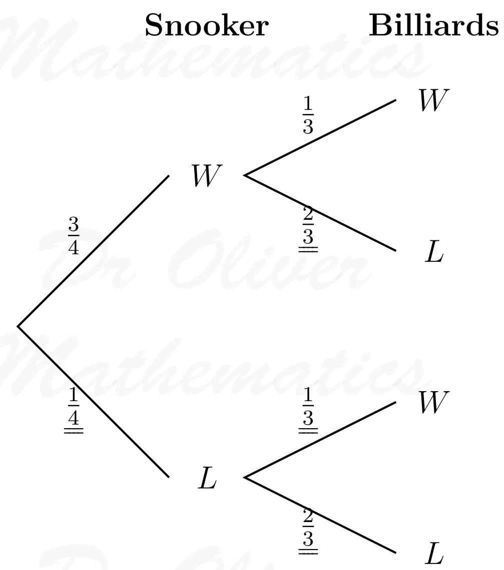
$$\underline{\underline{y = -2x + 5}}.$$

13. Amy is going to play one game of snooker and one game of billiards.
The probability that she will win the game of snooker is $\frac{3}{4}$.
The probability that she will win the game of billiards is $\frac{1}{3}$.

(a) Complete the probability tree diagram. (2)



Solution



(b) Work out the probability that Amy will win **exactly** one game. (3)

Solution

$$\begin{aligned}\mathrm{P}(WL) + \mathrm{P}(LW) &= (\tfrac{3}{4} \times \tfrac{2}{3}) + (\tfrac{1}{4} \times \tfrac{1}{3}) \\ &= \tfrac{6}{12} + \tfrac{1}{12} \\ &= \underline{\underline{\tfrac{7}{12}}}.\end{aligned}$$

Amy played one game of snooker and one game of billiards on a number of Fridays.
She won at **both** snooker and billiards on 21 Fridays.

(c) Work out an estimate for the number of Fridays on which Amy did not win either game. (3)

Solution

$$\mathrm{P}(WW) = \tfrac{3}{4} \times \tfrac{1}{3} = \tfrac{1}{4}$$

and

$$\mathrm{P}(LL) = \tfrac{1}{4} \times \tfrac{2}{3} = \tfrac{1}{6}.$$

Now, the number of Fridays on which Amy did not win either game is

$$21 \times \frac{\frac{1}{6}}{\frac{1}{4}} = 21 \times \frac{4}{6}$$
$$= \frac{84}{6}$$
$$= \underline{\underline{14}}$$

14. In the diagram, A, B, and C are points on the circumference of a circle, centre O.

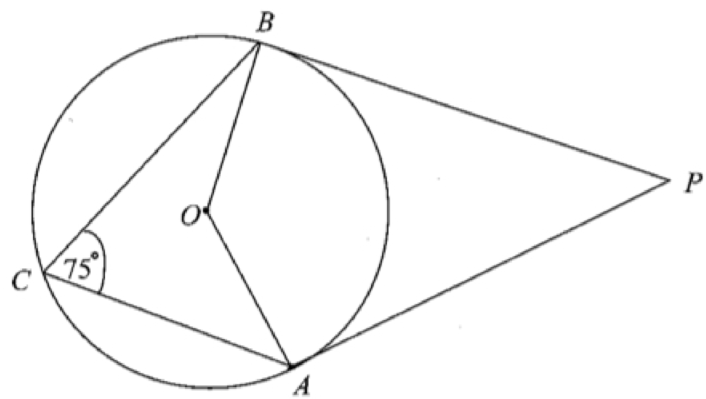


Diagram **NOT** accurately drawn

PA and PB are tangents to the circle.
Angle $ACB = 75°$.

(a) (i) Work out the size of angle AOB. (2)

Solution
Now, which is it: the obtuse angle $\underline{\underline{150°}}$ or the reflex angle $\underline{\underline{210°}}$? Either is possible...

(ii) Give a reason for your answer.

Solution
The angle at the centre is twice the angle at the circumference.

