

**Dr Oliver Mathematics**  
**GCSE Mathematics**  
**2022 November Paper 3H: Calculator**  
**1 hour 30 minutes**

The total number of marks available is 80.  
You must write down all the stages in your working.

1. Make  $a$  the subject of the formula

$$p = 3a - 9.$$

(2)

**Solution**

$$p = 3a - 9 \Rightarrow p + 9 = 3a$$

$$\Rightarrow a = \underline{\underline{\frac{p+9}{3}}}$$

2. Rob has been asked to divide 120 in the ratio 3 : 5.

(1)

Here is his working.

$$120 \div 3 = 40 \quad 120 \div 5 = 24.$$

Rob's working is not correct.

Describe what Rob has done wrong.

**Solution**

He should have done

$$\left(\frac{3}{3+5}\right) \times 120 : \left(\frac{5}{3+5}\right) \times 120.$$

3. 200 students chose one language to study.

(3)

Each student chose one language from French or Spanish or German.

Of the 200 students,

- 90 are boys and the rest of the students are girls,
- 70 chose Spanish,
- 60 of the 104 students who chose French are boys, and
- 18 girls chose German.

Work out how many boys chose Spanish.

**Solution**

Well,

- “90 are boys” which means 110 are girls.
- “60 of the 104 students who chose French are boys” which means 44 girls also chose French.
- “18 girls chose German.”

And we get the following table:

Subject	Boys	Girls
French	60	44
Spanish		
German		18
Total	90	110

Now,

$$110 - (44 + 18) = 48$$

of the girls chose Spanish and that means

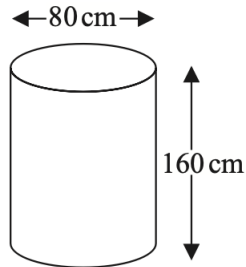
$$70 - 48 = \underline{\underline{22}}$$

boys chose Spanish.

Subject	Boys	Girls
French	60	44
Spanish	<u>22</u>	48
German	8	18
Total	90	110

4. Karina has 4 tanks on her tractor.  
Each tank is a cylinder with diameter 80 cm and height 160 cm.

(4)



The 4 tanks are to be filled completely with a mixture of fertiliser and water.

The fertiliser has to be mixed with water in the ratio 1 : 100 by volume.  
Karina has 32 litres of fertiliser.

1 litre = 1 000 cm<sup>3</sup>.

Has Karina enough fertiliser for the 4 tanks?  
You must show how you get your answer.

**Solution**

$$\begin{aligned}\text{Total fertiliser} &= 4 \times (\pi \times 40^2 \times 160) \\ &= 3\,216\,990.877 \text{ cm}^3 \text{ (FCD)} \\ &= 3\,216.990\,877 \text{ l (FCD)}.\end{aligned}$$

Now,

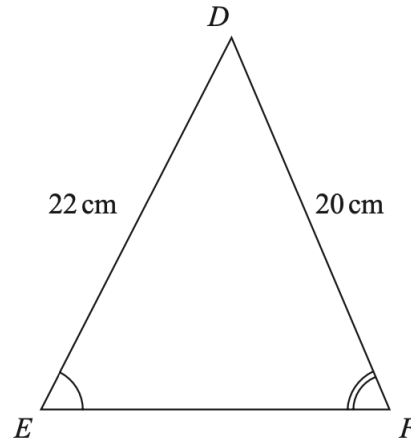
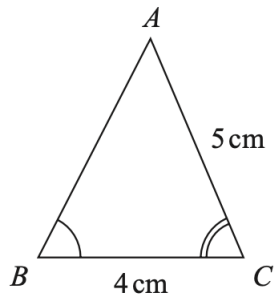
$$1 + 100 = 101$$

and the volume of mixture that 32 litres of fertiliser will make is

$$32 \times 101 = 3\,232 \text{ l.}$$

Is it more or less? It is more! Hence, Karina has got enough fertiliser for the 4 tanks.

5. Triangle  $ABC$  and triangle  $DEF$  are similar.



- (a) Work out the length of  $EF$ . (2)

**Solution**  
Well,

$$\frac{EF}{BC} = \frac{DF}{AC} \Rightarrow \frac{EF}{4} = \frac{20}{5}$$

$$\Rightarrow EF = 4 \times 4$$

$$\Rightarrow \underline{\underline{EF = 16 \text{ cm.}}}$$

- (b) Work out the length of  $AB$ . (2)

