Dr Oliver Mathematics
GCSE Mathematics
2014 June Paper 1H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Work out (1)

$$\frac{1}{7} \times \frac{2}{3}.$$

Solution

$$\frac{1}{7} \times \frac{2}{3} = \frac{2}{\underline{\underline{21}}}.$$

(b) Work out (2)

$$\frac{3}{5} - \frac{1}{3}.$$

Solution

$$\begin{aligned} \frac{3}{5} - \frac{1}{3} &= \frac{9}{15} - \frac{5}{15} \\ &= \frac{9-5}{15} \\ &= \frac{4}{\underline{\underline{21}}}. \end{aligned}$$

2. Here are the times, in minutes, that 20 children took to walk to school. (3)

13	21	19	27	31	5	23	29	18	25
34	15	28	23	22	40	16	19	32	9

Draw an ordered stem and leaf diagram for these times.

Solution

4	0							
3	1	2	4					
2	1	2	3	3	5	7	8	9
1	3	5	6	8	9	9		
0	5	9						

Key: 4|0 means 40 minutes.

3. 50 people each did one activity at a sports centre. (4)

Some of the people went swimming.
Some of the people played squash.
The rest of the people used the gym.

21 of the people were female.
6 of the 8 people who played squash were male.
18 of the people used the gym.
9 males went swimming.

Work out the number of females who used the gym.

Solution

Now,

$$\text{male} = 50 - 21 = 29;$$

the number of females who went swimming is

$$50 - 8 - 18 = 24,$$

and 9 of these were men. Do two subtractions to work out the number of those who used the gym:

	Swimming	Squash	Gym	Total
Male	9	6	14	29
Female	15	2	4	21
Total	24	8	18	50

Hence, 4 women used the gym.

4. Mr Brown and his 2 children are going to London by train. (4)

An adult ticket costs £24.
A child ticket costs £12.

Mr Brown has a Family Railcard.

Family Railcard gives

$\frac{1}{3}$ off adult tickets

60% off child tickets

Work out the total cost of the tickets when Mr Brown uses his Family Railcard.

Solution

He has to pay

$$24 \times \tfrac{2}{3} = £16$$

and each of his two children cost

$$12 \times 0.4 = £4.80.$$

Hence, the total cost of the tickets is

$$\begin{aligned} 16 + 2 \times 4.8 &= 16 + 9.6 \\ &= £25.60. \end{aligned}$$

5. Rebecca wants to find out how many books people buy. (2)
She is going to use a questionnaire.

Design a suitable question for Rebecca to use in her questionnaire.

Solution

A suitable question with a time frame, e.g., "Did you buy any books today/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0, 1-3, 4-6, 7 or more.

6. (a) Expand (1)

$$2m(m+3).$$

Solution

$$2m(m+3) = \underline{\underline{2m^2+6m}}.$$

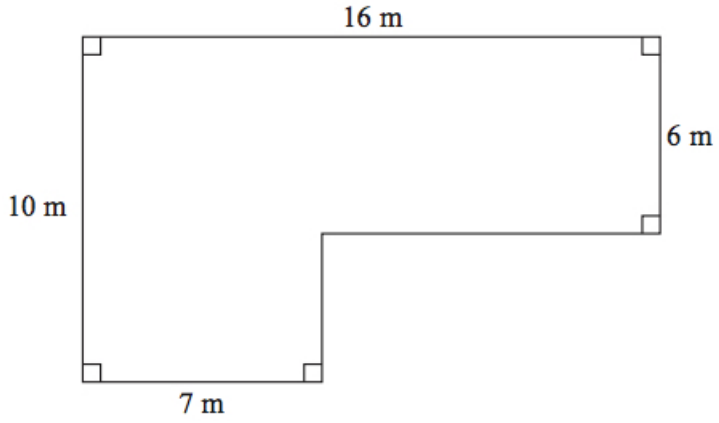
(b) Factorise fully (2)

$$3xy^2 - 6xy.$$

Solution

$$3xy^2 - 6xy = \underline{\underline{3xy(y-2)}}.$$

7. The diagram shows the plan of a small field. (4)



Kevin is going to keep some pigs in the field.
Each pig needs an area of 36 square metres.

Work out the greatest number of pigs Kevin can keep in the field.

Solution

$$\begin{aligned}\text{Area} &= 16 \times 10 - (10 - 6) \times (16 - 7)\\ &= 160 - 4 \times 9\\ &= 160 - 36\\ &= 124 \text{ m}^2.\end{aligned}$$

Now,

$$36 \times 3 = 108$$

and

$$36 \times 4 = 144$$

so he can keep 3 pigs.

8. The diagram shows a garden in the shape of a rectangle. (3)

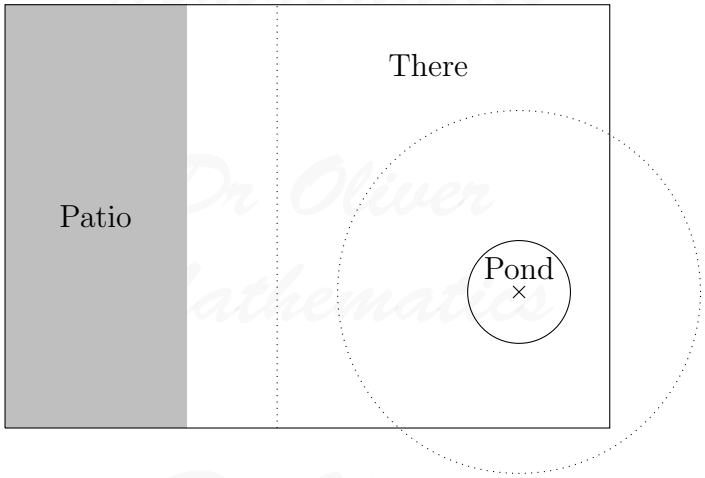
The scale of the diagram is 1 cm represents 2 m.

Scale: 1 cm represents 2 m.

