

**Dr Oliver Mathematics**  
**GCSE Mathematics**  
**2011 June Paper 4H: Calculator**  
**1 hour 45 minutes**

The total number of marks available is 100.

You must write down all the stages in your working.

1. Each student at a college studies one of four languages.

The table shows the probability a student chosen at random studies German or Russian or French.

Language	German	Spanish	Russian	French
Probability	0.2		0.1	0.5

A student is chosen at random.

- (a) Work out the probability that the student studies Spanish. (2)

**Solution**

$$\begin{aligned}\text{Probability} &= 1 - (0.2 + 0.1 + 0.5) \\ &= 1 - 0.8 \\ &= \underline{0.2}.\end{aligned}$$

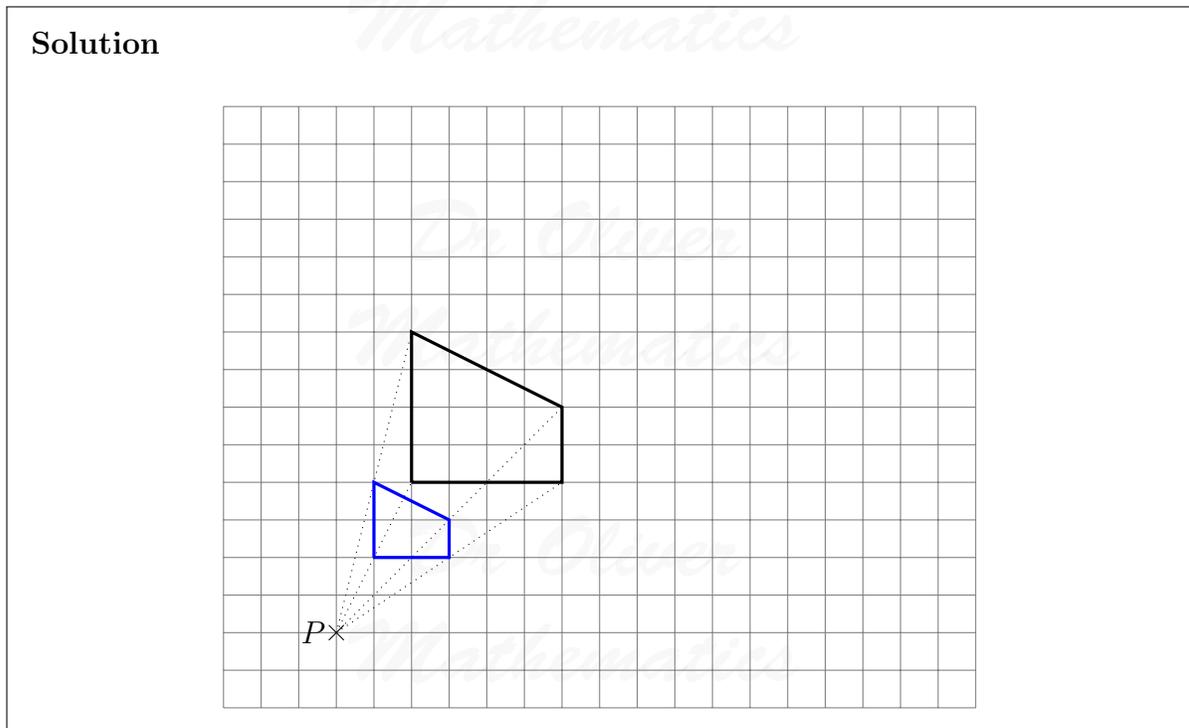
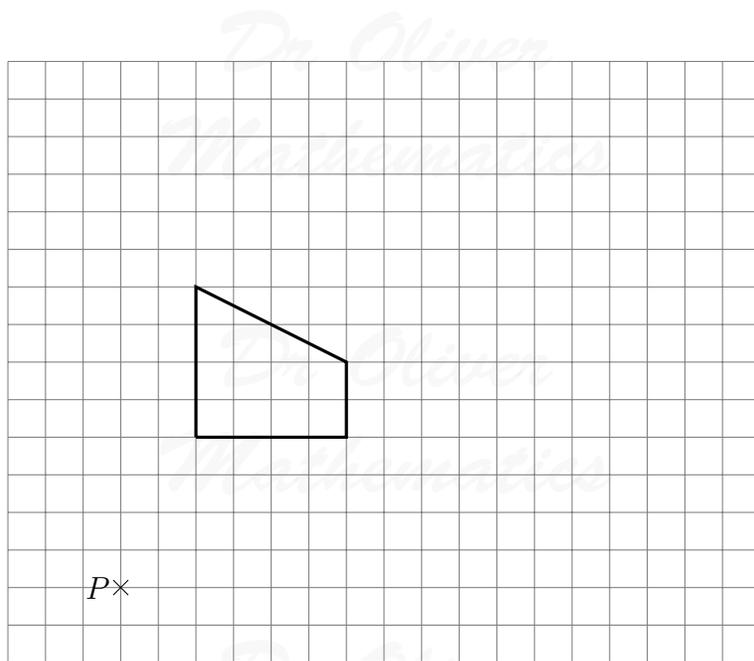
There are 800 students at the college.

