# Dr Oliver Mathematics
# Cambridge O Level Additional Mathematics
# 2013 June Paper 2 Variant 2: Calculator
# 2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Variables $x$ and $y$ are such that when $\sqrt{y}$ is plotted against $x^2$ a straight line graph passing through the points $(1, 3)$ and $(4, 18)$ is obtained. (4)

   Express $y$ in terms of $x$.

2. (a) Solve the equation (3)

$$3^{p+1} = 0.7,$$

   giving your answer to 2 decimal places.

   (b) Express (3)

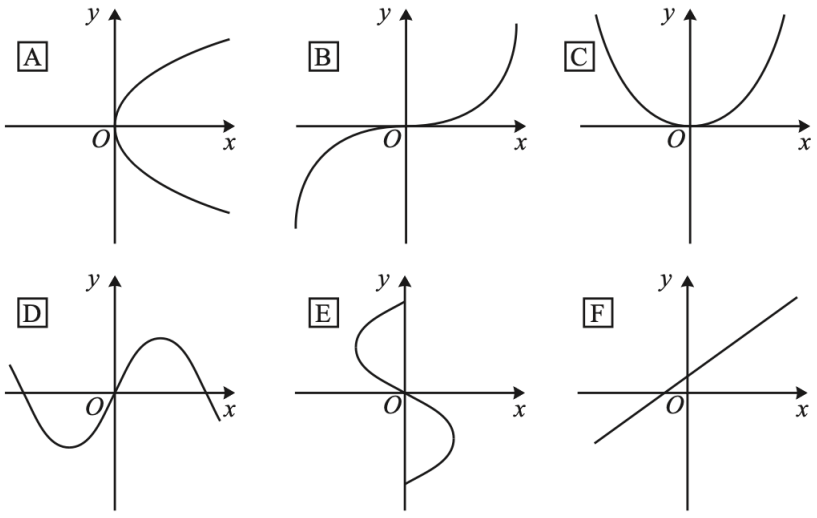
$$\frac{y \times (4x^3)^2}{\sqrt{8y^3}}$$

   in the form

$$2^a \times x^b \times y^c,$$

   where $a$, $b$, and $c$ are constants.

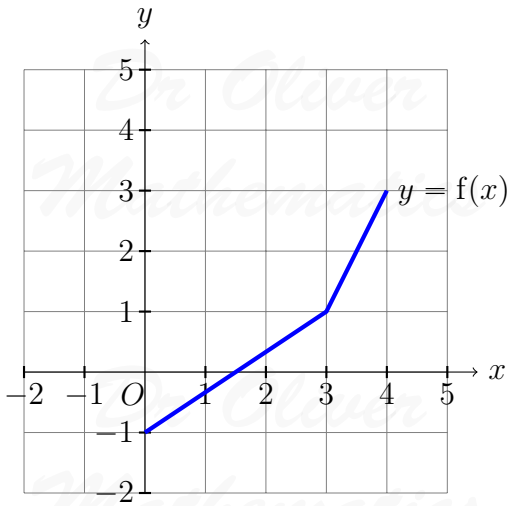
3. (a) Here are 6 graphs.



(i) Write down the letter of each graph which does **not** represent a function. (2)

(ii) Write down the letter of each graph which represents a function that does **not** have an inverse. (2)

(b) The diagram shows the graph of a function $y = \mathrm{f}(x)$. (2)




On the same axes sketch the graph on $y = \mathrm{f}^{-1}(x)$.

4. The position vectors of the points $A$ and $B$, relative to an origin $O$, are $(4\mathbf{i} - 21\mathbf{j})$ and $(22\mathbf{i} - 30\mathbf{j})$ respectively.

The point $C$ lies on $AB$ such that

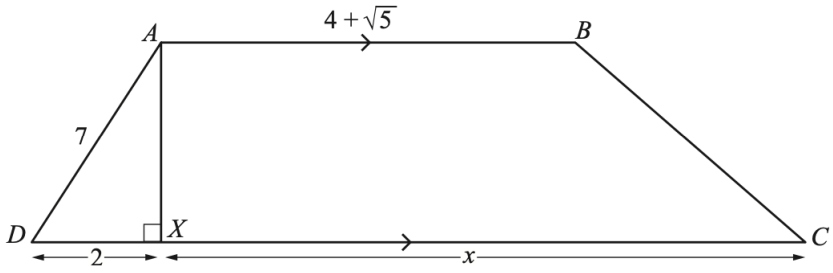
$$\overrightarrow{AB} = 3\overrightarrow{AC}.$$

(a) Find the position vector of $C$ relative to $O$. (4)

(b) Find the unit vector in the direction $\overrightarrow{OC}$. (2)

5. **Calculators must not be used in this question.** (6)

The diagram shows a trapezium $ABCD$ in which $AD = 7$ cm and $AB = (4 + \sqrt{5})$ cm.