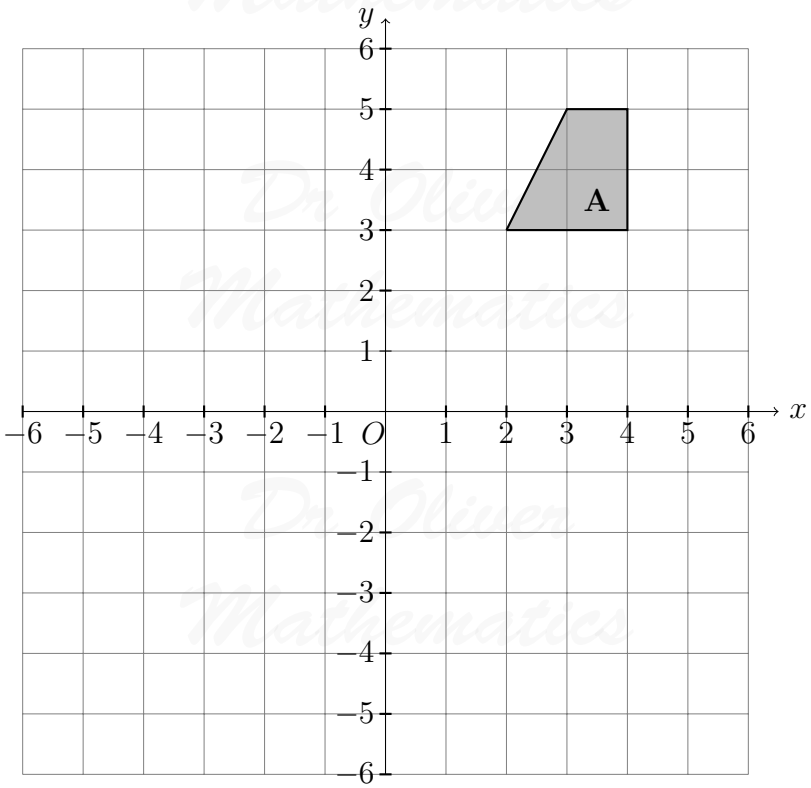
Irfan is going to plant a tree in the garden.
The tree must be more than 3 metres from the patio and more than 6 metres from the centre of the pond.

On the diagram, shade the region where Irfan can plant the tree.

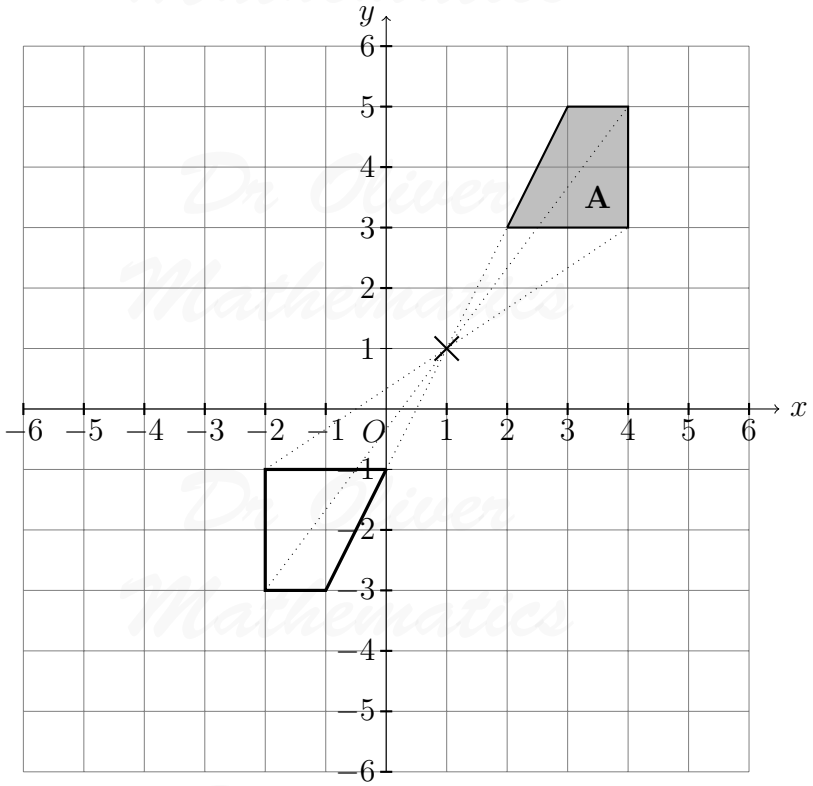
Solution



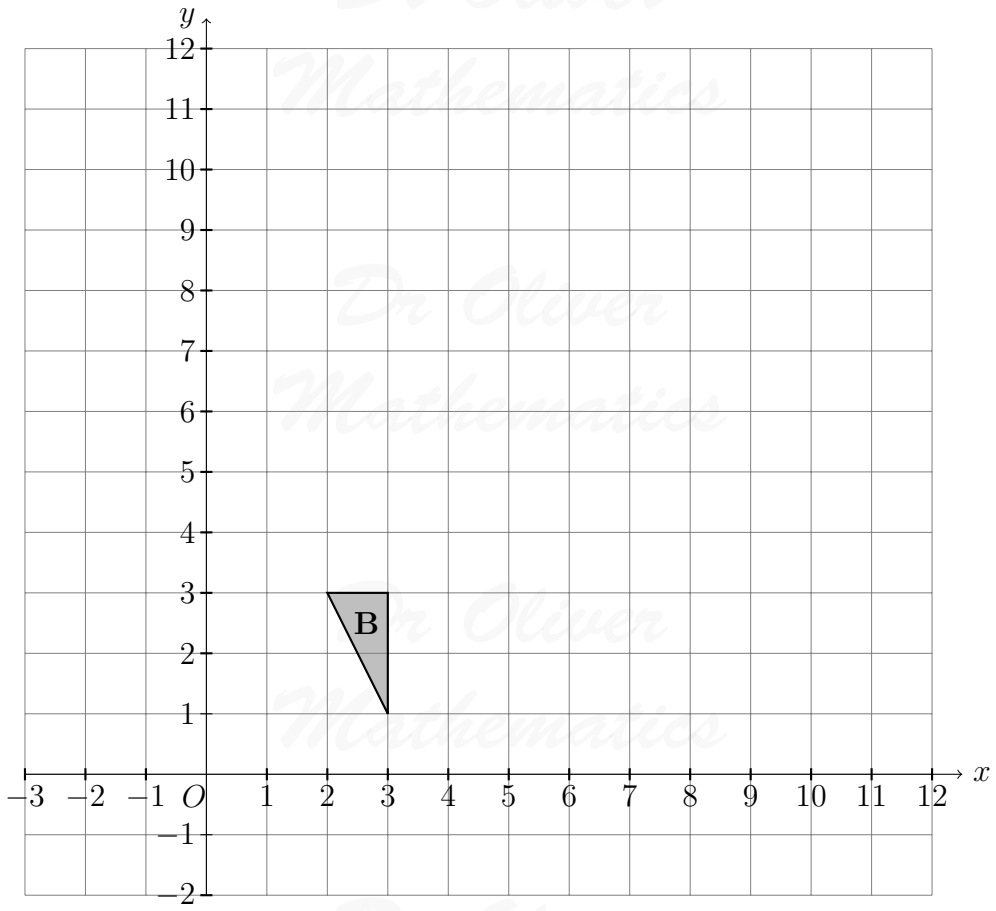
9. (a) On the grid, rotate shape **A** 180° about the point $(1, 1)$. (2)