- (b) Work out the number of students who study German. (2)

**Solution**

$$800 \times 0.2 = \underline{160 \text{ students}}.$$

2. On the grid, enlarge the shape with a scale factor of  $\frac{1}{2}$ , centre  $P$ . (3)



3. (a) Express 45 as a product of its prime factors.

(2)

**Solution**

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$$\begin{array}{r|l} & 45 \\ 3 & 15 \\ 3 & 5 \\ 5 & 1 \\ \hline \end{array}$$

So

$$45 = 3 \times 3 \times 5 = \underline{\underline{3^2 \times 5}}.$$

(b) Find the Highest Common Factor (HCF) of 45 and 30. (2)

**Solution**

$$\begin{array}{r|l} & 30 \\ 2 & 15 \\ 3 & 5 \\ 5 & 1 \\ \hline \end{array}$$

So

$$30 = 2 \times 3 \times 5.$$

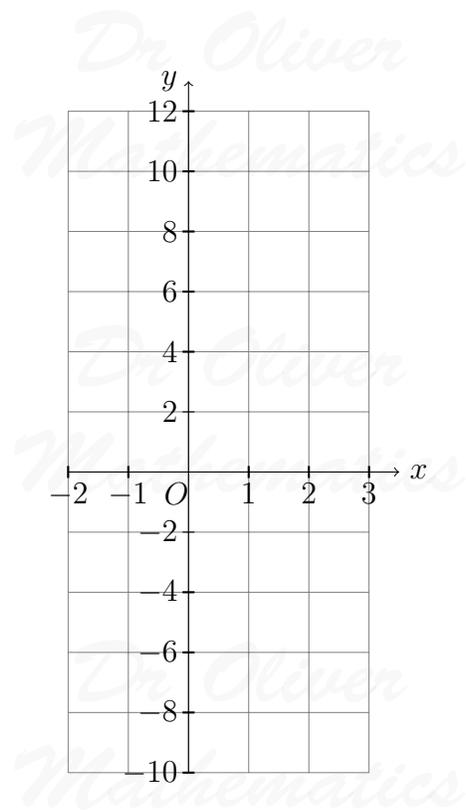
Hence,

$$\text{HCF}(45, 30) = 3 \times 5 = \underline{\underline{15}}.$$

4. On the grid, draw the graph of  $y = 4x - 2$ . (3)

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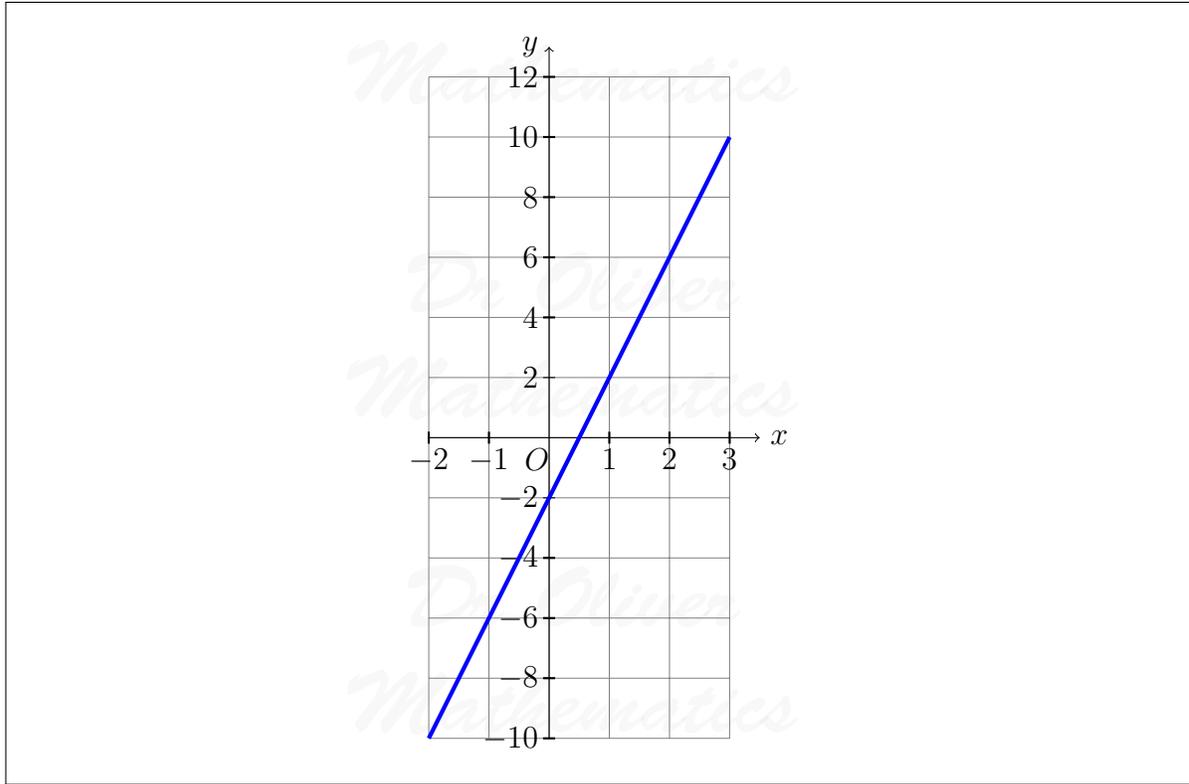


**Solution**

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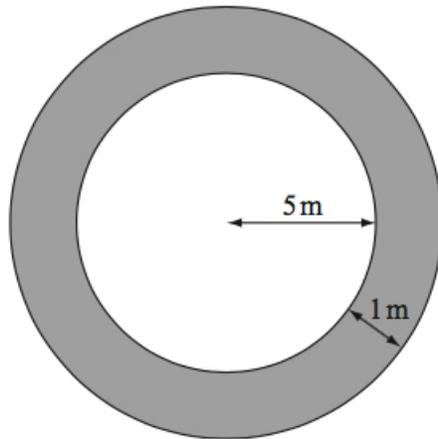