**Solution**

$$\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{AB}{22} = \frac{5}{20}$$

$$\Rightarrow AB = 22 \times \frac{1}{4}$$

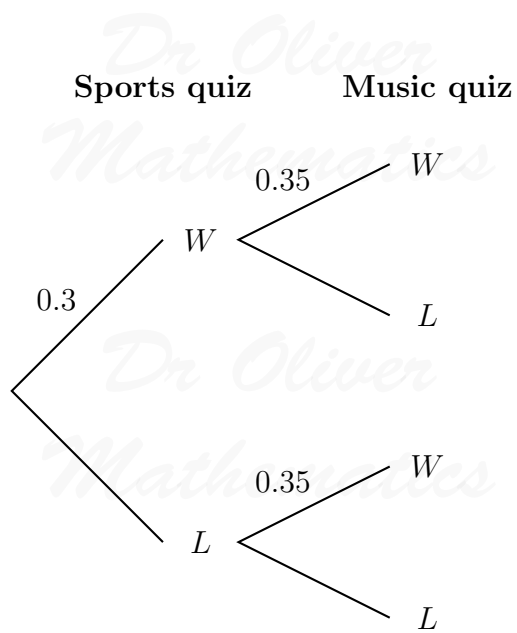
$$\Rightarrow \underline{\underline{AB = 5\frac{1}{2} \text{ cm.}}}$$

6. One weekend the Keddie family is going to do a sports quiz and a music quiz.

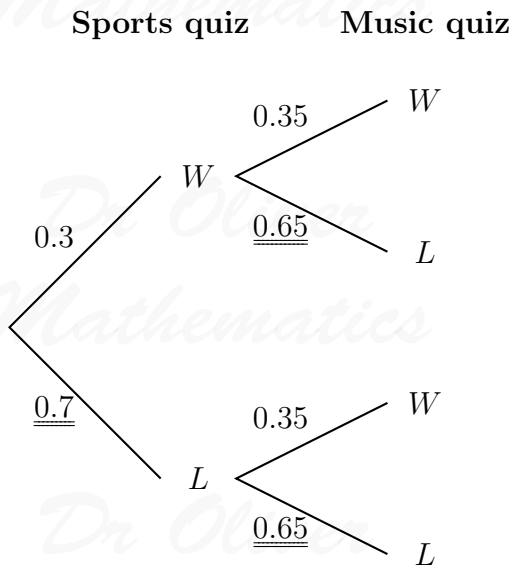
The probability that the family will win the sports quiz is 0.3.

The probability that the family will win the music quiz is 0.35.

- (a) Complete the probability tree diagram. (2)



**Solution**



- (b) Work out the probability that the Keddie family will win both the sports quiz and the music quiz. (2)

**Solution**

$$\begin{aligned}
 P(\text{sports quiz, music quiz}) &= 0.3 \times 0.35 \\
 &= \underline{0.105}.
 \end{aligned}$$

7. (a) Change  $8\,000\text{ cm}^3$  to  $\text{m}^3$ . (1)

**Solution**

$$\begin{aligned}8\,000\text{ cm}^3 &= 8\,000 \times 1\text{ cm}^3 \\ &= 8\,000 \times (1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}) \\ &= 8\,000 \times (0.01\text{ m} \times 0.01\text{ m} \times 0.01\text{ m}) \\ &= 8\,000 \times 0.000\,001\text{ m}^3 \\ &= \underline{\underline{0.008\text{ m}^3}}.\end{aligned}$$

- (b) Change a speed of  $180\text{ km per hour}$  to metres per second. (3)

**Solution**

Well,

$$\begin{aligned}180\text{ km per hour} &= \frac{180\text{ km}}{1\text{ hr}} \\ &= \frac{180\,000\text{ m}}{60\text{ mins}} \\ &= \frac{180\,000\text{ m}}{(60 \times 60)\text{ s}} \\ &= \underline{\underline{50\text{ m/s}}}.\end{aligned}$$

8. There are 30 women and 20 men at a gym. (3)

The mean height of all 50 people is  $167.6\text{ cm}$ .

The mean height of the 20 men is  $182\text{ cm}$ .

Work out the mean height of the 30 women.

**Solution**

Let  $x$  cm be the mean height of the 30 women. Now,

$$\begin{aligned}\frac{(20 \times 182) + (30 \times x)}{50} &= 167.6 \Rightarrow \frac{3640 + 30x}{50} = 167.6 \\ &\Rightarrow 3640 + 30x = 8380 \\ &\Rightarrow 30x = 4740 \\ &\Rightarrow \underline{x = 158}.\end{aligned}$$

9. (a) Write

$$6.75 \times 10^{-4}$$

(1)

as an ordinary number.

**Solution**

$$6.75 \times 10^{-4} = \underline{0.000675}.$$

(b) Work out

$$\frac{(2.56 \times 10^6) \times (4.12 \times 10^3)}{1.6 \times 10^{-2}}.$$

(2)

Give your answer in standard form.

**Solution**

$$\begin{aligned}\frac{(2.56 \times 10^6) \times (4.12 \times 10^3)}{1.6 \times 10^{-2}} &= \frac{1.05472 \times 10^{10}}{1.6 \times 10^{-2}} \\ &= \underline{6.592 \times 10^{11}}.\end{aligned}$$

10. Peter has to subtract  $(x^2 - 2x - 4)$  from  $(x^2 + 3x + 5)$ .

(1)

Here is his working.

$$\begin{aligned}(x^2 + 3x + 5) - (x^2 - 2x - 4) \\ &= x^2 + 3x + 5 - x^2 - 2x - 4 \\ &= x + 1.\end{aligned}$$

Explain what is wrong with Peter's working.

