

Dr Oliver Mathematics

Worked Examples

Right-Angled Triangle 1

1. A right-angled triangle ABC is drawn, as shown in Figure 1.

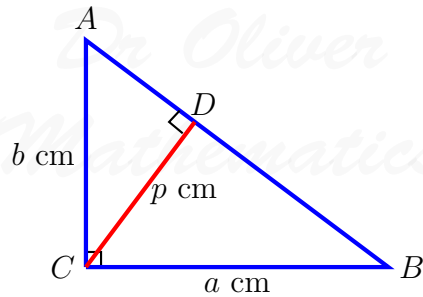


Figure 1: a right-angled triangle ABC

- Let $BC = a$ cm.
- Let $AC = b$ cm.
- Let $\angle ACB = 90^\circ$.
- The line CD is perpendicular to the line AB .

Prove

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

Solution

Well, we know that

$$\text{area of } ABC = \frac{1}{2} \times AC \times BC$$

and

$$\text{area of } ABC = \frac{1}{2} \times CD \times AB.$$

Now, the two area are equal:

$$\begin{aligned} \frac{1}{2} \times AC \times BC &= \frac{1}{2} \times CD \times AB \\ \Rightarrow AC \times BC &= CD \times AB \\ \Rightarrow (AC \times BC)^2 &= (CD \times AB)^2 \\ \Rightarrow AC^2 \times BC^2 &= CD^2 \times AB^2 \\ \Rightarrow AC^2 \times BC^2 &= CD^2 \times (AC^2 + BC^2) \\ \Rightarrow a^2 b^2 &= p^2 (a^2 + b^2) \end{aligned}$$

divide each side by $a^2b^2p^2$:

$$\begin{aligned}\Rightarrow \frac{1}{p^2} &= \frac{a^2 + b^2}{a^2b^2} \\ \Rightarrow \frac{1}{p^2} &= \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2} \\ \Rightarrow \frac{1}{p^2} &= \frac{1}{a^2} + \frac{1}{b^2},\end{aligned}$$

as required.