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5. The diagram shows a circular pond with a path around it.

(3)



The pond has a radius of 5 m.

The path has a width of 1 m.

Work out the area of the path.

Give your answer correct to 3 significant figures.

**Solution**

$$\begin{aligned}\text{Area of the path} &= \pi \times 6^2 - \pi \times 5^2 \\ &= (36 - 25)\pi \\ &= 11\pi \\ &= 34.55751919 \text{ (FCD)} \\ &= \underline{\underline{34.6 \text{ cm}^2}} \text{ (3 sf)}.\end{aligned}$$

6. Here are the ages, in years, of 16 people.

36 48 18 25 36 28 45 30  
38 27 41 16 36 48 28 21

- (a) Draw an ordered stem and leaf diagram to show this information. (3)  
You must include a key.

**Solution**

4		1	5	8	8	
3		0	6	6	6	8
2		1	5	7	8	8
1		6	8			

Key: 1|6 means 16 years of age

- (b) Find the median age. (2)

**Solution**

We have two middle numbers – 30 and 36 – and we take the average:

$$\frac{30 + 36}{2} = \underline{\underline{33}}.$$

7. Bob has 120 beads. (3)  
The beads are either red or green.

Bob gives  $\frac{3}{4}$  of the beads to his friend.  
 $\frac{2}{3}$  of the beads Bob now has are red.  
Work out how many green beads Bob now has.