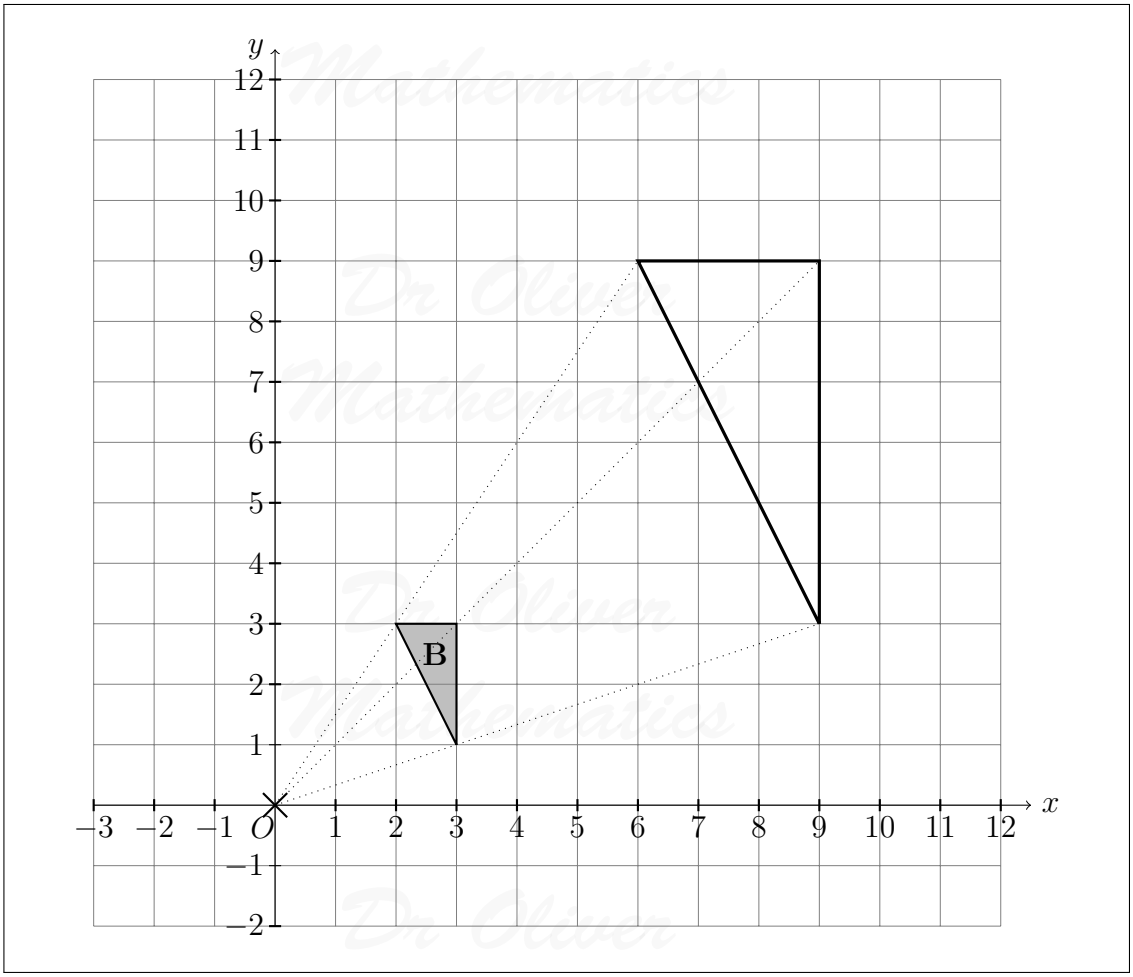
Solution



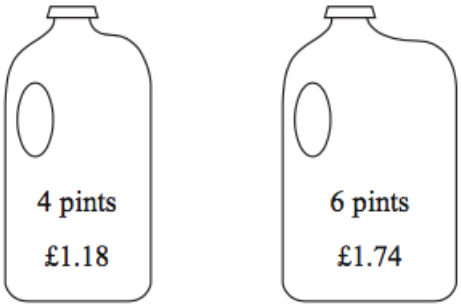
(b) On the grid, enlarge triangle **B** by scale factor 3, centre $(0, 0)$. (2)



Solution



10. Milk is sold in two sizes of bottle. (3)



A 4 pint bottle of milk costs £1.18.
A 6 pint bottle of milk costs £1.74.

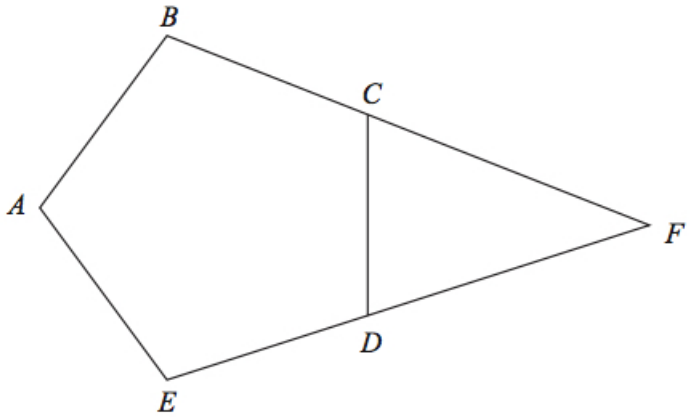
Which bottle of milk is the best value for money?
You must show all your working.

