

Dr Oliver Mathematics
Mathematics: Advanced Higher
2026 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. Differentiate

(2)

$$f(x) = 3 \sin^{-1} 7x.$$

2. Write down the binomial expansion of

(4)

$$\left(x^2 - \frac{5}{x}\right)^4$$

and simplify your answer.

3. (a) Find and simplify the Maclaurin expansion, up to and including the term in x^3 , for:

(i) e^{3x} ,

(2)

(ii) $\ln(1+x)$.

(2)

- (b) Hence find and simplify the Maclaurin expansion, up to and including the term in x^3 , for

(2)

$$e^{3x} \ln\left(\frac{1}{1+x}\right).$$

4. (a) Use the Euclidean algorithm to find d , the greatest common divisor of 1 428 and 567.

(1)

- (b) Find integers a and b such that

(2)

$$1\,428a + 567b = d.$$

5. Given

(5)

$$y = x^{4x},$$

use logarithmic differentiation to find $\frac{dy}{dx}$.

Write your answer in terms of x .

6. (a) An arithmetic sequence has terms

$$u_3 = 6 \text{ and } u_{11} = 10.$$

For this sequence, find the:

- (i) common difference, (1)
- (ii) first term, and (1)
- (iii) sum of the first 109 terms. (1)

(b) The terms

$$v_3 = 18 \text{ and } v_4 = 27$$

form part of a geometric sequence.

For this sequence, find:

- (i) common ratio, (1)
 - (ii) first term, and (1)
 - (iii) an expression, in terms of n , for the sum of the first n terms. (1)
- (c) Find **algebraically** the least value of n such that the sum of the geometric series exceeds the sum of the first 109 terms in the arithmetic sequence (1)

7. A curve is defined parametrically by

$$x = 3 \ln(2t + 1), y = t - \frac{1}{2}t^2, \text{ where } t > 0.$$

- (a) Find an expression for $\frac{dy}{dx}$. (2)
- Simplify your answer.

- (b) Find the coordinates of the stationary point on the curve. (2)

8. The volume, V cubic metres, of water held in a reservoir is given by (4)

$$V = 18(2 + \sqrt{h})^6 - 1\,152,$$

where h metres is the depth of water.

Water is pumped out of the reservoir at a constant rate of $0.6 \text{ m}^3 \text{ s}^{-1}$.

Find the rate of change of the depth of water in the reservoir when the depth is 9 metres.

9. Express (3)

$$3\,442_5$$

in base 9.

10. A curve is defined by the equation

$$x^2 e^{6y} + x^2 + y^5 = 50.$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y . (4)

(b) Given $x > 0$, explain why the derivative is never zero. (1)

11. Find (4)

$$\int \left(\frac{2x+4}{x^2+4} \right) dx.$$

12. Prove by induction that (5)

$$7^n + 2$$

is divisible by 3 for all $n \in \mathbb{N}$.

13. (a) Express using partial fractions (2)

$$\frac{x+3}{(x+7)(x+5)}.$$

(b) Hence find the particular solution of the differential equation (7)

$$\frac{dy}{dx} + \left(\frac{2}{x+3} \right) y = \frac{1}{(x+7)(x+5)(x+3)}, \text{ where } x \geq 0,$$

given that $y = 0$ when $x = 3$.

14. The plane π contains the points $P(2, 3, -4)$, $Q(3, 5, 1)$, and $R(6, 0, -6)$.

(a) Determine the Cartesian equation of π . (4)

The line L has symmetric equations

$$\frac{x-8}{2} = \frac{y+2}{-1} = \frac{z+2}{3}.$$

L intersects π at the point S .

(b) Find the coordinates of S . (3)

(c) Calculate the size of the acute angle between L and π . (3)

15. Let r be a positive real number and consider the following statement:

If r is irrational, then $\sqrt{r}$ is irrational.

(a) Write down the contrapositive of the statement. (1)

(b) Hence prove that the statement is true. (3)

16. (a) Given (2)

$$y = \ln(\cos x), \quad 0 \leq x < \frac{1}{2}\pi,$$

show that

$$\frac{dy}{dx} = -\tan x.$$

For a function $g(x)$, it is known that

$$\int x g(x) \, dx = 2x \tan 2x - \int 2 \tan 2x \, dx.$$

- (b) (i) Determine the exact value of (2)

$$\int_0^{\frac{1}{6}\pi} x g(x) \, dx.$$

- (ii) Find an expression for $g(x)$ in terms of x . (1)