**Solution**

It should be

$$x^2 + 3x + 5 - \underline{\underline{x^2 + 2x + 4}},$$

giving  $5x + 5$  as his answer.

11.  $x$  and  $y$  are integers such that

- $3 < x < 8$ ,
- $4 < y < 10$ , and
- $x + y = 14$ .

Find all the possible values of  $x$ .

**Solution**

Well,

$$x + y = 14 \Rightarrow y = 14 - x$$

and

$$\begin{aligned} 4 < y < 10 &\Rightarrow 4 < 14 - x < 10 \\ &\Rightarrow -4 > x - 14 > -10 \\ &\Rightarrow 10 > x > 4; \end{aligned}$$

but we know that  $3 < x < 8$ !

Hence, all the possible values of  $x$  are

$$\underline{\underline{5, 6, \text{ and } 7.}}$$

12. Martin used his calculator to work out the value of a number  $P$ .  
He wrote down the first two digits of the answer on his calculator.

He wrote down 1.2.

Complete the error interval for  $P$ :

$$\dots \leq P < \dots$$



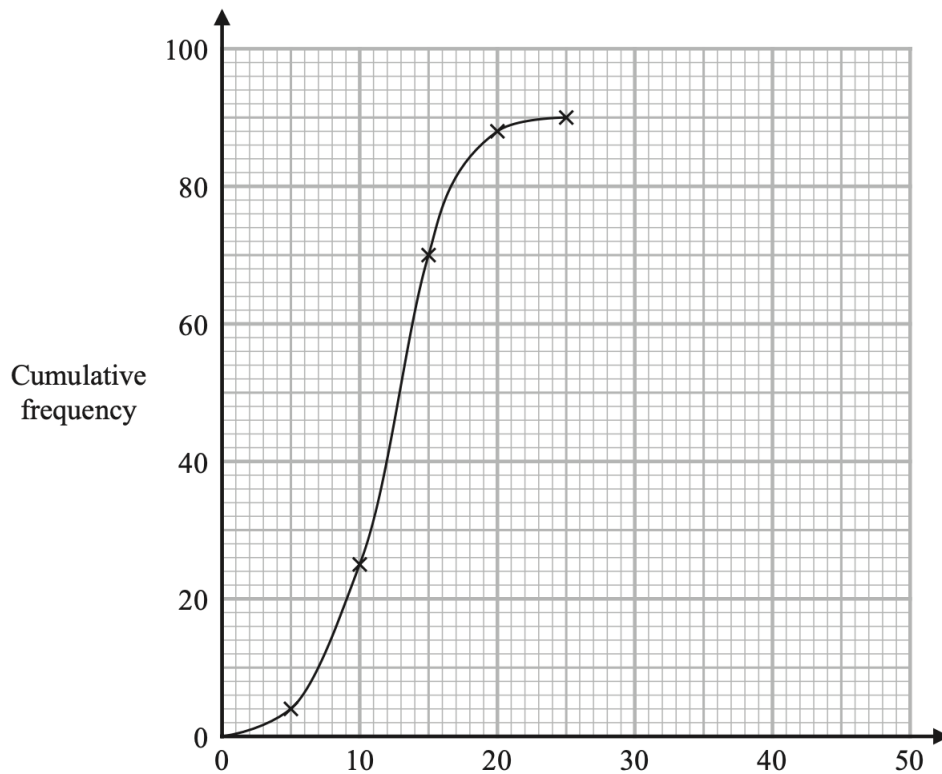
**Solution**

$$\underline{\underline{1.2 \leq P < 1.3}}$$

13. Chen has this information about the time that it took an operator at a call centre to answer each of 90 calls. (2)

Time ( $t$ seconds)	Cumulative frequency
$0 < t \leq 10$	4
$0 < t \leq 20$	25
$0 < t \leq 30$	70
$0 < t \leq 40$	88
$0 < t \leq 50$	90

Chen draws this cumulative frequency graph for the information in the table.



Write down two different things that are wrong with this graph.

**Solution**

E.g., there is no scale on the horizontal axis, Chen has plotted (5, 4), (10, 25), ... instead of (10, 4), (20, 25), ...