(b) Work out the size of angle APB. (3)

Solution

$$360 - 150 - 90 - 90 = 360 - 330 = \underline{\underline{30^\circ}}.$$

15. (a) Change $\frac{3}{11}$ to a decimal. (1)

Solution

$$\begin{aligned}\tfrac{3}{11} &= \tfrac{3}{11} \times \tfrac{9}{9} \\ &= \tfrac{27}{99} \\ &= \underline{\underline{0.\dot{2}\dot{7}}}.\end{aligned}$$

(b) Prove that the recurring decimal (3)

$$0.\dot{3}\dot{9} = \tfrac{13}{33}.$$

Solution

Let

$$x = 0.\dot{3}\dot{9} \quad (1)$$

and

$$100x = 39.\dot{3}\dot{9} \quad (2).$$

Now, (2) − (1):

$$99x = 39 \Rightarrow x = \tfrac{39}{99} = \underline{\underline{\tfrac{13}{33}}}.$$

16. d is directly proportional to the square of t.
$d = 80$ when $t = 4$.

(a) Express d in terms of t. (2)

Solution

$$d \propto t^2 \Rightarrow d = kt^2$$

for some k. Now,

$$80 = k \times 4^2 \Rightarrow k = 5$$

and so

$$\underline{\underline{d = 5t^2}}.$$

(b) Work out the value of d when $t = 7$. (2)

Solution

$$d = 5 \times 7^2 = 5 \times 49 = \underline{\underline{245}}.$$

(c) Work out the positive value of t when $d = 45$. (2)

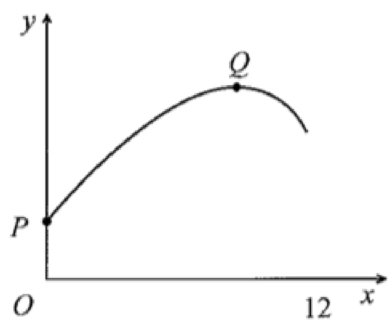
Solution

$$\begin{aligned} 45 = 5t^2 &\Rightarrow t^2 = 9 \\ &\Rightarrow \underline{\underline{t = 3}}, \end{aligned}$$

as we want $t > 0$.

17. Here is a sketch of the graph of

$$y = 25 - \frac{(x-8)^2}{4} \text{ for } 0 \leqslant x \leqslant 12.$$



P and Q are points on the graph.
P is the point at which the graph meets the y-axis.
Q is the point at which y has its maximum value.

(a) Find the coordinates of (3)

(i) P,

Solution

$$\begin{aligned} 25 - \frac{(0-8)^2}{4} &= 25 - \frac{64}{4} \\ &= 25 - 16 \\ &= 9 \end{aligned}$$

so $\underline{\underline{P(0,9)}}$.

(ii) Q.

Solution
$\underline{\underline{Q(8,25)}}$.

(b) Show that (3)

$$25 - \frac{(x-8)^2}{4} = \frac{(2+x)(18-x)}{4}.$$

Solution

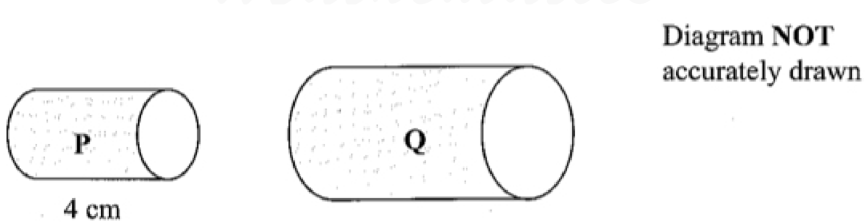
$$\begin{aligned} 25 - \frac{(x-8)^2}{4} &= \frac{100 - (x^2 - 16x + 64)}{4} \\ &= \frac{36 + 16x - x^2}{4} \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & +16 \\ \text{multiply to:} & (+36) \times (-1) = -36 \end{array}\right\} +18,\ -2$$

$$\begin{aligned} &= \frac{36 + 18x - 2x - x^2}{4} \\ &= \frac{18(2+x) - 2(2+x)}{4} \\ &= \underline{\underline{\frac{(2+x)(18-x)}{4}}}, \end{aligned}$$

as required.