Solution

$$\begin{aligned}\frac{6}{4} \times 1.18 &= \frac{3}{2} \times 1.18 \\ &= 3 \times 0.59 \\ &= 1.77;\end{aligned}$$

hence, 6 pint bottle is the best value for money.

11. $ABCDE$ is a regular pentagon. (3)
BCF and EDF are straight lines.



Work out the size of angle CFD.
You must show how you got your answer.

Solution

The angles in a regular pentagon is

$$\begin{aligned}\frac{3}{5} \times 180 &= 3 \times 36 \\ &= 108^\circ\end{aligned}$$

and

$$\angle DEF = 180 - 108 = 72^\circ.$$

Finally,

$$\angle CFD = 180 - 72 - 72 = \underline{36^\circ}.$$

12. You can change temperatures from °F to °C by using the formula

$$C = \frac{5(F-32)}{9}.$$

F is the temperature in °F.
C is the temperature in °C.

The minimum temperature in an elderly person's home should be 20°C.

Mrs Smith is an elderly person.
The temperature in Mrs Smith's home is 77°F.

(a) Decide whether or not the temperature in Mrs Smith's home is lower than the minimum temperature should be. (3)

Solution

$$\begin{aligned}\frac{5(77-32)}{9} &= \frac{5\times 45}{9}\\ &= 5\times 5\\ &= 25°\text{C};\end{aligned}$$

hence, Mrs Smith's home is warmer than it should be (by 5°C).

(b) Make F the subject of the formula (3)

$$C = \frac{5(F-32)}{9}.$$

Solution

$$\begin{aligned}C = \frac{5(F-32)}{9} &\Rightarrow F - 32 = \frac{9C}{5}\\ &\Rightarrow \underline{\underline{F = \frac{9C}{5} + 32.}}\end{aligned}$$

13. In a competition, a prize is won every 2014 seconds. (4)

Competition

a prize every 2014 seconds

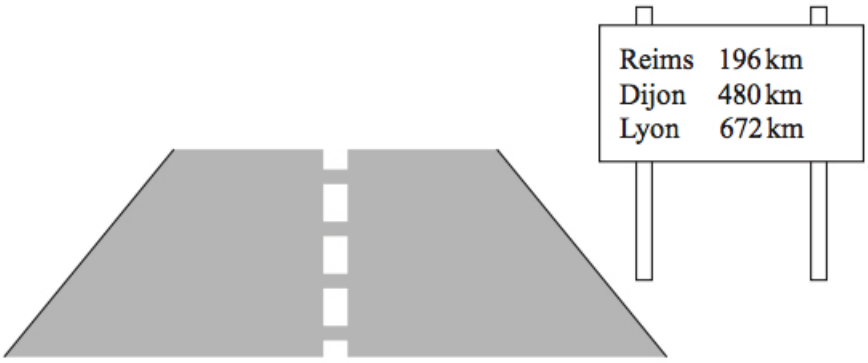
Work out an estimate for the number of prizes won in 24 hours.
You must show your working.

Solution

Round to 1 significant figure:

$$\begin{aligned}\frac{24 \times 60 \times 60}{2\,014} &\approx \frac{20 \times 60 \times 60}{2\,000} \\ &= \frac{20 \times 3\,600}{2\,000} \\ &= \frac{72\,000}{2\,000} \\ &= \underline{\underline{36}}.\end{aligned}$$

14. Emily is driving in France. (4)
She sees this sign.



Emily is going to drive to Dijon.
She plans to drive at an average speed of 50 miles per hour.

Work out how long it should take Emily to drive to Dijon.

Solution

$$\begin{aligned}\frac{480 \text{ km}}{50 \text{ mph}} &= \frac{480 \times \frac{5}{8} \text{ miles}}{50 \text{ mph}} \\ &= \frac{60 \times 5 \text{ miles}}{50 \text{ mph}} \\ &= \frac{300 \text{ miles}}{50 \text{ mph}} \\ &= \underline{\underline{6 \text{ hours}}}.\end{aligned}$$

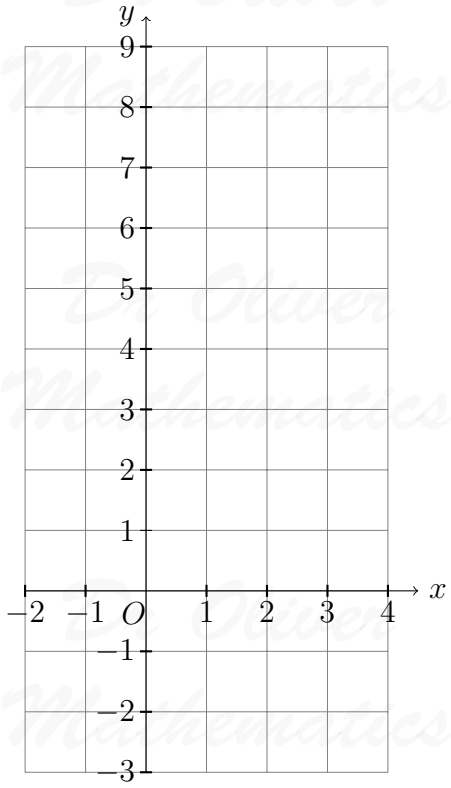
15. (a) Complete the table of values for $y = x^2 - 2x - 1$. (2)