14. (a) Simplify fully

$$(3x^5y^6)^4.$$

(2)

**Solution**

$$(3x^5y^6)^4 = \underline{\underline{81x^{20}y^{24}}}.$$

(b) Expand and simplify

$$(x + 2)(x - 3)(x + 4).$$

(3)

**Solution**

Well,

$$\begin{array}{r|rr} \times & x & +2 \\ \hline x & x^2 & +2x \\ -3 & -3x & -6 \\ \hline \end{array}$$

so

$$(x + 2)(x - 3) = x^2 - x - 6.$$

Now,

$$\begin{array}{r|rrr} \times & x^2 & -x & -6 \\ \hline x & x^3 & -x^2 & -6x \\ +4 & +4x^2 & -4x & -24 \\ \hline \end{array}$$

so

$$(x + 2)(x - 3)(x + 4) = \underline{\underline{x^3 + 3x^2 - 10x - 24}}.$$

15. A pet shop has

(2)

- 7 guppy fish,
- 13 tetra fish, and
- 5 angel fish.

David is going to choose one of the following combinations of fish

- a guppy fish and an angel fish,
- **or** a tetra fish and an angel fish,
- **or** a guppy fish, a tetra fish, and an angel fish.

Show that there are 555 different ways for David to choose his fish.

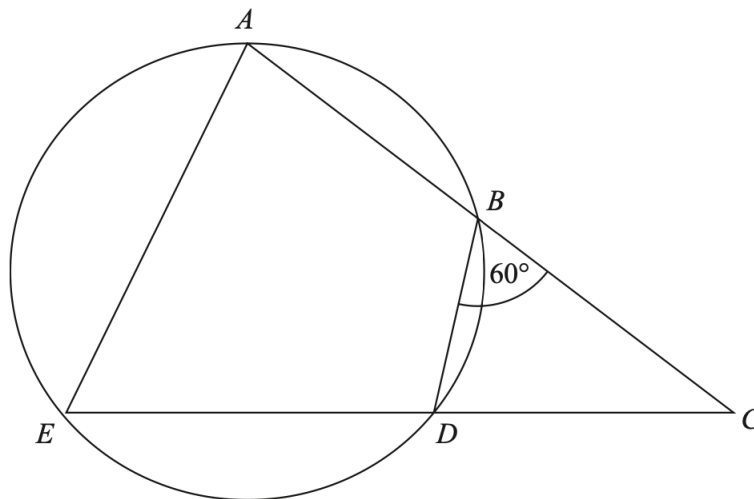
**Solution**

$$\begin{aligned} \text{Total number of ways} &= (7 \times 5) + (13 \times 5) + (7 \times 13 \times 5) \\ &= 35 + 65 + 455 \\ &= \underline{555 \text{ ways}}, \end{aligned}$$

as required.

16.  $ABDE$  is a cyclic quadrilateral.  
 $ABC$  and  $EDC$  are straight lines.  
 Angle  $DBC = 60^\circ$ .

(4)



Given that

$$\text{size of angle } EAB : \text{size of angle } BCD = 2 : 1,$$

work out the size of angle  $BCD$ .  
You must show all your working.

**Solution**

Let  $\angle EAB = 2x$  and  $\angle BCD = x$ .

Well,  $\angle DBA = 180 - 60 = 120^\circ$  (supplementary angles)  
 $\angle AED = 180 - 120 = 60^\circ$  (opposite angles in a cyclic quadrilateral).

Then, because all the angles in  $\triangle ACE$  add up to 180,

$$\begin{aligned}2x + x + 60 &= 180 \Rightarrow 3x = 120 \\ &\Rightarrow x = 40;\end{aligned}$$

hence,  $\angle BCD = 40^\circ$ .

17. There are four boxes on a shelf: **A**, **B**, **C**, and **D**.

(4)

The total weight of **A** and **B** is 3 times the total weight of **C** and **D**.

The weight of **A** is  $\frac{2}{3}$  of the weight of **B**.  
The weight of **C** is 75% of the weight of **D**.

Find the ratio

weight of **A** : weight of **B** : weight of **C** : weight of **D**.

**Solution**

Well,

$$\mathbf{A + B = 3(C + D)}$$

$$\mathbf{A = \frac{2}{3}B}$$

$$\mathbf{C = \frac{3}{4}D.}$$

Now,

$$\begin{aligned}A + B &= 3(C + D) \Rightarrow \frac{2}{3}B + B = 3\left(\frac{3}{4}D + D\right) \\ &\Rightarrow \frac{5}{3}B = 3\left(\frac{7}{4}D\right) \\ &\Rightarrow \frac{5}{3}B = \frac{21}{4}D \\ &\Rightarrow \frac{20}{3}B = 21D \\ &\Rightarrow \frac{20}{63}B = D.\end{aligned}$$

Next,

$$\begin{aligned}C &= \frac{3}{4}D \Rightarrow \frac{4}{3}C = D \\ &\Rightarrow \frac{20}{63}B = \frac{4}{3}C \\ &\Rightarrow \frac{5}{21}B = C.\end{aligned}$$