18. Two cylinders, **P** and **Q**, are mathematically similar.



The total surface area of cylinder **P** is 90π cm^2.
The total surface area of cylinder **Q** is 810π cm^2.
The length of cylinder **P** is 4 cm.

(a) Work out the length of cylinder **Q**. (3)

Solution
The area scale ratio (ASR) is

$$\frac{810\pi}{90\pi} = 9 = 3^2$$

and the length scale ratio (LSR) is 3; hence,

$$4 \times 3 = \underline{\underline{12 \text{ cm}}}.$$

The volume of cylinder **P** is 100π cm^3.

(b) Work out the volume of cylinder **Q**.
Give your answer as a multiple of π . (2)

Solution
The volume scale ratio (VSR) is

$$3^3 = 27$$

and the volume of cylinder **Q** is

$$100\pi \times 27 = \underline{\underline{2\,700\pi \text{ cm}^3}}.$$

19. (a) Find the value of (4)

(i) 64^0,

Solution

$$64^0 = \underline{\underline{1}}.$$

(ii) $64^{\frac{1}{2}}$,

Solution

$$64^{\frac{1}{2}} = \sqrt{64} = \underline{\underline{8}}.$$

(iii) $64^{-\frac{2}{3}}$.

Solution

$$\begin{aligned} 64^{-\frac{2}{3}} &= \frac{1}{64^{\frac{2}{3}}} \\ &= \frac{1}{(64^{\frac{1}{3}})^2} \\ &= \frac{1}{4^2} \\ &= \underline{\underline{\tfrac{1}{16}}}. \end{aligned}$$

(b) $3 \times \sqrt{27} = 3^n$. (2)

Find the value of n.

Solution

$$\begin{aligned} 3 \times \sqrt{27} &= 3 \times 3\sqrt{3} \\ &= 3 \times 3^{\frac{3}{2}} \\ &= 3^{\frac{5}{2}}; \end{aligned}$$

hence, $\underline{\underline{n = \tfrac{5}{2}}}$.

20. Diagram 1 is a sketch of part of the graph of $y = \sin x°$.

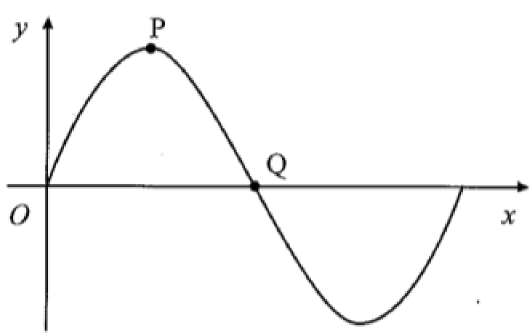


Diagram 1

(a) Write down the coordinates of (2)

(i) P,

Solution
$P(90, 1)$.

(ii) Q.

Solution
$Q(180, 0)$.

Diagram 2 is a sketch of part of the graph of $y = 3\cos 2x°$.

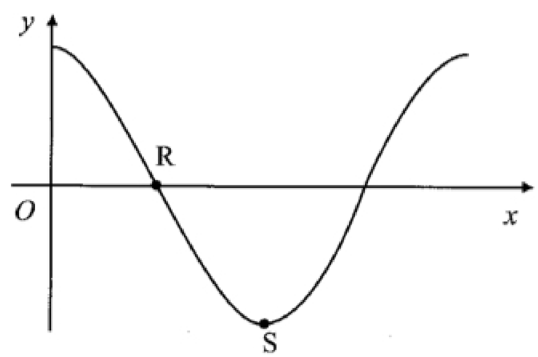


Diagram 2

(b) Write down the coordinates of (2)

(i) R,

Solution
$R(45, 0)$.

(ii) S.

Solution
$S(90, -3)$.