x	-2	-1	0	1	2	3	4
y	7			-2	-1		

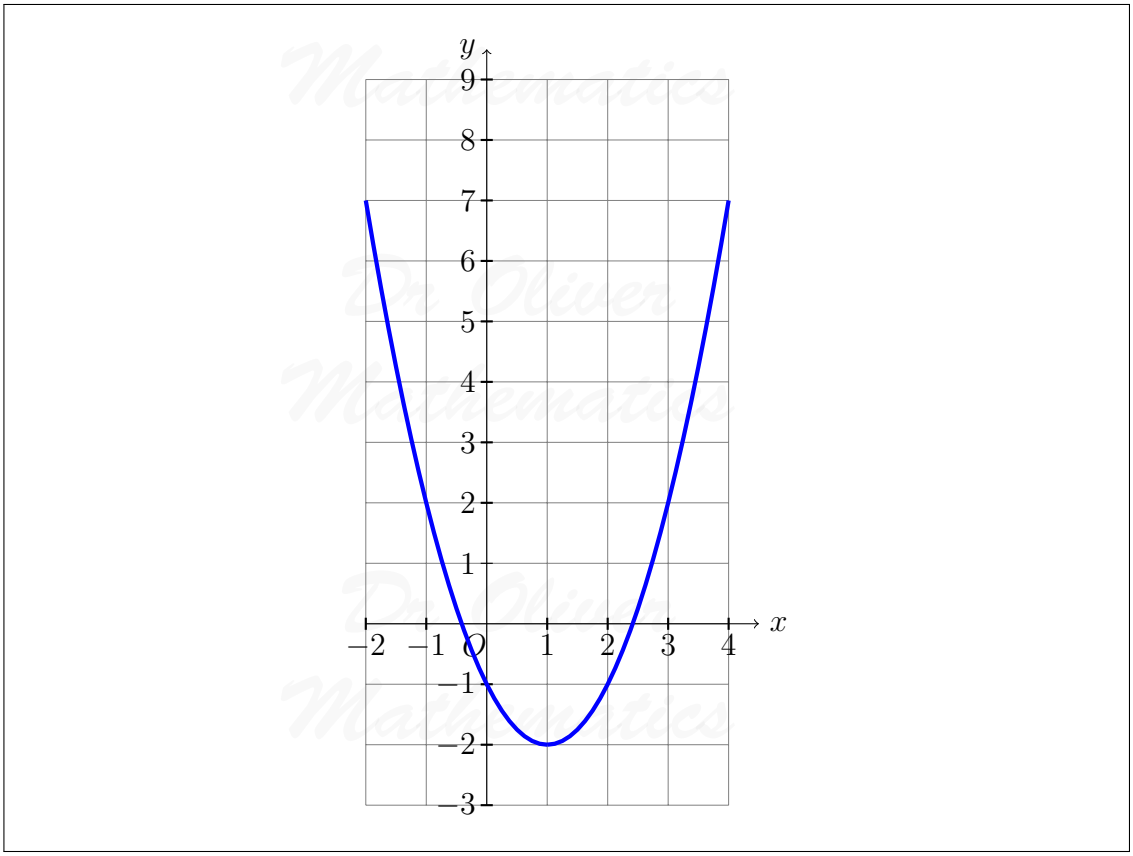
Solution

x	-2	-1	0	1	2	3	4
y	7	$\underline{\underline{2}}$	$\underline{\underline{-1}}$	-2	-1	$\underline{\underline{2}}$	$\underline{\underline{7}}$

(b) On the grid, draw the graph of $y = x^2 - 2x - 1$ for values of x from -2 to 4. (2)



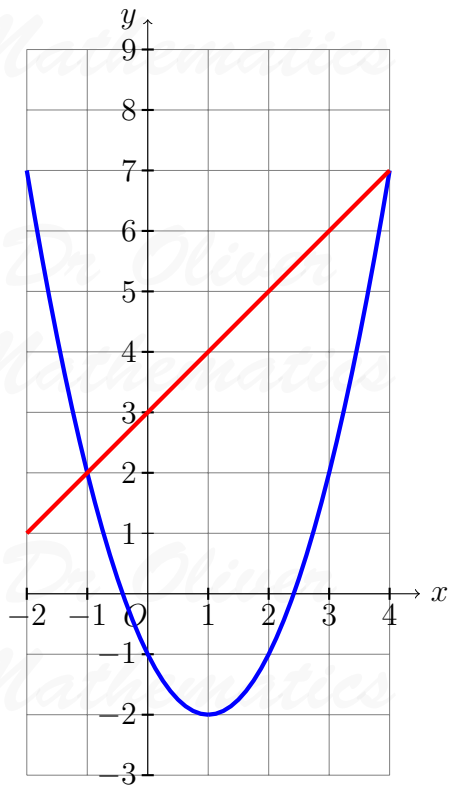
Solution



(c) Solve (2)

$$x^2 - 2x - 1 = x + 3.$$

Solution

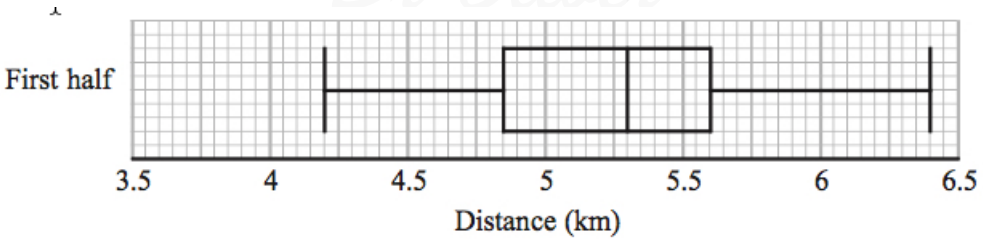


Correct read-off: $x = -1$ or $x = 4$.

16. Colin took a sample of 80 football players.

He recorded the total distance, in kilometres, each player ran in the first half of their matches on Saturday.

Colin drew this box plot for his results.



(a) Work out the interquartile range. (2)

Solution

$$\begin{aligned} \text{IQR} &= \text{UQ} - \text{LQ} \\ &= 5.6 - 4.85 \\ &= \underline{\underline{0.75 \text{ km}}}. \end{aligned}$$

There were 80 players in Colin's sample.

