Dr Oliver Mathematics
Mathematics: Advanced Higher
2026 Paper 1: Non-Calculator
1 hour

The total number of marks available is 35.
You must write down all the stages in your working.

1. Differentiate:

   (a) $y = 3x^4 \sec 2x$, (2)

Solution
Product rule:

$$u = 3x^4 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 12x^3$$
$$v = \sec 2x \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = 2 \sec 2x \tan 2x,$$

and so

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= (3x^4)(2 \sec 2x \tan 2x) + (12x^3)(\sec 2x) \\ &= \underline{\underline{6x^4 \sec 2x \tan 2x + 12x^3 \sec 2x}}.\end{aligned}$$

   (b) $\mathrm{f}(x) = \dfrac{\mathrm{e}^{5x}}{2x+1}$. (3)

Solution
Quotient rule:

$$u = \mathrm{e}^{5x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 5\mathrm{e}^{5x}$$
$$v = 2x + 1 \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = 2,$$

and so

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{(2x+1)(5\mathrm{e}^{5x}) - (2)(\mathrm{e}^{5x})}{(2x+1)^2} \\ &= \frac{5(2x+1)\mathrm{e}^{5x} - 2\mathrm{e}^{5x}}{(2x+1)^2} \\ &= \frac{[10x+5-2]\mathrm{e}^{5x}}{(2x+1)^2} \\ &= \underline{\underline{\frac{(10x+3)\mathrm{e}^{5x}}{(2x+1)^2}}}.\end{aligned}$$

2. A system of equations is given by (4)

$$\begin{aligned} x + y - z &= 9 \\ 2x - y + z &= -2 \\ 3x + 2y - 2z &= 21. \end{aligned}$$

Use Gaussian elimination to solve this system of equations.

Solution

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 2 & -1 & 3 & -2 \\ 3 & 2 & -2 & 21 \end{array}\right)$$

Do $R_2 - 2R_1$ and $R_3 - 3R_1$:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & -3 & 5 & -20 \\ 0 & -1 & 1 & -6 \end{array}\right)$$

Do $R_3 - \frac{1}{3}R_1$:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & -3 & 3 & -20 \\ 0 & 0 & -\frac{2}{3} & \frac{2}{3} \end{array}\right)$$

Now,

$$-\tfrac{2}{3}z = \tfrac{2}{3} \Rightarrow \underline{\underline{z = -1}},$$

$$\begin{aligned} -3y + 5(-1) = -20 &\Rightarrow -3y - 5 = -20 \\ &\Rightarrow -3y = -15 \\ &\Rightarrow \underline{\underline{y = 5}}, \end{aligned}$$

and

$$\begin{aligned} x + 5 - (-1) = 9 &\Rightarrow x + 6 = 9 \\ &\Rightarrow \underline{\underline{x = 3}}. \end{aligned}$$

3. A complex number is defined by

$$z = \sqrt{3} + \mathrm{i}.$$

(a) Express z in polar form. (2)

Solution
The modulus is

$$\sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

and

$$\begin{aligned} \text{argument} &= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\ &= \tfrac{1}{6}\pi. \end{aligned}$$

Hence,

$$z = \sqrt{3} + \mathrm{i} = \underline{\underline{2(\cos\tfrac{1}{6}\pi + \mathrm{i}\sin\tfrac{1}{6}\pi)}}.$$

(b) Use de Moivre's theorem to show that z^3 is purely imaginary. (2)

Solution
Now,

$$\begin{aligned} z^3 &= \left[2(\cos\tfrac{1}{6}\pi + \mathrm{i}\sin\tfrac{1}{6}\pi)\right]^3 \\ &= 8(\cos\tfrac{1}{2}\pi + \mathrm{i}\sin\tfrac{1}{2}\pi) \\ &= \underline{\underline{8\mathrm{i}}}; \end{aligned}$$

hence, z^3 is $\underline{\underline{\text{purely imaginary}}}$.

4. Find the particular solution of the differential equation (5)

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 3\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0,$$

given that $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$ and $x = 0$.

Solution

Auxiliary Equation:

$$\left.\begin{array}{ll} \text{add to:} & -3 \\ \text{multiply to:} & (+2) \times (+1) = +2 \end{array}\right\} -1,\ -2$$

and so

$$\begin{aligned} 2m^2 - 3m + 1 = 0 &\Rightarrow 2m^2 - m - 2m + 1 = 0 \\ &\Rightarrow m(2m-1) - 1(2m-1) = 0 \\ &\Rightarrow (m-1)(2m-1) = 0 \\ &\Rightarrow m - 1 = 0 \text{ or } 2m - 1 = 0 \\ &\Rightarrow m = 1 \text{ or } m = \tfrac{1}{2}; \end{aligned}$$

hence,

$$\text{CF} : y = A\mathrm{e}^{x} + B\mathrm{e}^{\frac{1}{2}x}.$$

Now,

$$y = A\mathrm{e}^{x} + B\mathrm{e}^{\frac{1}{2}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = A\mathrm{e}^{x} + \tfrac{1}{2}B\mathrm{e}^{\frac{1}{2}x}$$

and

$$\begin{aligned} x = 0, y = 2 &\Rightarrow \boxed{2 = A + B \quad (1)} \\ x = 0, \frac{\mathrm{d}y}{\mathrm{d}x} = -1 &\Rightarrow \boxed{-1 = A + \tfrac{1}{2}B \quad (2)}. \end{aligned}$$