Finally, we have

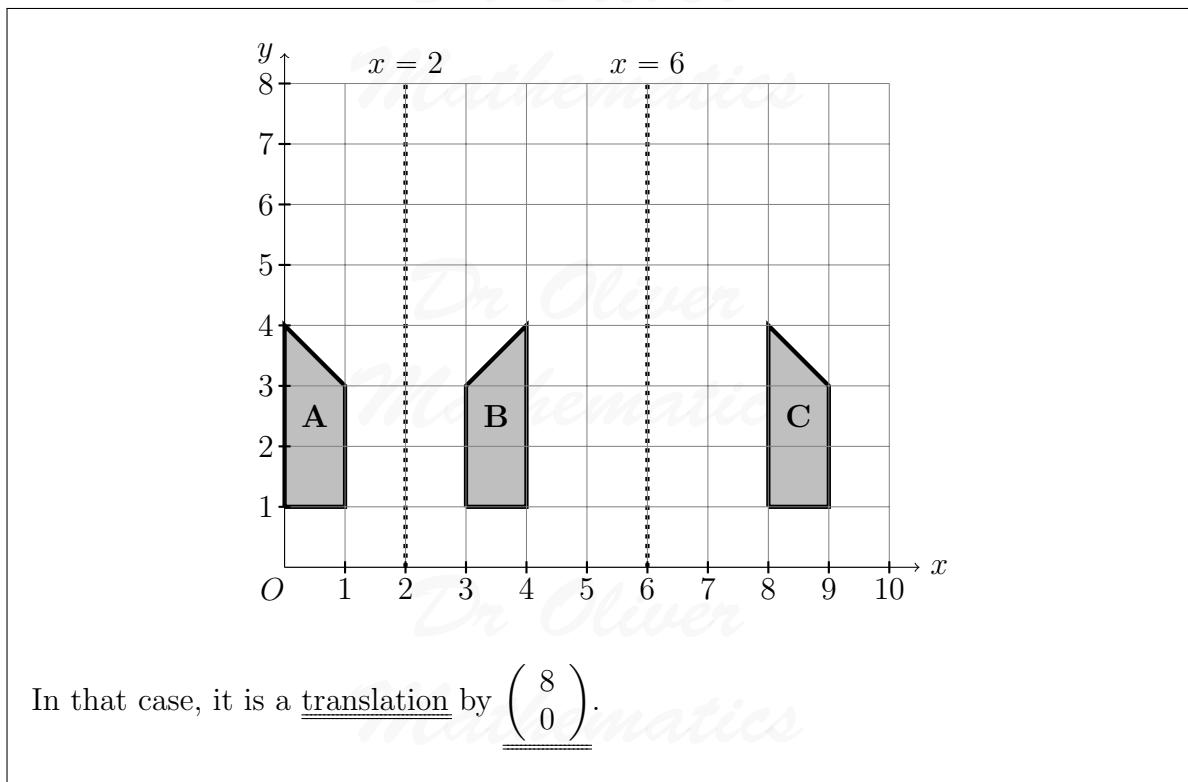
$$\begin{aligned}&\text{weight of A : weight of B : weight of C : weight of D} \\ &= \frac{2}{3}B : B : \frac{5}{21}B : \frac{20}{63}B \\ &= \frac{2}{3} : 1 : \frac{5}{21} : \frac{20}{63} \\ &= \frac{42}{63} : 1 : \frac{15}{63} : \frac{20}{63} \\ &= \underline{\underline{42 : 63 : 15 : 20}}.\end{aligned}$$

18. Shape **A** is reflected in the line with equation  $x = 2$  to give shape **B**.  
Shape **B** is reflected in the line with equation  $x = 6$  to give shape **C**. (2)

Describe fully the single transformation that maps shape **A** onto shape **C**.

**Solution**

Well, we will make up a shape that has a corner cut off of it:



19. There are only blue counters, red counters, and green counters in a box. (3)

The probability that a counter taken at random from the box will be blue is 0.4.  
The ratio of the number of red counters to the number of green counters is 7 : 8.

Sameena takes at random a counter from the box.  
She records its colour and puts the counter back in the box.  
Sameena does this a total of 50 times.

Work out an estimate for the number of times she takes a green counter.

**Solution**

The probability that it is a green counter is

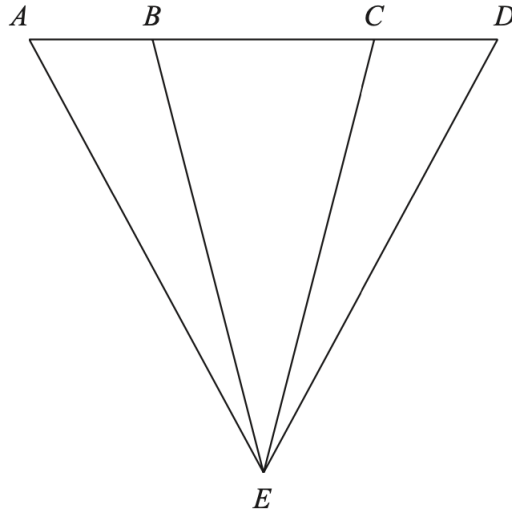
$$\begin{aligned} \left(\frac{8}{7+8}\right) \times (1 - 0.4) &= \frac{8}{15} \times 0.6 \\ &= 0.32 \end{aligned}$$

an an estimate is

$$50 \times 0.32 = \underline{\underline{16}}.$$

20. The diagram shows a triangle  $ADE$ .

(3)



$$AE = DE.$$

$$AB : BC : CD = 1 : 2 : 1.$$

Prove that triangle  $ACE$  is congruent to triangle  $DBE$ .