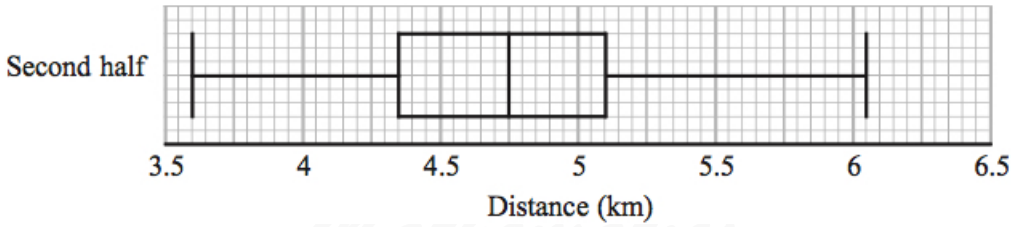
(b) Work out the number of players who ran a distance of more than 5.6 km. (2)

Solution

$80 \times \frac{1}{4} = \underline{\underline{20 \text{ players}}}$.

Colin also recorded the total distance each player ran in the second half of their matches.

He drew the box plot below for this information.



(c) Compare the distribution of the distances run in the first half with the distribution of the distances run in the second half. (2)

Solution

Average

Since the median for the first half (5.35) is higher than the median for the second half (4.75), they more running in the first half on average.

Spread

Since the range for the first half ($6.4 - 4.2 = 2.2$) is smaller than the range for the second half ($6.05 - 3.6 = 2.45$), the first half were more consistent.

OR

Since the IQR for the first half ($5.6 - 4.85 = 0.75$) is the same as the IQR for

the first half ($5.1 - 4.35 = 0.75$), they were consistent across the first and second halves.

17. (a) Write down the value of 10^0. (1)

Solution

$$10^0 = \underline{\underline{1}}.$$

(b) Write down the value of 10^{-2}. (1)

Solution

$$10^{-2} = \frac{1}{10^2} = \underline{\underline{\frac{1}{100}}}.$$

(c) Write these numbers in order of size. (2)
Start with the smallest number.

$$2.73 \times 10^3 \qquad 27.3 \times 10^{-3} \qquad 273 \times 10^2 \qquad 0.002\,73$$

Solution

$27.3 \times 10^{-3} = 2.73 \times 10^{-2}$.
$273 \times 10^2 = 2.73 \times 10^4$.
$0.002\,73 = 2.73 \times 10^{-3}$.
Hence, the order is

$$\underline{\underline{0.002\,73, 27.3 \times 10^{-3}, 2.73 \times 10^3, \text{ and } 273 \times 10^2}}.$$

18. Solve the simultaneous equations (3)

$$4x + y = 25$$
$$x - 3y = 16.$$

Solution

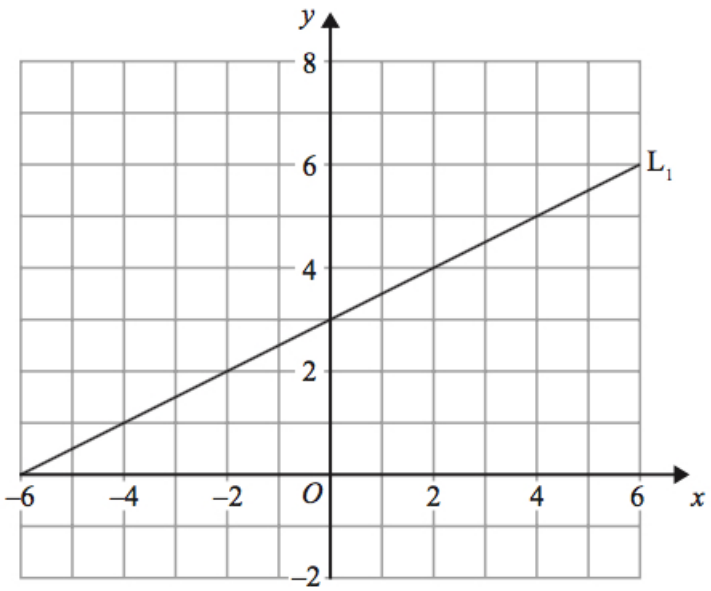
$$4x + y = 25 \quad (1)$$
$$x - 3y = 16 \quad (2)$$

$$(1): \quad 4x + y = 25$$
$$4 \times (2): \quad 4x - 12y = 64 \quad (3)$$

Now, (1) − (3):

$$13y = -39 \Rightarrow \underline{\underline{y = -3}}$$
$$\Rightarrow x + 9 = 16$$
$$\Rightarrow \underline{\underline{x = 7}}.$$

19. The diagram shows a straight line, L_1, drawn on a grid. (3)



A straight line, L_2, is parallel to the straight line L_1 and passes through the point $(0, -5)$.

Find an equation of the straight line L_2.

Solution

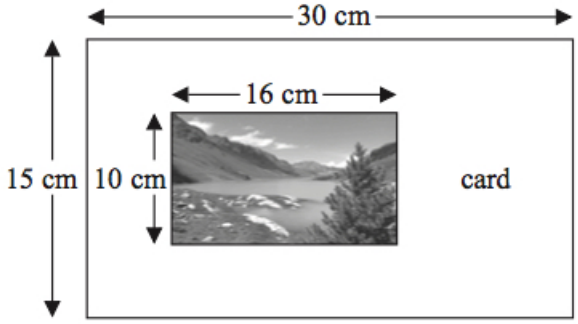
For L_1, choose $(0, 3)$ and $(-6, 0)$:

$$\text{gradient} = \frac{3-0}{0-(-6)}$$
$$= -\tfrac{1}{2}$$

and this must be the same for L_2.
Finally, L_2:

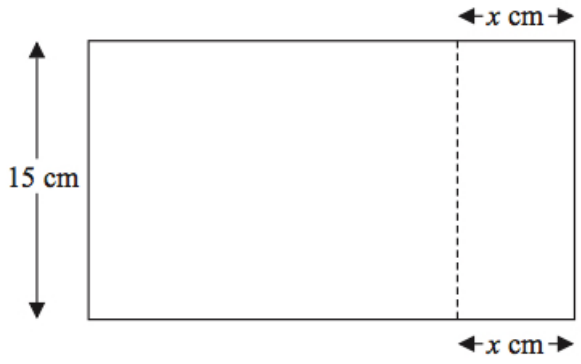
$$\underline{\underline{y = -\tfrac{1}{2}x - 5}}.$$

20. Steve has a photo and a rectangular piece of card. (3)



The photo is 16 cm by 10 cm.
The card is 30 cm by 15 cm.