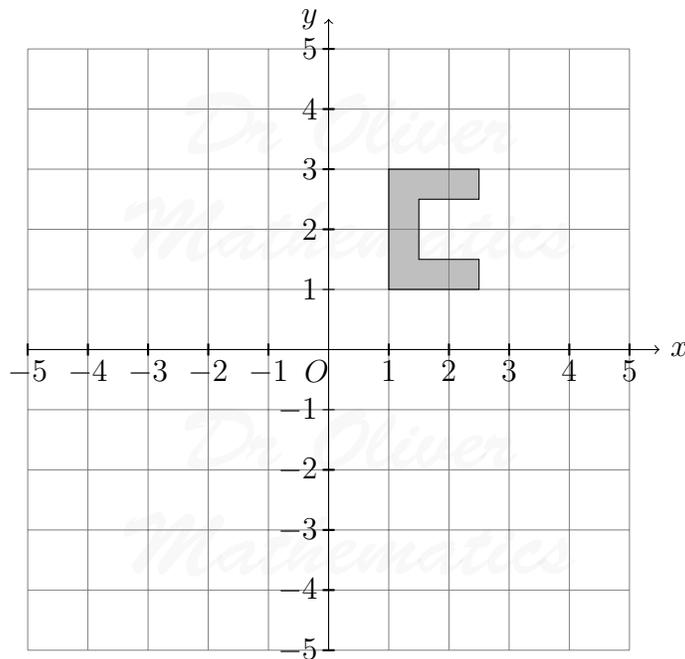
**Solution**

He has  $\frac{1}{4} \times 120 = 30$  beads left.

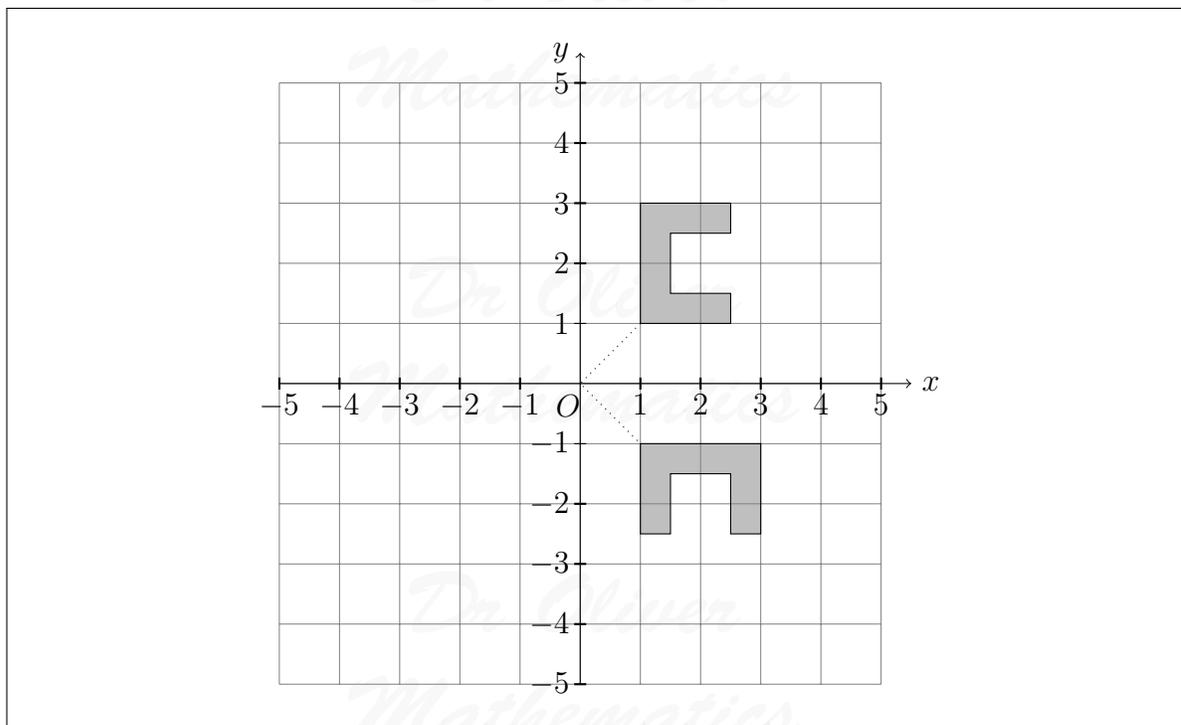
Now, after his friend takes the beads, Bob is left with  $\frac{1}{3} \times 30 = \underline{\underline{10}}$  green.