**Solution**

$$AE = DE \text{ (given)}$$

$$AC = AB + BC = BC + CD = BD.$$

$$\angle CAE = \angle BDE \text{ (base angles).}$$

So, triangles  $\triangle ACE$  and  $\triangle BDE$  are congruent (SAS).

21. The equation of a curve is

(3)

$$y = 4x^2 - 56x.$$

The curve has one turning point.

By completing the square, show that the coordinates of the turning point are  $(7, -196)$ .

You must show all your working.

**Solution**

$$y = 4x^2 - 56x$$

$$= 4[x^2 - 14x]$$

$$= 4[(x^2 - 14x + 49) - 49]$$

$$= 4[(x - 7)^2 - 49]$$

$$= 4(x - 7)^2 - 196.$$

So, the  $x$ -point of the turning point is  $x = 7$  and  $y$ -component is  $y = -196$ .

Hence, the coordinates of the turning point are  $(7, -196)$ .

22.

(3)

$$\frac{2x + 3}{x - 5} + \frac{x - 4}{x + 5} - 3$$

can be written in the form

$$\frac{ax + b}{x^2 - 25},$$

where  $a$  and  $b$  are integers.

Work out the value of  $a$  and the value of  $b$ .

You must show all your working.

**Solution**

Now,

$$\begin{aligned} & \frac{2x + 3}{x - 5} + \frac{x - 4}{x + 5} - 3 \\ = & \frac{(2x + 3)(x + 5)}{(x - 5)(x + 5)} + \frac{(x - 4)(x - 5)}{(x + 5)(x - 5)} - \frac{3(x - 5)(x + 5)}{(x - 5)(x + 5)} \end{aligned}$$

$\times$	$2x$	$+3$
$x$	$2x^2$	$+3x$
$+5$	$+10x$	$+15$



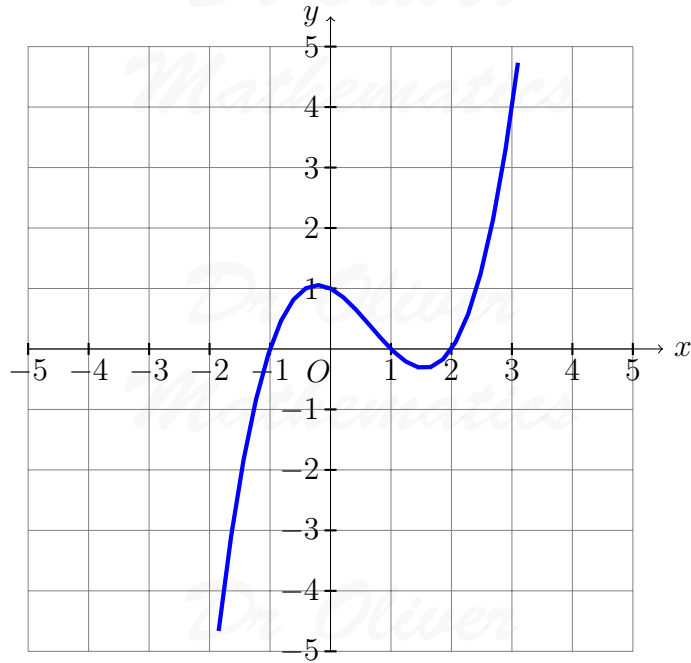
$$\begin{array}{r|rr} \times & x & -4 \\ \hline x & x^2 & -4x \\ -5 & -5x & +20 \\ \hline \end{array}$$

$$\begin{array}{r|rr} \times & x & +5 \\ \hline x & x^2 & +5x \\ -5 & -5x & -25 \\ \hline \end{array}$$

$$\begin{aligned} &= \frac{(2x^2 + 13x + 15)}{x^2 - 25} + \frac{(x^2 - 9x + 20)}{x^2 - 25} - \frac{3(x^2 - 25)}{x^2 - 25} \\ &= \frac{(2x^2 + 13x + 15) + (x^2 - 9x + 20) - (3x^2 - 75)}{x^2 - 25} \\ &= \frac{4x + 110}{x^2 - 25}; \end{aligned}$$

hence,  $a = 4$  and  $b = 110$ .