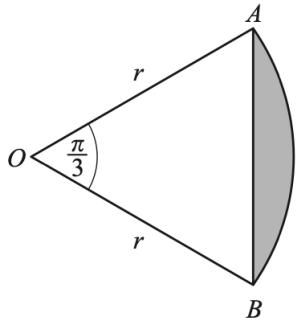
$AX$ is perpendicular to $DC$ with $DX = 2$ cm and $XC = x$ cm.

Given that the area of trapezium $ABCD$ is

$$15(\sqrt{5} + 2) \text{ cm}^2,$$

obtain an expression for $x$ in the form $a + b\sqrt{5}$, where $a$ and $b$ are integers.

6. The shaded region in the diagram is a segment of a circle with centre $O$ and radius $r$ cm.




Angle $AOB = \frac{1}{3}\pi$ radians.

(a) Show that the perimeter of the segment is (2)

$$r\left(\frac{3+\pi}{3}\right) \text{ cm}.$$

(b) Given that the perimeter of the segment is 26 cm, find the value of $r$ and the area of the segment. (5)

7. Differentiate, with respect to $x$,

(a) $(3 - 5x)^{12}$, (2)

(b) $x^2 \sin x$, (2)

(c) $\dfrac{\tan x}{1 + \mathrm{e}^{2x}}$. (4)

8. **Solutions to this question by accurate drawing will not be accepted.**
The points $A(-6, 2)$, $B(2, 6)$, and $C$ are the vertices of a triangle.

(a) ) Find the equation of the line $AB$ in the form $y = mx + c$. (2)

(b) Given that angle $ABC = 90°$, find the equation of $BC$. (2)

(c) Given that the length of $AC$ is 10 units, find the coordinates of each of the two possible positions of point $C$. (4)

9. (a) The graph of (3)

$$y = k(3^x) + c$$

passes through the points $(0, 14)$ and $(-2, 6)$.

Find the value of $k$ and of $c$.

(b) The variables $x$ and $y$ are connected by the equation

$$y = e^x + 25 - 24e^{-x}.$$

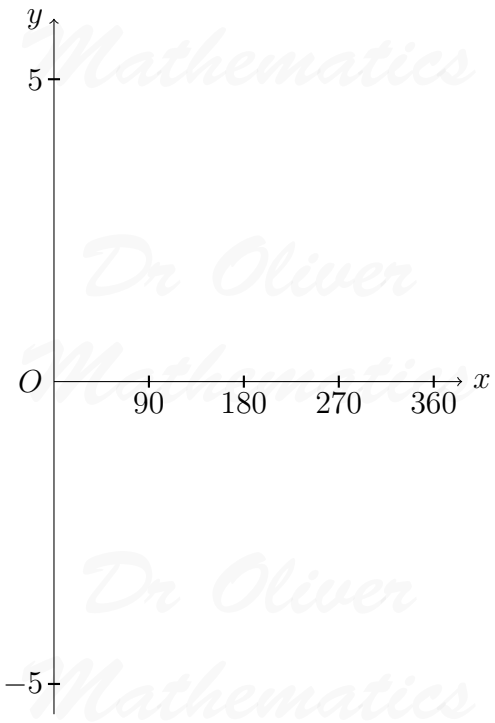
(i) Find the value of $y$ when $x = 4$. (1)

(ii) Find the value of $e^x$ when $y = 20$ and hence find the corresponding value of $x$. (4)

10. (a) The function f is defined, for $0^\circ \leqslant x \leqslant 360^\circ$, by

$$f(x) = 1 + 3\cos 2x.$$

(i) Sketch the graph of $y = f(x)$ on the axes below. (4)




(ii) State the amplitude of f. (1)

(iii) State the period of f. (1)

(b) Given that (3)

$$\cos x = p,$$

where $270^\circ < x < 360^\circ$, find $\operatorname{cosec} x$ in terms of $p$.

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11. A curve has equation

$$y = 3x + \frac{1}{(x-4)^3}.$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (4)

(b) Show that the coordinates of the stationary points of the curve are $(5, 16)$ and $(3, 8)$. (2)

(c) Determine the nature of each of these stationary points. (2)

(d) Find (2)

$$\int \left(3x + \frac{1}{(x-4)^3}\right) \mathrm{d}x.$$

(e) Hence find the area of the region enclosed by the curve, the line $x = 5$, the $x$-axis, and the line $x = 6$. (2)

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