Dr Oliver Mathematics
Mathematics: National Qualifications N5
2015 Paper 1: Non-Calculator
1 hour

The total number of marks available is 40.
You must write down all the stages in your working.

1. Evaluate (2)
$$6\tfrac{1}{5} - 2\tfrac{1}{3}.$$

Solution

$$6\tfrac{1}{5} - 2\tfrac{1}{3} = 4 + \tfrac{3}{15} - \tfrac{5}{15}$$
$$= 4 - \tfrac{2}{15}$$
$$= 3\tfrac{13}{15}.$$

2. Solve algebraically the inequality (3)
$$11 - 2(1 + 3x) < 39.$$

Solution

$$11 - 2(1 + 3x) < 39 \Rightarrow -2(1 + 3x) < 28$$
$$\Rightarrow 1 + 3x > -14$$
$$\Rightarrow 3x > -15$$
$$\Rightarrow \underline{\underline{x > -5}}.$$

3. AC is a tangent to the circle, centre O, with point of contact B. (3)
DE is a diameter of the circle and F is a point on the circumference.
Angle ABD is $77°$ and angle DEF is $64°$.
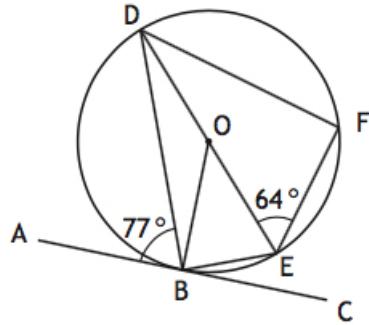
Calculate the size of angle BDF.

Solution

$\angle DEB = 77°$ (alternate segment theorem) and

$$\angle BDF = 180 - \angle BEF \text{ (supplementary)}$$
$$= 180 - (64 + 77)$$
$$= 180 - 141$$
$$= \underline{\underline{39°}}$$

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4. Multiply out the brackets and collect like terms (3)

$$(x - 4)(x^2 + x - 2).$$

Solution

$\times$	x^2	$+x$	-2
x	x^3	$+x^2$	$-2x$
-4	$-4x^2$	$-4x$	$+8$

Hence,

$$(x - 4)(x^2 + x - 2) = \underline{\underline{x^3 - 3x^2 - 6x + 8}}.$$

5. The standard deviation of 1, 2, 2, 2, and 8 is equal to $\sqrt{a}$. (3)
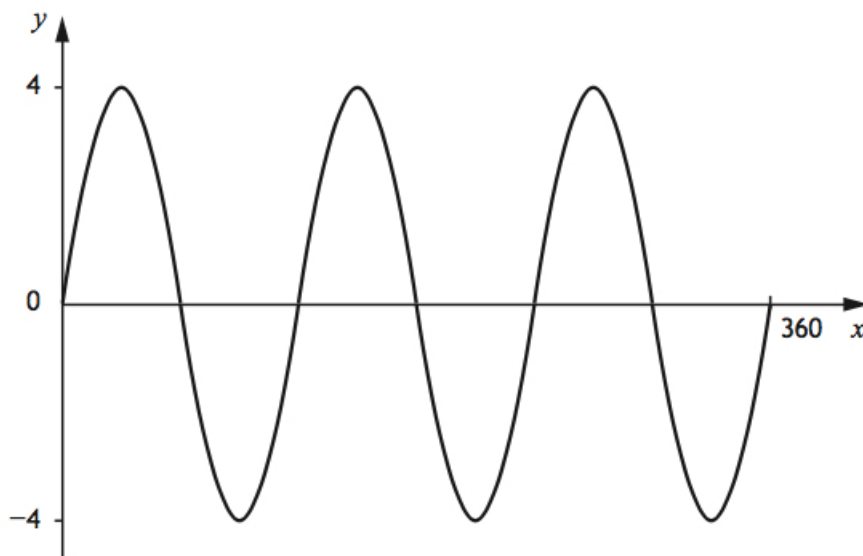
   Find the value of a.

Solution

x	x^2
1	1
2	4
2	4
2	4
8	64
$\sum x = 15$	$\sum x^2 = 77$

Hence,

$$\sqrt{a} = \sqrt{\frac{77 - (15)^2/5}{4}} \Rightarrow a = \frac{77 - (15)^2/5}{4}$$

$$\Rightarrow a = \frac{77 - 225/5}{4}$$

$$\Rightarrow a = \frac{77 - 45}{4}$$

$$\Rightarrow a = \frac{32}{4}$$

$$\Rightarrow \underline{\underline{a = 8}}.$$

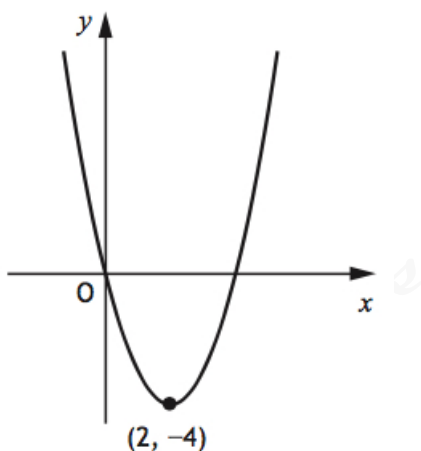6. Part of the graph of $y = a \sin bx°$ is shown in the diagram. (2)

3

State the values of a and b.