8. Rotate the shape  $90^\circ$  clockwise, centre  $O$ .

(2)

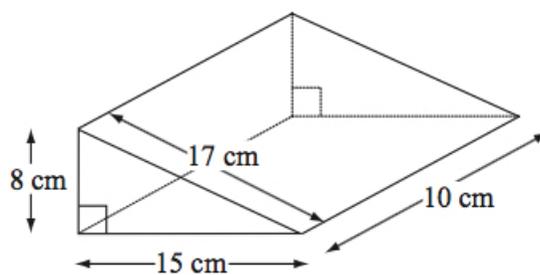


**Solution**



9. Work out the **total** surface area of the triangular prism.

(4)



**Solution**

$$2 \times \frac{1}{2} \times 8 \times 15 = 120$$

$$10 \times 15 = 150$$

$$10 \times 17 = 170$$

$$8 \times 10 = 80.$$

Add them:

$$120 + 150 + 170 + 80 = \underline{\underline{520 \text{ cm}^2}}.$$

10. (a) Simplify

$$6e + 5f + e - 3f.$$

(2)

**Solution**

$$6e + 5f + e - 3f = \underline{\underline{7e + 2f.}}$$

(b) Solve

$$4(2x - 1) = 3x - 19.$$

(3)

**Solution**

$$\begin{aligned} 4(2x - 1) = 3x - 19 &\Rightarrow 8x - 4 = 3x - 19 \\ &\Rightarrow 5x = -15 \\ &\Rightarrow \underline{\underline{x = -3.}} \end{aligned}$$

(c) Solve

$$\frac{y + 4}{5} = 30.$$

(2)

**Solution**

$$\begin{aligned} \frac{y + 4}{5} = 30 &\Rightarrow y + 4 = 150 \\ &\Rightarrow \underline{\underline{y = 146.}} \end{aligned}$$

11. Bianca asked 32 women about the number of children they each had. The table shows information about her results.

Number of children	Frequency
0	9
1	6
2	7
3	8
4	2
more than 4	0

(a) Find the mode.

(1)

**Solution**

0.

(b) Calculate the mean.

(3)

**Solution**

Number of children	Frequency	Number $\times$ Freq
0	9	0
1	6	6
2	7	14
3	8	24
4	2	8
more than 4	0	2
Total	32	52

The mean is

$$\frac{52}{32} = \underline{\underline{1\frac{5}{8}}}.$$

12. The equation

(4)

$$x^3 + 5x = 67$$

has a solution between 3 and 4.

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show **ALL** your working.

**Solution**

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

**F(X)=** and you type in  $X^3 + 5X$ ; then you press  $\boxed{=}$ .

**Start?** and you enter 3; then you press  $\boxed{=}$ .

**End?** and you enter 4; then you press  $\boxed{=}$ .

**Step?** and enter 0.05 – 1 decimal place divided by 2; then you press  $\boxed{=}$ .

$x$	$f(x)$	Comment
3.65	66.877	too low
3.7	69.153	too high

Clearly,

$$3.65 < x < 3.7$$

and the answer is

$$\underline{\underline{x = 3.7 \text{ (1 dp)}}}$$

13. Use your calculator to work out

$$\sqrt{\frac{920 - 170 \tan 65^\circ}{0.012 + 0.034}}$$

- (a) Write down all the figures on your calculator display. (2)  
 You must write your answer as a decimal.

**Solution**

$$\sqrt{\frac{920 - 170 \tan 65^\circ}{0.012 + 0.034}} = \underline{\underline{109.8847047 \text{ (FCD)}}}$$

- (b) Give your answer to part (a) correct to 3 significant figures. (1)

**Solution**

$$\underline{\underline{110 \text{ (3 sf)}}}$$

14. The table shows six expressions:

$$2n - 3 \quad 3n - 2 \quad 3(n + 4) \quad 4n + 1 \quad 4(3n + 1) \quad 2n + 1.$$

$n$  is a positive integer.

- (a) From the table, write the expression whose value is (2)  
 (i) always even,

**Solution**

$$\underline{\underline{4(3n + 1)}}.$$

(ii) always a multiple of 3.

