Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2015 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

Section A

1. Find the equation of the line which is perpendicular to the line (3)

$$2x + 3y = 5$$

and which passes through the point $(3, 4)$.

2. (a) Find α in the range $0^\circ \leqslant \alpha \leqslant 180^\circ$ such that (2)

$$\tan \alpha = -1.5.$$

(b) Find β in the range $0^\circ \leqslant \beta \leqslant 180^\circ$ such that (2)

$$\sin \beta = 0.2.$$

3. Find the equation of the tangent to the curve (5)

$$y = x^3 + 3x - 5$$

at the point $(2, 9)$.

4. (a) Find (4)

$$\int_1^2 (x^2 + 2x + 3)\,\mathrm{d}x.$$

(b) Interpret your answer geometrically. (1)

5. A train accelerates from rest from a point O such that at t seconds the displacement, s metres from O, is given by the formula

$$s = \tfrac{3}{2}t^2 - 2t + 3.$$

(a) Show by calculus that the acceleration is constant. (3)

(b) Find the velocity after 5 seconds. (2)

6. You are given that n is a positive integer and $(n-1)$, n, $(n+1)$ are three consecutive integers.

In each of the following cases form an equation in n and solve it.

(a) The three integers add up to 99. (2)

(b) When the product of the first integer and third integer is added to 5 times the second integer the sum is 203. (4)

7. (a) Solve algebraically the simultaneous equations (4)

$$y = 3 + 5x - x^2 \text{ and } andy = x + 7.$$

(b) Interpret your answer geometrically. (1)

8. The cubic polynomial

$$\mathrm{f}(x) = x^3 + ax + 6,$$

where a is a constant, has a factor of $(x + 3)$.

(a) Find the value of a. (2)

(b) Hence or otherwise, solve the equation $\mathrm{f}(x) = 0$ for this value of a. (4)

9. The equation of the circle C is

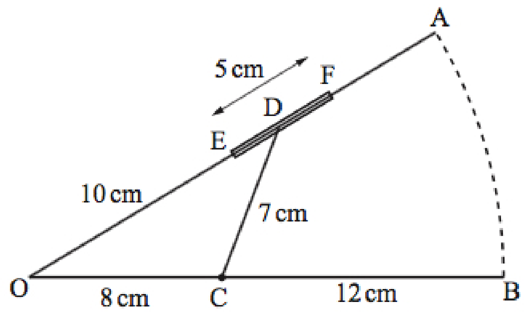
$$x^2 + y^2 - 8x + 2y - 19 = 0.$$

(a) Express the equation of C in the form (4)

$$(x - a)^2 + (y - b)^2 = r^2.$$

(b) Hence or otherwise, use an algebraic method to decide whether the point $(8, 3)$ lies inside, outside or on the circumference of the circle. (2)
Show all your working.

10. The figure shows a partly open window OA, viewed from above. The window is hinged at O. When the window is closed, the end A is at point B. The window is kept open by a rod CD, where C is a fixed point on the line OB.



The point D slides along a fixed bar EF. When the window is closed, D is at F. When the window is fully open, D is at E.

$OA = OB = 20$ cm, $OC = 8$ cm, $CD = 7$ cm, $EF = 5$ cm, and $OE = 10$ cm.

Find

(a) angle EOC when the window is fully open, (3)

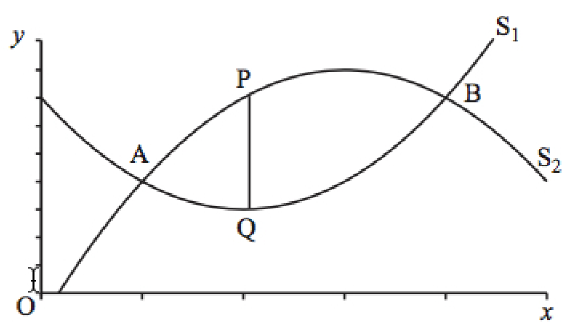
(b) the distance OD when angle EOC is 30°. (4)

Section B

11. Two curves, S_1 and S_2 have equations

$$y = x^2 - 4x + 7 \text{ and } y = 6x - x^2 - 1$$

respectively.



The curves meet at A and at B.

(a) Show that the coordinates of A and B are $(1, 4)$ and $(4, 7)$ respectively. (2)

Points P and Q lie on S_1 and S_2 between A and B. P and Q have the same x-coordinate so that PQ is parallel to the y-axis, as shown in the above figure.

(b) Find an expression, in its simplest form, for the length PQ as a function of x. (2)

(c) Use calculus to find the greatest length of PQ. (4)

(d) Find the area between the two curves. (4)

12. A distributor of flower bulbs has a large number of tulip bulbs and daffodil bulbs, mixed in the ratio 1 : 3 respectively. He packs the bulbs in boxes. He puts 10 bulbs, chosen at random, into each box.

(a) Find the probability that a box, chosen at random, contains

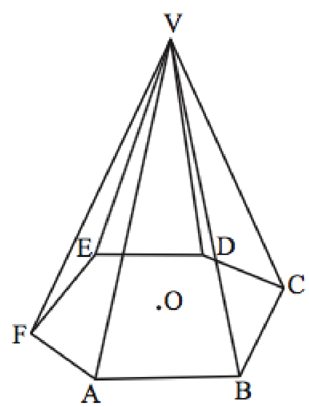
(i) exactly 4 daffodil bulbs, (4)

(ii) at least 1 tulip bulb. (3)

Two boxes of bulbs are chosen at random.

(b) Find the probability that there is a total of 3 tulip bulbs in the two boxes. (5)

13. A gardener marks out a regular hexagon $ABCDEF$ on his horizontal garden. Each side of the hexagon is 0.5 m. The gardener sticks a cane in the ground at each point of the hexagon. He joins the six canes at V where V is vertically above the centre, O, of the hexagon, as shown below. Each cane has a length of 2.4 m from the ground to V.



Calculate, giving your answers to 3 significant figures,

(a) the vertical height of V above the ground, (3)

(b) the angle between each cane and the ground, (2)

(c) the angle between the plane VAB and the ground. (4)

The gardener stretches a horizontal wire around the structure to strengthen it. He fixes the wire to each cane at a point 1 m vertically above the ground.

(d) Find the length of the wire. (3)

14. A company produces bottles of two liquids, X and Y. There are two ingredients, A and B, in each liquid.

The table shows the quantities, in centilitres (cl), of A and B needed for each bottle of liquid.

	A	B
X	4	2
Y	3	5

Each day the company can use 84 cl of A and 90 cl of B.

From this information an analyst writes down the inequality

$$4x + 3y \leqslant 84.$$

(a) Explain what x and y stand for in this inequality and explain what the inequality models. (2)

(b) Use the information given to write down another inequality, other than $x \geqslant 0$ and $y \geqslant 0$. (1)

(c) Illustrate your two inequalities. Shade the region that is **not** required. (3)

The company needs to produce the same number of bottles of X and of Y each day.

(d) Find the maximum number of bottles of X and of Y that the company can produce. (2)

On one day the company does not have to produce the same numbers of bottles of X and of Y.

(e) Write down the maximum number of bottles that can be produced and all the combinations that will give this maximum. (4)