Dr Oliver Mathematics Cambridge O Level Additional Mathematics 2005 June Paper 1: Calculator 2 hours

The total number of marks available is 80. You must write down all the stages in your working.

1. Given that

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix},$$

(4)

(5)

find $(A^2)^{-1}$.

2. A student has a collection of 9 CDs, of which 4 are by the Beatles, 3 are by Abba, and 2 are by the Rolling Stones.

She selects 4 of the CDs from her collection.

Calculate the number of ways in which she can make her selection if

- (a) her selection must contain her favourite Beatles CD, (2)
- (b) her selection must contain 2 CDs by one group and 2 CDs by another. (3)
- 3. Given that θ is acute and that

$$\sin\theta = \frac{1}{\sqrt{3}},$$

express, without using a calculator,

$$\frac{\sin\theta}{\cos\theta - \sin\theta}$$

in the form $a + \sqrt{b}$, where a and b are integers.

4. The position vectors of points A and B relative to an origin O are $-3\mathbf{i} - \mathbf{j}$ and $\mathbf{i} + 2\mathbf{j}$ (6) respectively.

The point C lies on AB and is such that $\overrightarrow{AC} = \frac{3}{5}\overrightarrow{AB}$.

Find the position vector of C and show that it is a unit vector.

5. The function f is defined, for $0^{\circ} \leq x \leq 180^{\circ}$, by

$$\mathbf{f}(x) = A + 5\cos Bx,$$

where A and B are constants.

(a) Given that the maximum value of f is 3, state the value of A .	(1)
(b) State the amplitude of f.	(1)
(c) Given that the period of f is 120° , state the value of B.	(1)
(d) Sketch the graph of f.	(3)

- 6. Given that each of the following functions is defined for the domain $-2 \leq x \leq 3$, find the range of
 - (a) $f: x \mapsto 2 3x$, (1)
 - (b) $g: x \mapsto |2 3x|,$ (2)
 - (c) $h: x \mapsto 2 |3x|$.
 - (d) State which of the functions f, g, and h has an inverse.
- 7. Variables l and t are related by the equation

$$l = l_0 (1 + \alpha)^t,$$

where l_0 and α are constants.

- (a) Given that $l_0 = 0.64$ and $\alpha = 2.5 \times 10^{-3}$, find the value of t for which l = 0.66. (3)
- (b) Solve the equation

$$1 + \log(8 - x) = \log(3x + 2)$$

8. The table below shows experimental values of the variables x and y which are related by an equation of the form

$$y = kx^n$$
,

where k and n are constants.

x	10	100	1000	10000
y	1 900	250	31	4

- (a) Using draw the graph of $\log y$ against $\log x$.
- (b) Use your graph to estimate the value of k and of n.
- 9. (a) Determine the set of values of k for which the equation (5)

$$x^2 + 2x + k = 3kx - 1$$

has no real roots.

(b) Hence state, giving a reason, what can be deduced about the curve $y = (x + 1)^2$ (2)and the line y = 3x - 1. $Mathematics_2$

(3)

(4)

(2)

(2)

(4)

10. The remainder when

$2x^3 + 2x^2 - 13x + 12$

is divided by (x + a) is three times the remainder when it is divided by (x - a).

(a) Show that

$$2a^3 + a^2 - 13a + 6 = 0.$$

- (b) Solve this equation completely.
- 11. A particle travels in a straight line so that, t seconds after passing a fixed point A on the line, its acceleration, $a \text{ ms}^{-2}$, is given by

$$a = -2 - 2t.$$

It comes to rest at a point B when t = 4.

- (a) Find the velocity of the particle at A.
 (b) Find the distance AB.
 (3)
- (c) Sketch the velocity-time graph for the motion from A to B. (1)

EITHER

12. The diagram, which is not drawn to scale, shows part of the graph of (10)

$$y = 8 - e^{2x},$$

crossing the y-axis at A.



The tangent to the curve at A crosses the x-axis at B.

Find the area of the shaded region bounded by the curve, the tangent, and the *x*-axis. **OR**

(5)

(3)

13. A piece of wire, of length 2 m, is divided into two pieces.

One piece is bent to form a square of side x m and the other is bent to form a circle of radius r m.

(a) Express r in terms of x and show that the total area, $A m^2$, of the two shapes is (4) given by

$$A = \frac{(\pi + 4)x^2 - 4x + 1}{\pi}.$$

Given that x can vary, find

- (b) the stationary value of A,
- (c) the nature of this stationary value.

(2)

(4)