**Solution**

$$\underline{\underline{3(n + 4)}}.$$

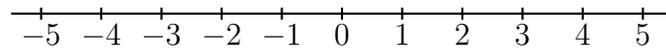
(b) From the table, write the expression which is a factor of  $4n^2 - 1$ .

(1)

**Solution**

$$\underline{\underline{2n + 1}}.$$

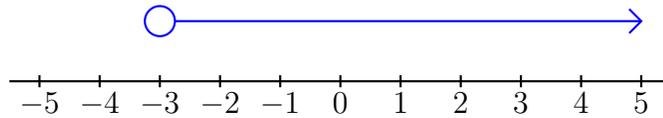
15.  $x > -3$ .



(a) Show this inequality on the number line.

(2)

**Solution**



(b) Solve the inequality

(2)

$$7y + 36 \leq 8.$$

**Solution**

$$\begin{aligned} 7y + 36 \leq 8 &\Rightarrow 7y \leq -28 \\ &\Rightarrow \underline{\underline{y \leq -4}}. \end{aligned}$$

16. In a sale the normal price of a book is reduced by 10%.

(3)

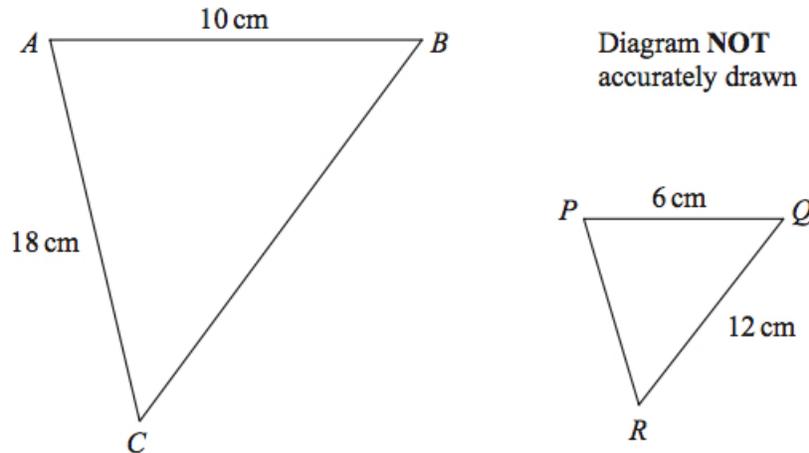
The sale price of the book is £4.86.

Calculate the normal price of the book.

**Solution**

$$\frac{4.86}{1 - 0.1} = \underline{\underline{\pounds 5.40}}$$

17. The diagram shows two similar triangles.



In triangle  $ABC$ ,  $AB = 10$  cm and  $AC = 18$  cm.

In triangle  $PQR$ ,  $PQ = 6$  cm and  $QR = 12$  cm.

Angle  $ABC =$  angle  $PQR$ .

Angle  $CAB =$  angle  $RPQ$ .

(a) Calculate the length of  $BC$ . (2)

**Solution**

$$BC = \frac{10}{6} \times 12 = \underline{\underline{20 \text{ cm}}}.$$

(b) Calculate the length of  $PR$ . (2)

**Solution**

$$PR = \frac{6}{10} \times 18 = \underline{\underline{10.8 \text{ cm}}}.$$

18. (a) Simplify  $(c^2k^5)^4$ . (1)

**Solution**

$$(c^2k^5)^4 = \underline{\underline{c^8k^{20}}}.$$

(b) Expand and simplify

$$(3x + 5)(4x - 1).$$

(2)

**Solution**

×	3x	+5
4x	12x <sup>2</sup>	+20x
-1	-3x	-5

Hence,

$$(3x + 5)(4x - 1) = \underline{\underline{12x^2 + 17x - 5}}.$$

(c) Solve

$$x^2 - 3x - 10 = 0.$$

(3)

**Solution**

$$\left. \begin{array}{l} \text{add to:} \quad -3 \\ \text{multiply to:} \quad -10 \end{array} \right\} -5, +2$$

$$\begin{aligned} x^2 - 3x - 10 = 0 &\Rightarrow (x - 5)(x + 2) = 0 \\ &\Rightarrow x - 5 = 0 \text{ or } x + 2 = 0 \\ &= \underline{\underline{x = -2 \text{ or } x = 5}}. \end{aligned}$$

19. The surface area of Earth is 510 072 000 km<sup>2</sup>.

The surface area of Jupiter is 6.21795 × 10<sup>10</sup> km<sup>2</sup>.