> **Solution**
>
> $\underline{\underline{a = 4}}$ (amplitude) and $\underline{\underline{b = 3}}$ (three graphs in the space of one).

7. The graph below shows part of the parabola with equation of the form

$$y = (x + a)^2 + b.$$

The minimum turning point $(2, -4)$ is shown in the diagram.

(a) State the values of

    (i) a,   (1)

> **Solution**
>
> $\underline{\underline{a = -2}}$.

    (ii) b.   (1)

> **Solution**
>
> $\underline{\underline{b = -4}}$.

(b) Write down the equation of the axis of symmetry of the graph.   (1)

> **Solution**
>
> $\underline{\underline{x = 2}}$.

8. Find the equation of the line joining the points $(-2, 5)$ and $(3, 15)$.   (3)

Give the equation in its simplest form.

> **Solution**
>
> $$\text{Gradient} = \frac{15 - 5}{3 - (-2)}$$
> $$= \frac{10}{5}$$
> $$= 2$$
>
> and the equation is
>
> $$y - 15 = 2(x - 3) \Rightarrow y - 15 = 2x - 6$$
> $$\Rightarrow \underline{\underline{y = 2x + 9}}.$$

9. Write the following in order of size starting with the smallest.   (2)

$$\cos 90° \quad \cos 100° \quad \cos 300°.$$

Justify your answer.

10. Ten couples took part in a dance competition.
    The couples were given a score in each round.
    The scores in the first round were

$$16 \quad 27 \quad 12 \quad 18 \quad 26 \quad 21 \quad 27 \quad 22 \quad 18 \quad 17$$

(a) Calculate the median and semi-interquartile range of these scores. (3)

Solution

$$
\begin{array}{c|ccccc}
1 & 2 & 6 & 7 & 8 & 8 \\
2 & 1 & 2 & 6 & 7 & 7 \\
\end{array}
$$

$$
\begin{aligned}
\text{Median} &= \frac{18 + 21}{2} \\
&= \frac{39}{2} \\
&= \underline{\underline{19\tfrac{1}{2}}}
\end{aligned}
$$

and

$$
\begin{aligned}
\text{SIQR} &= \tfrac{1}{2}\left[\tfrac{3}{4}(10 + 1)\text{th entry} - \tfrac{1}{4}(10 + 1)\text{th entry}\right] \\
&= \tfrac{1}{2}\left[8\tfrac{1}{4}\text{th entry} - 2\tfrac{3}{4}\text{th entry}\right] \\
&= \tfrac{1}{2}\left[8\text{th entry} - 3\text{th entry}\right] \\
&= \tfrac{1}{2}\left[26 - 17\right] \\
&= \tfrac{1}{2} \times 9 \\
&= \underline{\underline{4\tfrac{1}{2}}}.
\end{aligned}
$$

In the second round, the median was 26 and the semi-interquartile range was 2.5.

(b) Make two valid comparisons between the scores in the first and second rounds. (2)

> **Solution**
>
> <u>Median</u>: As the median in the first round was $19\frac{1}{2}$ and the the median in the first round was 26, the median has gone <u>up in the second round</u>.
>
> <u>SIQR</u>: As the SIQR in the first round was $4\frac{1}{2}$ and the the SIQR in the first round was 2.5, the SIQR has gone down in the second round, i.e., <u>they are less spread out during the second round</u>.

11. Solve algebraically the system of equations (3)

$$3x + 2y = 17$$
$$2x + 5y = 4.$$

> **Solution**
>
> $$3x + 2y = 17 \quad (1)$$
> $$2x + 5y = 4 \quad (2)$$
>
> Now, e.g.,
>
> $$5 \times (1): \quad 15x + 10y = 85 \quad (3)$$
> $$2 \times (2): \quad 4x + 10y = 8 \quad (4)$$
>
> Finally, $(3) - (4)$:
>
> $$11x = 77 \Rightarrow \underline{x = 7}$$
> $$\Rightarrow 21 + 2y = 17$$
> $$\Rightarrow 2y = -4$$
> $$\Rightarrow \underline{\underline{y = -2}}.$$

12. Simplify (3)

$$\frac{x^2 - 4x}{x^2 + x - 20}.$$

Solution

$$\begin{array}{r} \text{add to:} \quad +1 \\ \text{multiply to:} \quad -20 \end{array} \Big\} + 5, \, -4$$

$$\frac{x^2 - 4x}{x^2 + x - 20} = \frac{x(x-4)}{(x+5)(x-4)}$$
$$= \underline{\underline{\frac{x}{x+5}}}.$$

13. Express (3)

$$\frac{4}{\sqrt{8}}$$

with a rational denominator.
Give your answer in its simplest form.

Solution

$$\frac{4}{\sqrt{8}} = \frac{4}{\sqrt{4 \times 2}}$$
$$= \frac{4}{\sqrt{4} \times \sqrt{2}}$$
$$= \frac{4}{2\sqrt{2}}$$
$$= \frac{2}{\sqrt{2}}$$
$$= \underline{\underline{\sqrt{2}}}.$$

14. Evaluate (2)

$$8^{\frac{5}{3}}.$$

Solution

$$8^{\frac{5}{3}} = (8^{\frac{1}{3}})^5$$
$$= 2^5$$
$$= \underline{\underline{32}}.$$