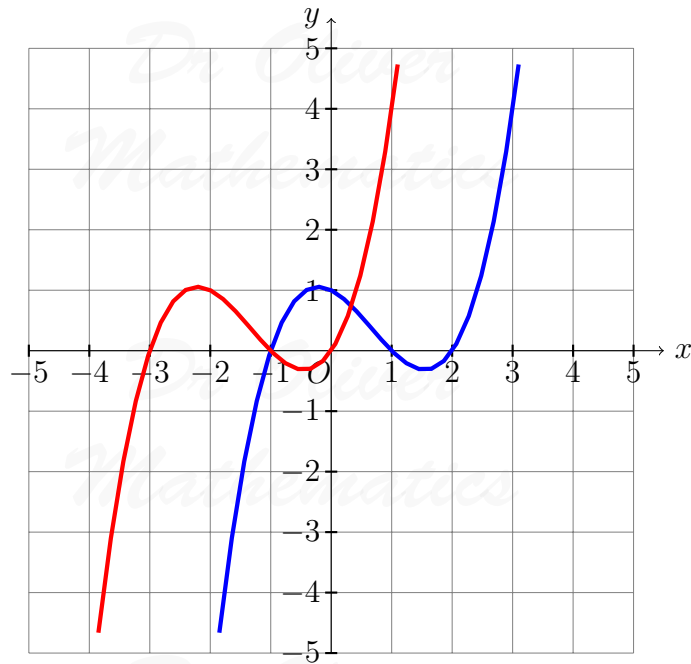
23. The graph of  $y = f(x)$  is shown on the grid below.

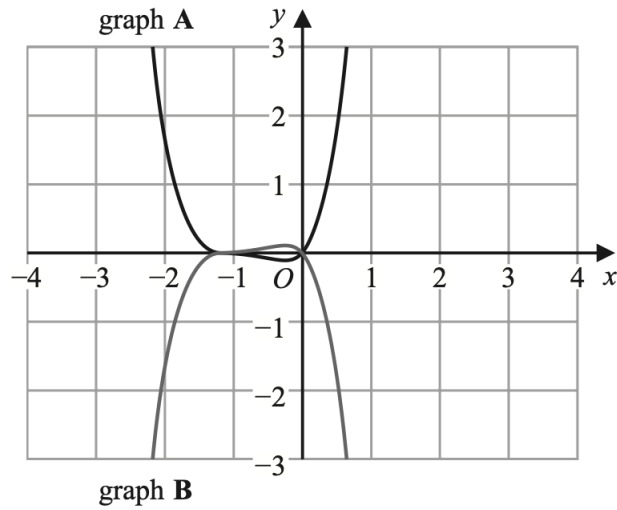


(a) On the grid above, sketch the graph of  $y = f(x + 2)$ .

(1)

**Solution**



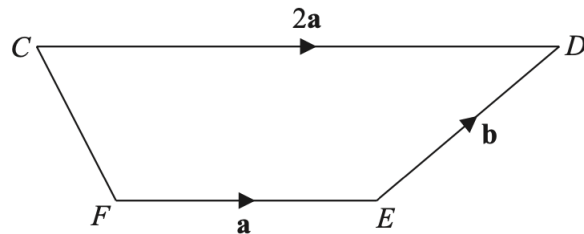


On this grid, graph **A** has been reflected to give graph **B**.  
 The equation of graph **A** is  $y = g(x)$ .

(b) Write down an equation of graph **B**. (1)

**Solution**  
 An equation is  $y = -g(x)$ .

24.  $CDEF$  is a quadrilateral. (4)



- $\overrightarrow{FE} = \mathbf{a}$ .
- $\overrightarrow{ED} = \mathbf{b}$ .
- $\overrightarrow{CD} = 2\mathbf{a}$ .

The point  $P$  is such that  $CEP$  is a straight line and that  $CE = EP$ .

Use a vector method to prove that  $CF$  is parallel to  $DP$ .

**Solution**

Well,

$$\begin{aligned}\overrightarrow{CE} &= \overrightarrow{CD} + \overrightarrow{DE} \\ &= \overrightarrow{CD} - \overrightarrow{ED} \\ &= 2\mathbf{a} - \mathbf{b}\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{CF} &= \overrightarrow{CE} + \overrightarrow{EF} \\ &= 2\overrightarrow{CE} \\ &= 2(2\mathbf{a} - \mathbf{b}) \\ &= 4\mathbf{a} - 2\mathbf{b}.\end{aligned}$$

Next,

$$\begin{aligned}\overrightarrow{CF} &= \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} \\ &= \overrightarrow{CD} - \overrightarrow{ED} - \overrightarrow{FE} \\ &= 2\mathbf{a} - \mathbf{b} - \mathbf{a} \\ &= \mathbf{a} - \mathbf{b}.\end{aligned}$$

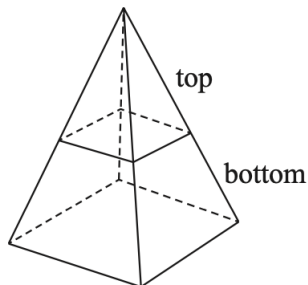
Finally,

$$\begin{aligned}\overrightarrow{DP} &= \overrightarrow{DC} + \overrightarrow{CP} \\ &= -\overrightarrow{CD} + \overrightarrow{CF} \\ &= -2\mathbf{a} + (4\mathbf{a} - 2\mathbf{b}) \\ &= 2\mathbf{a} - 2\mathbf{b} \\ &= 2(\mathbf{a} - \mathbf{b}) \\ &= 2\overrightarrow{CF};\end{aligned}$$

hence,  $CF$  is parallel to  $DP$ .