The surface area of Jupiter is greater than the surface area of Earth.

How many times greater?

Give your answer in standard form.

(3)

**Solution**

The surface area of Earth is  $5.10072 \times 10^8 \text{ km}^2$  and

$$\frac{6.21795 \times 10^{10}}{5.10072 \times 10^8} = 121.9033783 \text{ (FCD)}$$
$$= \underline{\underline{1.22 \times 10^2}} \text{ (3 sf).}$$

20. The table shows some expressions.

$a$ ,  $b$ ,  $c$ , and  $d$  represent lengths.

$\pi$  and 2 are numbers that have no dimensions.

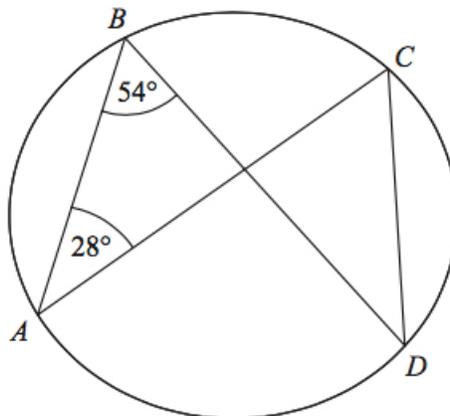
$c^2(b + d)$	$\pi a^2 c^2$	$\frac{a^3 b}{c^3}$	$\pi a^2 b$	$\frac{2a^3 d}{c}$	$d^2$	$2a + b^2$
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Tick ( $\checkmark$ ) the boxes underneath the **three** expressions which could represent volumes.

**Solution**

$c^2(b + d)$	$\pi a^2 c^2$	$\frac{a^3 b}{c^3}$	$\pi a^2 b$	$\frac{2a^3 d}{c}$	$d^2$	$2a + b^2$
$\checkmark$			$\checkmark$	$\checkmark$		

21.  $A$ ,  $B$ ,  $C$ , and  $D$  are points on the circumference of a circle.



Angle  $ABD = 54^\circ$ .

Angle  $BAC = 28^\circ$ .

- (a) Find the size of angle  $ACD$ .

**Solution**

$54^\circ$ .

- (b) Give a reason for your answer.

**Solution**

The angles in the same segment are equal.

22. There are three secondary schools in Banley.

(2)

The table shows the number of students in each of these schools.

Adis College	Greslow High	Fripp School
750	700	900

Germaine takes a sample of 50 students stratified by school.

Work out the number of students from Greslow High in the sample.

**Solution**

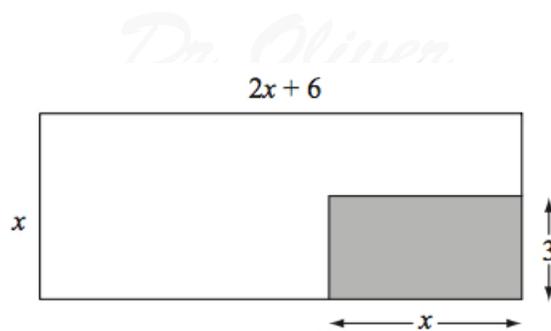
The total number of students is

$$750 + 700 + 900 = 2\,350$$

and the the number of students from Greslow High in the sample is

$$\begin{aligned} \frac{700}{2\,350} \times 50 &= 14\frac{42}{47} \\ &= \underline{\underline{15}} \text{ (nearest whole number)}. \end{aligned}$$

23. The diagram below shows a large rectangle of length  $(2x + 6)$  cm and width  $x$  cm. A smaller rectangle of length  $x$  cm and width 3 cm is cut out and removed.



The area of the shape that is left is  $100 \text{ cm}^2$ .

(a) Show that

$$2x^2 + 3x - 100 = 0.$$

(3)

**Solution**

$$\begin{aligned} x(2x + 6) - 3x &= 100 \Rightarrow 2x^2 + 6x - 3x = 100 \\ &\Rightarrow \underline{\underline{2x^2 + 3x - 100 = 0}}, \end{aligned}$$

as required.

(b) Calculate the length of the smaller rectangle.