Steve cuts the card along the dotted line shown in the diagram below.



Steve throws away the piece of card that is 15 cm by x cm.
The piece of card he has left is mathematically similar to the photo.

Work out the value of x.

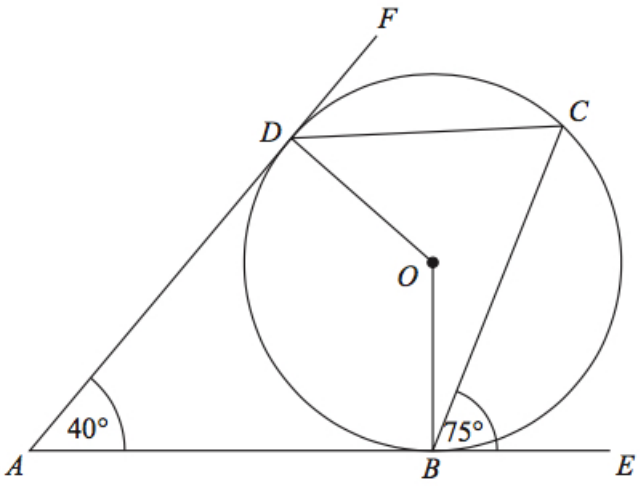
Solution

$$\frac{16}{10} \times 15 = \frac{16}{2} \times 3$$
$$= 8 \times 3$$
$$= 24$$

and so

$$x = 30 - 24 = \underline{\underline{6}}.$$

21. B, C, and D are points on the circumference of a circle, centre O. (3)



ABE and ADF are tangents to the circle.

Angle $DAB = 40°$.
Angle $CBE = 75°$.

Work out the size of angle ODC.

Solution

$\angle ADO = \angle ABO = 90^\circ$ (right angles)
$\angle DOB = 360 - 90 - 90 - 40 = 140^\circ$ (sum of the angles in a quadrilateral)
$\angle DCB = \frac{140}{2} = 70^\circ$ (the angle at the centre is twice the angle at the circumference)
$\angle OBC = 90 - 75 = 15^\circ$ (right angles)
$\angle ODC = \angle OCD = 70 - 15 = \underline{\underline{55^\circ}}$.

22. (a) Simplify (2)

$$(3x^2y^4)^3.$$

Solution

$$(3x^2y^4)^3 = \underline{\underline{27x^6y^{12}}}.$$

(b) Simplify (3)

$$\frac{x^2 - 9}{2x^2 + 5x - 3}.$$

Solution

$\underline{x^2 - 9}$:

$$\left.\begin{array}{ll} \text{add to:} & 0 \\ \text{multiply to:} & -9 \end{array}\right\} +3, -3$$

$$x^2 - 9 = (x + 3)(x - 3).$$

$\underline{2x^2 + 5x - 3}$:

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & (+2) \times (-3) = -6 \end{array}\right\} +6, -1$$

$$\begin{aligned} 2x^2 + 5x - 3 &= 2x^2 + 6x - x - 3 \\ &= 2x(x + 3) - 1(x + 3) \\ &= (2x - 1)(x + 3). \end{aligned}$$

Hence,

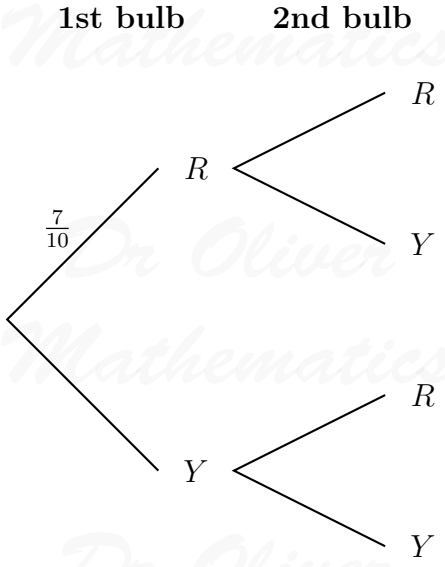
$$\begin{aligned} \frac{x^2 - 9}{2x^2 + 5x - 3} &= \frac{(x + 3)(x - 3)}{(2x - 1)(x + 3)} \\ &= \underline{\underline{\frac{x - 3}{2x - 1}}}. \end{aligned}$$

23. Yvonne has 10 tulip bulbs in a bag.

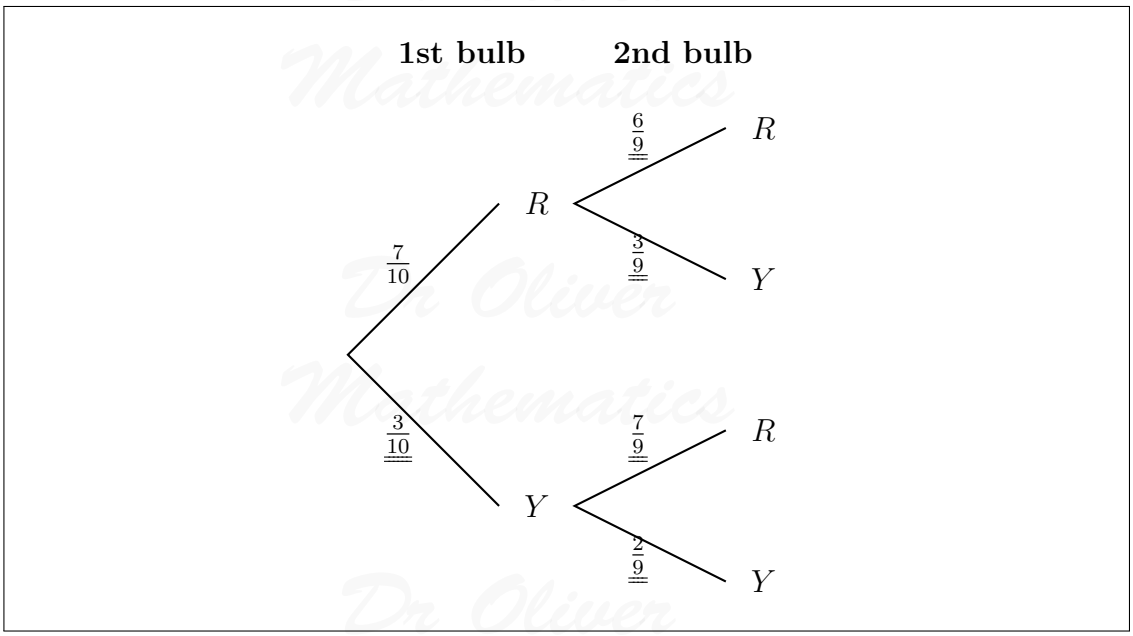
7 of the tulip bulbs will grow into red tulips.
3 of the tulip bulbs will grow into yellow tulips.

