

Dr Oliver Mathematics

Modulus Function

Core Mathematics 3, Chapter 5

The modulus function

The **modulus** (or **absolute value**) function, denoted by $|x|$, is defined to be

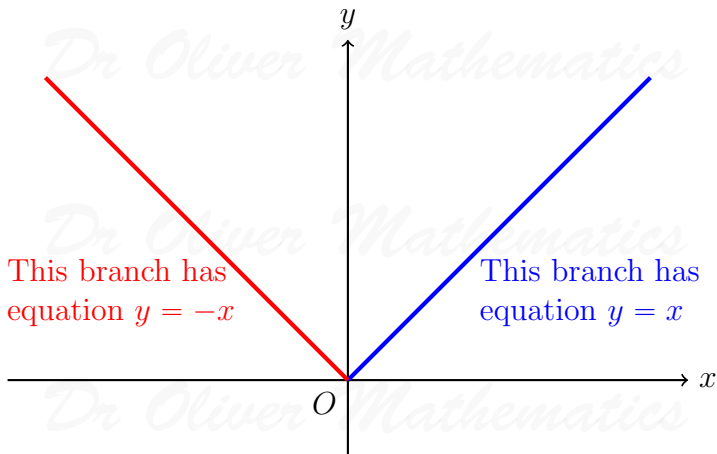
$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

So

$$|14.5| = 14.5 \text{ and } |-8.13| = 8.13.$$

Writing the function in the way that we have is called a **piecewise function** and this can be very useful when working with the modulus function.

The graph of $y = |x|$



Express $|x + 2|$ as a piecewise function

When $x + 2 \geq 0$ then

$$|x + 2| = x + 2$$

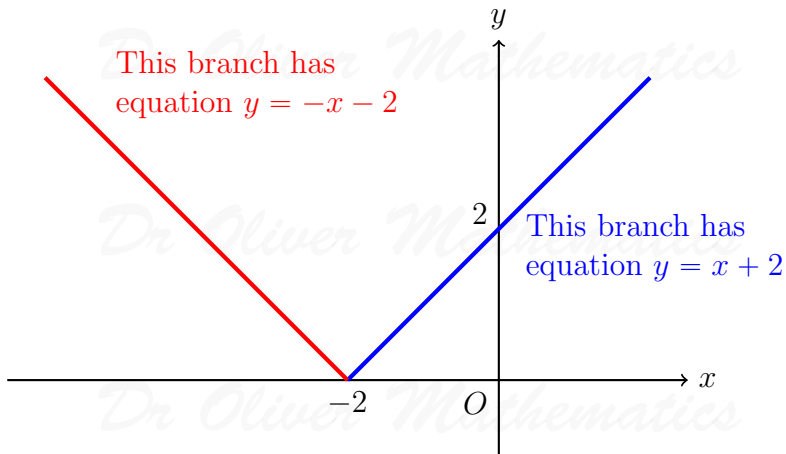
and when $x + 2 < 0$ then

$$|x + 2| = -(x + 2) = -x - 2.$$

Hence

$$|x + 2| = \begin{cases} x + 2 & \text{if } x \geq -2, \\ -x - 2 & \text{if } x < -2. \end{cases}$$

The graph of $y = |x + 2|$



Express $|3 - x|$ as a piecewise function

When $3 - x \geq 0$ then

$$|3 - x| = 3 - x$$

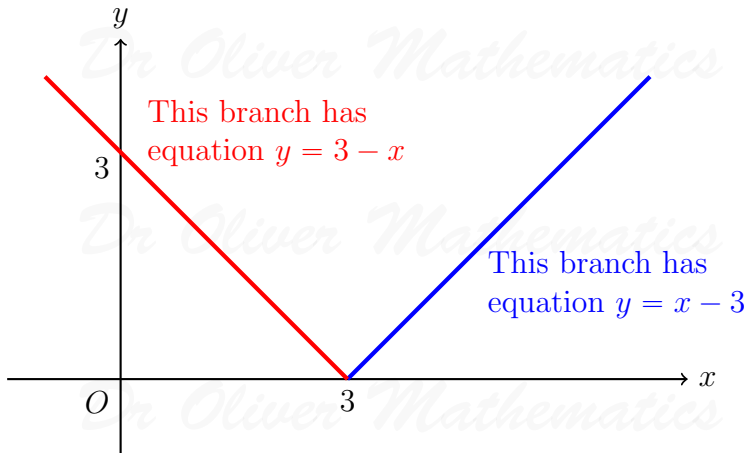
and when $3 - x < 0$ then

$$|3 - x| = -(3 - x) = x - 3.$$

Hence

$$|3 - x| = \begin{cases} 3 - x & \text{if } x \leq 3, \\ x - 3 & \text{if } x > 3. \end{cases}$$

The graph of $y = |3 - x|$



Express $|x + 2| + |x + 5|$ as a piecewise function

$$|x + 2| = \begin{cases} x + 2 & \text{if } x \geq -2, \\ -x - 2 & \text{if } x < -2, \end{cases}$$

and

$$|x + 5| = \begin{cases} x + 5 & \text{if } x \geq -5, \\ -x - 5 & \text{if } x < -5. \end{cases}$$

Hence

$$|x + 2| + |x + 5| = \begin{cases} -2x - 7 & \text{if } x < -5, \\ 3 & \text{if } -5 \leq x < -2, \\ 2x + 7 & \text{if } x \geq -2. \end{cases}$$

Express $|x - 1| - |x - 2| - |x - 3|$ as a piecewise function

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$$\underline{x < 1} : -(x - 1) - [-(x - 2)] - [-(x - 3)] = x - 4$$

$$\underline{1 \leq x < 2} : (x - 1) - [-(x - 2)] - [-(x - 3)] = 3x - 6$$

$$\underline{2 \leq x < 3} : (x - 1) - (x - 2) - [-(x - 3)] = x - 2$$

$$\underline{x \geq 3} : (x - 1) - (x - 2) - (x - 3) = -x + 4$$

$$|x - 1| - |x - 2| - |x - 3| = \begin{cases} x - 4 & \text{if } x < 1, \\ 3x - 6 & \text{if } 1 \leq x < 2, \\ x - 2 & \text{if } 2 \leq x < 3, \\ 4 - x & \text{if } x \geq 3. \end{cases}$$