Dr Oliver Mathematics
Worked Examples
Radius of a Circle 2

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1. A square, with side length x cm, is drawn.

   A circle, with radius r cm is drawn as follows: the circle is tangent to the bottom and left sides of the square and passes through its top-right corner, as shown in Figure 2.

   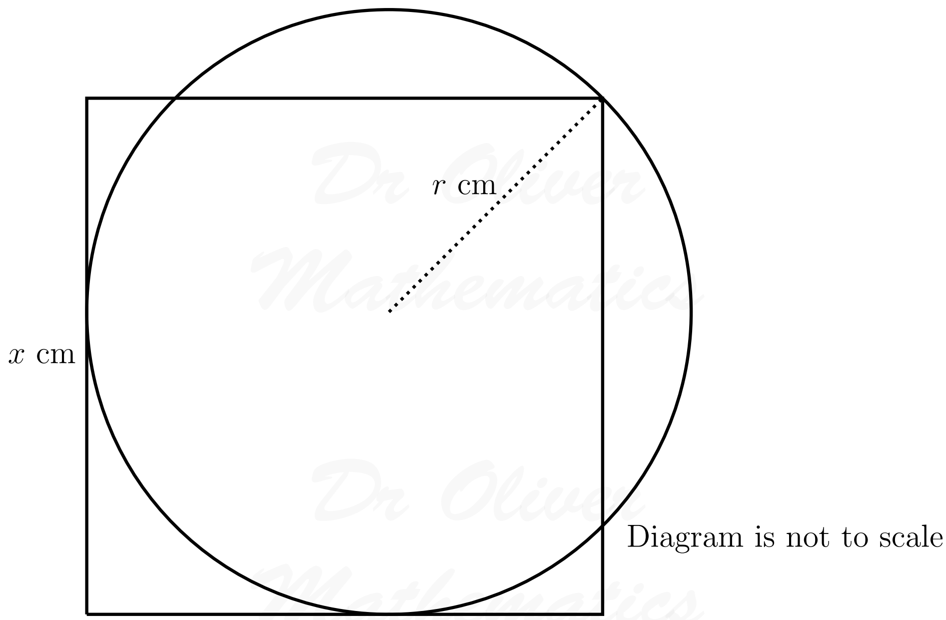

   Figure 1: a square and a circle

   Find the length of the radius of the circle.

   Solution

   Let O be the centre of the circle. We add in the new dimensions:

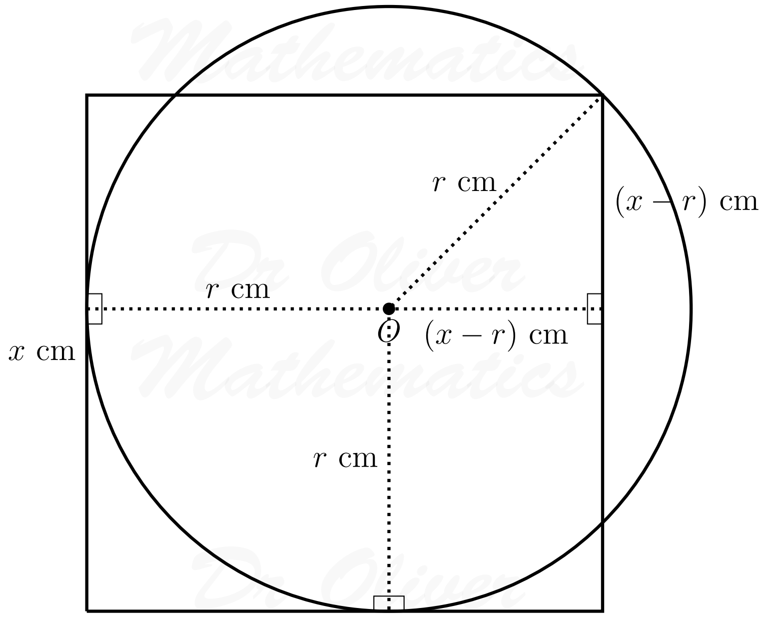

Figure 2: with the dimension added on

Pythagoras' Theorem:

$$\begin{aligned}
(x-r)^2 + (x-r)^2 = r^2 &\Rightarrow 2(x-r)^2 = r^2 \\
&\Rightarrow 2(x^2 - 2rx + r^2) = r^2 \\
&\Rightarrow 2x^2 - 4rx + 2r^2 = r^2 \\
&\Rightarrow 2x^2 - 4rx + r^2 = 0 \\
&\Rightarrow r^2 - 4rx = -2x^2 \\
&\Rightarrow r^2 - 4rx + (2x)^2 = -2x^2 + (2x)^2 \\
&\Rightarrow (r-2x)^2 = -2x^2 + 4x^2 \\
&\Rightarrow (r-2x)^2 = 2x^2 \\
&\Rightarrow r - 2x = \pm x\sqrt{2} \\
&\Rightarrow r = 2x \pm x\sqrt{2} \\
&\Rightarrow r = (2 \pm \sqrt{2})x.
\end{aligned}$$

$r = (2+\sqrt{2})x$? Look at picture: $(x-r)$ is a *length* which means it is bigger than zero:

$$x - r > 0 \Rightarrow x > r.$$

So $r \neq (2+\sqrt{2})x$. Hence, we need the other solution:

$$\underline{\underline{r = (2-\sqrt{2})x}}.$$