Dr Oliver Mathematics
The "Cover-Up" Method

As long division is a very accident-prone procedure, this is a useful check on the accuracy.

1. Express
$$\frac{6x-2}{(x-2)(x+3)}$$
as partial fractions.

Solution
$$\frac{6x-2}{(x-2)(x+3)} \equiv \frac{A}{x-2} + \frac{B}{x+3}.$$
Cover up the $(x-2)$ part of the denominator and substitute $x=2$:
$$A = \left.\frac{6x-2}{(x-2)(x+3)}\right|_{x=2} = \frac{6(2)-2}{2+3} = \frac{10}{5} = 2.$$
Cover up the $(x+3)$ part of the denominator and substitute $x=-3$:
$$B = \left.\frac{6x-2}{(x-2)(x+3)}\right|_{x=-3} = \frac{6(-3)-2}{(-3)-2} = \frac{-20}{-5} = 4.$$
Hence,
$$\frac{6x-2}{(x-2)(x+3)} \equiv \underline{\underline{\frac{2}{x-2} + \frac{4}{x+3}}}.$$

2. Express
$$\frac{2x+11}{(x+1)(x+4)}$$
as partial fractions.

Solution
$$\frac{2x+11}{(x+1)(x+4)} \equiv \frac{A}{x+1} + \frac{B}{x+4}.$$
Cover up the $(x+1)$ part of the denominator and substitute $x=-1$:
$$A = \left.\frac{2x+11}{(x+1)(x+4)}\right|_{x=-1} = \frac{2(-1)+11}{(-1)+4} = \frac{9}{3} = 3.$$

Cover up the $(x + 4)$ part of the denominator and substitute $x = -4$:

$$B = \left. \frac{2x + 11}{(x + 1)(x + 4)} \right|_{x=-4} = \frac{2(-4) + 11}{(-4) + 1} = \frac{3}{-3} = -1.$$

Hence,

$$\frac{2x + 11}{(x + 1)(x + 4)} \equiv \underline{\underline{\frac{3}{x + 1} - \frac{1}{x + 4}}}.$$

3. Express

$$\frac{-2x - 5}{(4 + x)(2 - x)}$$

as partial fractions.

Solution

$$\frac{-2x - 5}{(4 + x)(2 - x)} \equiv \frac{A}{4 + x} + \frac{B}{2 - x}.$$

Cover up the $(4 + x)$ part of the denominator and substitute $x = -4$:

$$A = \left. \frac{-2x - 5}{(4 + x)(2 - x)} \right|_{x=-4} = \frac{-2(-4) - 5}{2 - (-4)} = \frac{3}{6} = \frac{1}{2}.$$

Cover up the $(2 - x)$ part of the denominator and substitute $x = 2$:

$$B = \left. \frac{-2x - 5}{(4 + x)(2 - x)} \right|_{x=2} = \frac{-2(2) - 5}{4 + 2} = \frac{-9}{6} = -\frac{3}{2}.$$

Hence,

$$\frac{-2x - 5}{(4 + x)(2 - x)} \equiv \underline{\underline{\frac{\frac{1}{2}}{4 + x} - \frac{\frac{3}{2}}{2 - x}}}.$$

4. Express

$$\frac{x + 1}{x(3x - 1)}$$

as partial fractions.

Solution

$$\frac{x+1}{x(3x-1)} \equiv \frac{A}{x} + \frac{B}{3x-1}.$$

Cover up the x part of the denominator and substitute $x = 0$:

$$A = \frac{0+1}{3(0)-1} = \frac{1}{1} = -1.$$

Cover up the $(3x-1)$ part of the denominator and substitute $x = \frac{1}{3}$:

$$B = \frac{\frac{1}{3}+1}{\frac{1}{3}} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4.$$

Hence,

$$\frac{x+1}{x(3x-1)} \equiv \underline{\underline{-\frac{1}{x} + \frac{4}{3x-1}}}.$$

5. Express

$$\frac{2x^2-12x-26}{(x+1)(x-2)(x+5)}$$

as partial fractions.

Solution

$$\frac{2x^2-12x-26}{(x+1)(x-2)(x+5)} \equiv \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+5}.$$

$$x=-1 \Rightarrow A = \frac{2(-1)^2-12(-1)-26}{(-1-2)(-1+5)} = \frac{-12}{-12} = 1$$

$$x=2 \Rightarrow B = \frac{2(2)^2-12(2)-26}{(2+1)(2+5)} = \frac{-42}{21} = -2$$

$$x=-5 \Rightarrow C = \frac{2(-5)^2-12(-5)-26}{(-5+1)(-5-2)} = \frac{84}{28} = 3.$$

Hence,

$$\frac{2x^2-12x-26}{(x+1)(x-2)(x+5)} \equiv \underline{\underline{\frac{1}{x+1} - \frac{2}{x-2} + \frac{3}{x+5}}}.$$

6. Express

$$\frac{2x^2+39x+12}{(2x+1)^2(x-3)}$$

as partial fractions.

Solution

As you will recall,

$$\frac{2x^2+39x+12}{(2x+1)^2(x-3)} \equiv \frac{A}{(2x+1)^2} + \frac{B}{2x+1} + \frac{C}{x-3},$$

and *not* $(2x+1)^3(x-3)$ on the bottom line.

Now, we proceed on the A and C terms.

$$x = -\tfrac{1}{2} \Rightarrow A = \frac{2(-\frac{1}{2})^2 + 39(-\frac{1}{2}) + 12}{-\frac{1}{2} - 3} = \frac{-7}{-\frac{7}{2}} = 2$$
$$x = 3 \Rightarrow C = \frac{2(3)^2 + 39(3) + 12}{(2(3)+1)^2} = \frac{147}{49} = 3.$$

Next, just set $x = 0$:

$$\frac{2(0)+39(0)+12}{(2(0)+1)^2(x-3)} \equiv \frac{2}{(2(0)+1)^2} + \frac{B}{2(0)+1} + \frac{3}{0-3}$$
$$\Rightarrow \quad -4 = 2 + B - 1$$
$$\Rightarrow \quad B = -5.$$

Hence,

$$\frac{2x^2+39x+12}{(2x+1)^2(x-3)} \equiv \underline{\underline{\frac{2}{(2x+1)^2} - \frac{5}{2x+1} + \frac{3}{x-3}}}.$$