Yvonne takes at random two tulip bulbs from the bag.
She plants the bulbs.

(a) Complete the probability tree diagram. (2)



Solution

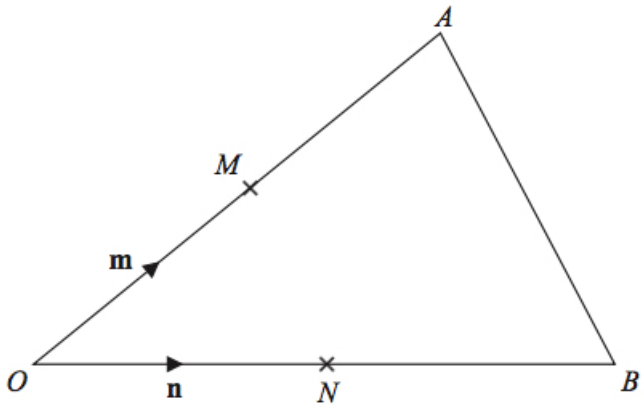


(b) Work out the probability that at least one of the bulbs will grow into a yellow tulip. (3)

Solution

$$\begin{aligned} \text{P(at least one will into a yellow tulip)} &= 1 - \text{P}(RR) \\ &= 1 - (\tfrac{7}{10} \times \tfrac{6}{9}) \\ &= 1 - \tfrac{42}{90} \\ &= \underline{\underline{\tfrac{48}{90}}}. \end{aligned}$$

24. OAB is a triangle. (3)



M is the midpoint of OA.
N is the midpoint of OB.
$\overrightarrow{OM} = \mathbf{m}$.
$\overrightarrow{ON} = \mathbf{n}$.
Show that AB is parallel to MN.

Solution

$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MO} + \overrightarrow{ON} \\ &= \mathbf{n} - \mathbf{m}\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= 2\mathbf{n} - 2\mathbf{m} \\ &= 2(\mathbf{n} - \mathbf{m}) \\ &= 2\overrightarrow{MN};\end{aligned}$$

hence, $\underline{\underline{AB \text{ is parallel to } MN}}$.

25. (a) Rationalise the denominator of (2)

$$\frac{12}{\sqrt{3}}.$$

Solution

$$\begin{aligned}\frac{12}{\sqrt{3}} &= \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{12\sqrt{3}}{3} \\ &= \underline{\underline{4\sqrt{3}}}.\end{aligned}$$

(b) Work out the value of (2)

$$(\sqrt{2} + \sqrt{8})^2.$$

Solution

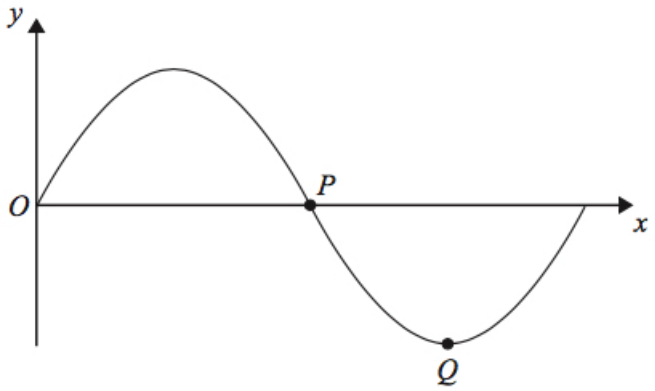
Now,

$$\begin{aligned}\sqrt{8} &= \sqrt{4 \times 2} \\ &= \sqrt{4} \times \sqrt{2} \\ &= 2\sqrt{2}.\end{aligned}$$

Finally,

$$\begin{aligned}(\sqrt{2} + \sqrt{8})^2 &= (\sqrt{2} + 2\sqrt{2})^2 \\ &= (3\sqrt{2})^2 \\ &= 9 \times 2 \\ &= \underline{\underline{18}}.\end{aligned}$$

26. The diagram shows part of a sketch of the curve $y = \sin x^\circ$.



(a) Write down the coordinates of the point P. (1)

Solution

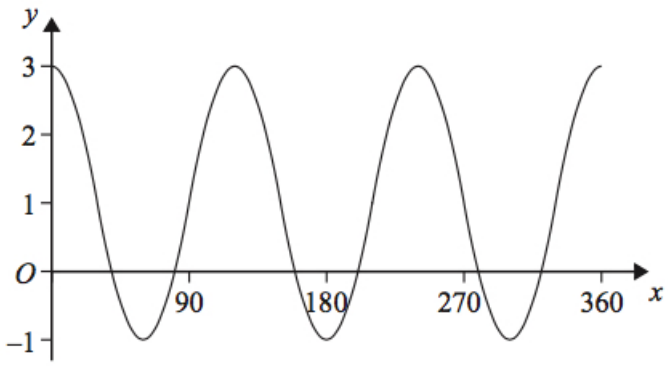
$\underline{\underline{P(180, 0)}}$.

(b) Write down the coordinates of the point Q. (1)

Solution

$\underline{\underline{Q(270, -1)}}$.

Here is a sketch of the curve $y = a\cos bx + c$, $0 \leqslant x \leqslant 360$.



(c) Find the values of a, b, and c. (3)

Solution

$\underline{\underline{a = 2}}$ (amplitude)

$\underline{\underline{b = 3}}$ (it is stretched three times in the x-direction)

$\underline{\underline{c = 1}}$ (translated by 1 in the y-direction)