Do (1) − (2):

$$\begin{aligned} 3 = \tfrac{1}{2}B &\Rightarrow B = 6 \\ &\Rightarrow A = -4; \end{aligned}$$

hence,

$$\underline{\underline{y = -4\mathrm{e}^{x} + 6\mathrm{e}^{\frac{1}{2}x}}}.$$

5. Matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{pmatrix} 3 & 5 \\ -2 & x \end{pmatrix}.$$

(a) State an expression for the determinant of $\mathbf{A}$ in terms of x. (1)

Solution

Well,

$$\begin{aligned}\det\ \mathbf{A} &= (3)(x) - (5)(-2)\\ &= \underline{\underline{3x + 10}}.\end{aligned}$$

Matrix $\mathbf{A}$ is multiplied by matrix $\mathbf{B}$ such that

$$\det(\mathbf{AB}) = 12x + 40.$$

(b) State the determinant of $\mathbf{B}$. (1)

Solution

Now,

$$\begin{aligned}\det\ (\mathbf{AB}) = \det\ \mathbf{A} \times \det\ \mathbf{B} &\Rightarrow 12x + 40 = (3x + 10) \times \det\ \mathbf{B}\\ &\Rightarrow \det\ \mathbf{B} = \frac{4(3x + 10)}{3x + 10)}\\ &\Rightarrow \underline{\underline{\det\ \mathbf{B} = 4}}.\end{aligned}$$

The inverse of matrix $\mathbf{B}$ is

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & -1 \\ -\frac{5}{4} & \frac{3}{2} \end{pmatrix}.$$

(c) Find matrix $\mathbf{B}$. (2)

Solution

Well,

$$\begin{aligned}\mathbf{B} &= 4\begin{pmatrix} \frac{3}{2} & 1 \\ \frac{5}{4} & 1 \end{pmatrix}\\ &= \underline{\underline{\begin{pmatrix} 6 & 4 \\ 5 & 4 \end{pmatrix}}}.\end{aligned}$$

6. (a) Use the substitution (3)

$$u = x - 1$$

to find

$$\int x(x-1)^4 \, \mathrm{d}x.$$

Solution

Well,

$$u = x - 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \Rightarrow \mathrm{d}u = \mathrm{d}x$$

and

$$\begin{aligned} \int x(x-1)^4 \, \mathrm{d}x &= \int (u+1)u^4 \, \mathrm{d}u \\ &= \int (u^5 + u^4) \, \mathrm{d}u \\ &= \tfrac{1}{6}u^6 + \tfrac{1}{5}u^5 + c \\ &= \underline{\underline{\tfrac{1}{6}(x-1)^6 + \tfrac{1}{5}(x-1)^5 + c}}. \end{aligned}$$

(b) Hence find the exact volume of the solid formed by rotating the curve with equation (4)

$$y = 2\sqrt{x}(x-1)^2,$$

about the x-axis through 2π radians, from $x = 0$ to $x = 1$.

Solution

Now,

$$\begin{aligned} \text{volume} &= \pi \int_0^1 y^2 \, \mathrm{d}x \\ &= \pi \int_0^1 [2\sqrt{x}(x-1)^2]^2 \, \mathrm{d}x \\ &= \pi \int_0^1 4x(x-1)^4 \, \mathrm{d}x \\ &= 4\pi \int_0^1 x(x-1)^4 \, \mathrm{d}x \\ &= 4\pi \left[\tfrac{1}{6}(x-1)^6 + \tfrac{1}{5}(x-1)^5\right]_{x=0}^{1} \\ &= 4\pi \left[(0+0) - \left(-\tfrac{1}{6} + \tfrac{1}{5}\right)\right] \\ &= \underline{\underline{\tfrac{2}{15}}}\pi. \end{aligned}$$

7. The complex number

$$z = 2 + \mathrm{i}$$

is a root of the polynomial equation

$$z^4 - 2z^3 - z^2 + 2z + 10 = 0.$$

(a) State a second root of the equation. (1)

Solution

It is the complex conjugate: $\underline{\underline{z = 2 - \mathrm{i}}}$.

(b) Find the remaining roots. (5)

Solution

Well,

$\times$	z	-2	$-\mathrm{i}$
z	z^2	$-2z$	$-z\mathrm{i}$
-2	$-2z$	$+4$	$+2\mathrm{i}$
$+\mathrm{i}$	$+z\mathrm{i}$	$-2\mathrm{i}$	$+1$

and so

$$(z - 2 - \mathrm{i})(z - 2 + \mathrm{i}) = z^2 - 4z + 5.$$

Now,

$$\begin{array}{r|rrrrr}
 & & & z^2 & +2z & +2 \\
\cline{2-6}
z^2 - 4z + 5 & z^4 & -2z^3 & -z^2 & +2z & +10 \\
 & z^4 & -4z^3 & +5z^2 & & \\
\cline{2-6}
 & & 2z^3 & -6z^2 & +2z & +10 \\
 & & 2z^3 & -8z^2 & +10z & \\
\cline{3-6}
 & & & 2z^2 & -8z & +10 \\
 & & & 2z^2 & -8z & +10 \\
\cline{4-6}
 & & & & & 0
\end{array}$$

and so

$$\frac{z^4 - 2z^3 - z^2 + 2z + 10}{z^2 - 4z + 5} = z^2 + 2z + 2.$$

We complete the square:

add to:	$+2$
half it:	$+1$
square it:	$(+1)^2 = +1$

and

$$\begin{aligned} z^2 + 2z + 2 = 0 &\Rightarrow z^2 + 2z + 1 = -1 \\ &\Rightarrow (z+1)^2 = -1 \\ &\Rightarrow z + 1 = \pm \mathrm{i} \\ &\Rightarrow \underline{\underline{z = -1 \pm \mathrm{i}}}. \end{aligned}$$