25. The pyramid  $\mathbf{P}$  is formed from two parts made of different materials.

(5)



- The top part of **P** has a mass of 92.8 g and is made from material with a density of  $2.9 \text{ g/cm}^3$ .
- The bottom part of **P** has a mass of 972.8 g.
- The average density of **P** is  $4.7 \text{ g/cm}^3$ .

Calculate the volume of the top part of **P** as a percentage of the total volume of **P**.  
Give your answer correct to 1 decimal place.  
You must show all your working.

### Solution

Well,

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and

$$\begin{aligned} \text{density}_{\text{top}} &= \frac{\text{mass}_{\text{top}}}{\text{volume}_{\text{top}}} \Rightarrow \text{volume}_{\text{top}} = \frac{\text{mass}_{\text{top}}}{\text{density}_{\text{top}}} \\ &\Rightarrow \text{volume}_{\text{top}} = \frac{92.8}{2.9} \\ &\Rightarrow \text{volume}_{\text{top}} = 32 \text{ cm}^3. \end{aligned}$$

Now,

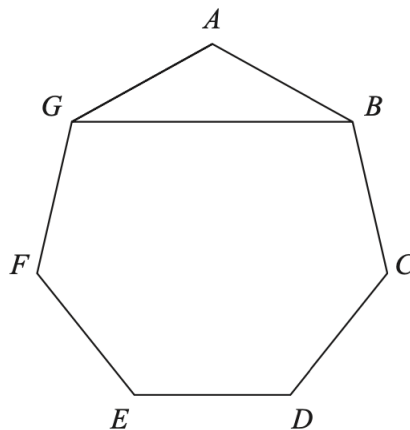
$$\begin{aligned} \frac{\text{mass}_{\text{top}} + \text{mass}_{\text{bottom}}}{\text{volume}_{\text{top}} + \text{volume}_{\text{bottom}}} &= \text{average density} \\ \Rightarrow \frac{92.8 + 972.8}{32 + \text{volume}_{\text{bottom}}} &= 4.7 \\ \Rightarrow \frac{1065.6}{32 + \text{volume}_{\text{bottom}}} &= 4.7 \\ \Rightarrow \frac{1065.6}{4.7} &= 32 + \text{volume}_{\text{bottom}} \\ \Rightarrow 226\frac{34}{47} &= 32 + \text{volume}_{\text{bottom}} \\ \Rightarrow \text{volume}_{\text{bottom}} &= 194\frac{34}{47} \text{ cm}^3. \end{aligned}$$

Finally,

$$\begin{aligned}\text{percentage} &= \left( \frac{32}{32 + 194\frac{34}{47}} \right) \times 100\% \\ &= 14.\dot{1}1\dot{4} \text{ (exact!)} \\ &= \underline{\underline{14.1\%}} \text{ (1 dp).}\end{aligned}$$

26.  $ABCDEFGG$  is a regular heptagon.

(5)



The area of triangle  $ABG$  is  $30 \text{ cm}^2$ .

Calculate the length of  $GB$ .

Give your answer correct to 3 significant figures.

You must show all your working.

### Solution

Each interior angle is

$$\angle GAB = 180 - \frac{360}{7} = 128\frac{4}{7}^\circ.$$

Let  $x = AG$  cm and  $y = GB$  cm. Then

$$\begin{aligned}\frac{1}{2} \times x \times x \times \sin 128\frac{4}{7}^\circ &= 30 \Rightarrow x^2 = \frac{60}{\sin 128\frac{4}{7}^\circ} \\ &\Rightarrow x = 8.760\ 301\ 391 \text{ cm (FCD).}\end{aligned}$$

Cosine rule:

$$\begin{aligned}y^2 &= x^2 + x^2 - 2(x)(x) \cos 128\frac{4}{7}^\circ \Rightarrow y^2 = 249.182\,567\,6 \text{ (FCD)} \\ &\Rightarrow y = 15.785\,517\,65 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{y = 15.8 \text{ (3 sf)}}}.\end{aligned}$$

*Dr Oliver  
Mathematics*

*Dr Oliver  
Mathematics*

*Dr Oliver  
Mathematics*

*Dr Oliver  
Mathematics*

*Dr Oliver  
Mathematics*