Give your answer correct to 3 significant figures.

(4)

**Solution**

$a = 2$ ,  $b = 3$ , and  $c = -100$ :

$$\begin{aligned} x &= \frac{3 \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-100)}}{2 \times 2} \\ &= \frac{-3 \pm \sqrt{809}}{4} \\ &= -7.860\ 731\ 327 \text{ or } 6.360\ 731\ 327 \text{ (FCD);} \end{aligned}$$

hence,  $\underline{\underline{x = 6.36 \text{ cm (3 sf)}}}$ .

24. There are 5 red pens, 3 blue pens, and 2 green pens in a box.

Gary takes at random a pen from the box and gives the pen to his friend.

Gary then takes at random another pen from the box.

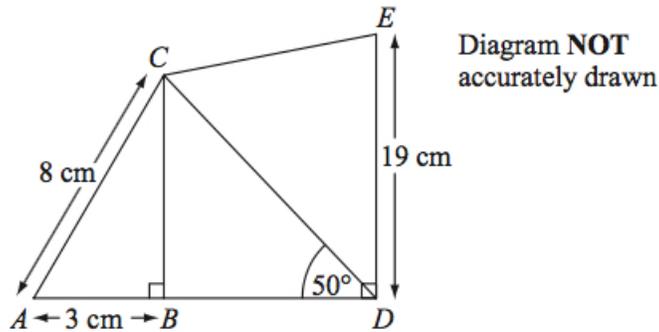
Work out the probability that both pens are the same colour.

(4)

**Solution**

$$\begin{aligned} P(\text{same colour}) &= P(RR) + P(BB) + P(GG) \\ &= \left(\frac{5}{10} \times \frac{4}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{2}{10} \times \frac{1}{9}\right) \\ &= \frac{20}{90} + \frac{6}{90} + \frac{2}{90} \\ &= \frac{28}{90} \\ &= \frac{14}{45} \end{aligned}$$

25. Here is a picture.



$$AC = 8 \text{ cm.}$$

$$AB = 3 \text{ cm.}$$

$$DE = 19 \text{ cm.}$$

$$\text{Angle } ABC = \text{angle } CBD = \text{angle } BDE = 90^\circ.$$

$$\text{Angle } BDC = 50^\circ.$$

(a) Calculate the length of  $CD$ .

(4)

Give your answer correct to 3 significant figures.

**Solution**

$$\begin{aligned} BC &= \sqrt{AC^2 - AB^2} \\ &= \sqrt{8^2 - 3^2} \\ &= \sqrt{55}. \end{aligned}$$

Finally,

$$\begin{aligned}\text{hyp} &= \frac{\text{opp}}{\sin} \Rightarrow CD = \frac{\sqrt{55}}{\sin 50^\circ} \\ &\Rightarrow CD = 9.681\,159\,564 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{CD = 9.68 \text{ cm (3 sf)}}}.\end{aligned}$$

(b) Calculate the length of  $CE$ .

Give your answer correct to 3 significant figures.

(3)

**Solution**

$\angle CDE = 90 - 50 = 40^\circ$ . Now,

$$\begin{aligned}CE &= \sqrt{CD^2 + DE^2 - 2 \cdot CD \cdot CE \cdot \cos CDE} \\ &= \sqrt{9.681\dots^2 + 19^2 - 2 \cdot 9.681\dots \cdot 19 \cdot \cos 40^\circ} \\ &= 13.149\,498\,39 \text{ (FCD)} \\ &= \underline{\underline{13.1 \text{ cm (3 sf)}}}.\end{aligned}$$

26. The voltage  $V$  of an electronic circuit is given by the formula

$$V = IR$$

(3)

where  $I$  is the current in amps and  $R$  is the resistance in ohms.

Given that  $V = 218$ , correct to 3 significant figures,  $R = 12.6$ , correct to 3 significant figures, calculate the lower bound of  $I$ .

**Solution**

$$217.5 \leq V < 218.5$$

and

$$12.55 \leq R < 12.65.$$

Finally, the lower bound of  $I$  is

$$\begin{aligned}\frac{217.5}{12.65} &= 17.193\,675\,89 \text{ (FCD)} \\ &= \underline{\underline{17.2 \text{ amps (3 sf)}}}.\end{aligned}$$