21. The radius of the base of a cone is x cm and its height is h cm. (3)

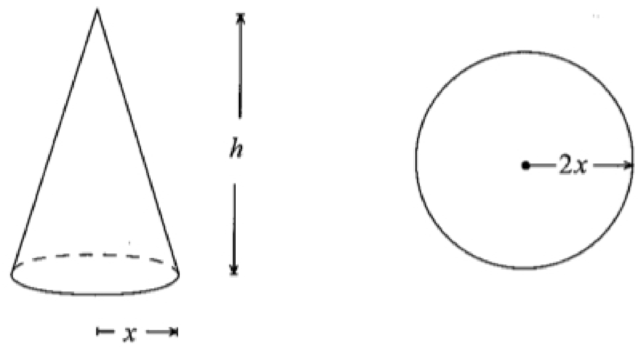


Diagram **NOT** accurately drawn

The radius of a sphere is $2x$ cm.
The volume of the cone and the volume of the sphere are equal.
Express h in terms of x.
Give your answer in its simplest form.

Solution

$$\begin{aligned}\text{Volume of the cone} = \text{Volume of the sphere} &\Rightarrow \tfrac{1}{3}\pi x^2 h = \tfrac{4}{3}\pi (2x)^3 \\ &\Rightarrow \tfrac{1}{3}\pi x^2 h = \tfrac{32}{3}\pi x^3 \\ &\Rightarrow \underline{\underline{h = 32x}}.\end{aligned}$$

22. $OPQR$ is a trapezium with PQ parallel to OR.

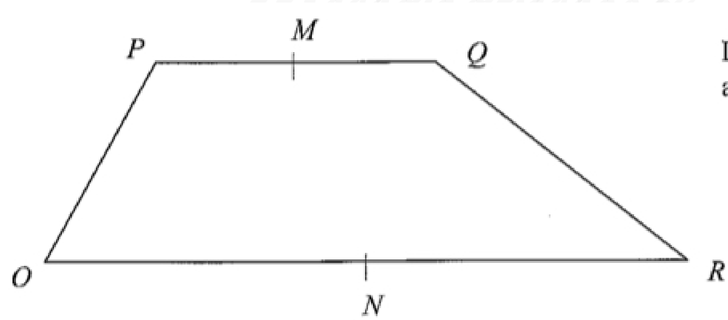


Diagram **NOT** accurately drawn

$\overrightarrow{OP} = 2\mathbf{b}$.
$\overrightarrow{PQ} = 2\mathbf{a}$.
$\overrightarrow{OR} = 6\mathbf{a}$.
M is the midpoint of PQ and N is the midpoint of OR.

(a) Find the vector $\overrightarrow{MN}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (2)

Solution

$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MP} + \overrightarrow{PO} + \overrightarrow{ON} \\ &= \tfrac{1}{2}\overrightarrow{QP} + \overrightarrow{PO} + \tfrac{1}{2}\overrightarrow{OR} \\ &= -\mathbf{a} - 2\mathbf{b} + 3\mathbf{b} \\ &= \underline{\underline{2\mathbf{a} - 2\mathbf{b}}}.\end{aligned}$$

X is the midpoint of MN and Y is the midpoint of QR.

(b) Prove that XY is parallel to OR. (2)

Solution

$$\begin{aligned}\overrightarrow{XY} &= \overrightarrow{XN} + \overrightarrow{NR} + \overrightarrow{RY} \\ &= \tfrac{1}{2}\overrightarrow{MN} + \overrightarrow{NR} + \tfrac{1}{2}\overrightarrow{RQ} \\ &= \tfrac{1}{2}(2\mathbf{a} - 2\mathbf{b}) + 3\mathbf{a} + \tfrac{1}{2}(-6\mathbf{a} + 2\mathbf{b} + 2\mathbf{a}) \\ &= \mathbf{a} - \mathbf{b} + 3\mathbf{a} + \tfrac{1}{2}(-4\mathbf{a} + 2\mathbf{b}) \\ &= 2\mathbf{a} \\ &= \tfrac{1}{3}\overrightarrow{OR};\end{aligned}$$

hence, $\underline{\underline{XY \text{ is parallel to } OR}}$.