Dr Oliver Mathematics
Algebra: Part 1

1. Find the set of real values of x satisfying the equation

$$\frac{x+5}{x+4} - \frac{x+6}{x+5} = \frac{x+7}{x+6} - \frac{x+8}{x+7}.$$

Solution

Well,

$$\frac{x+5}{x+4} - \frac{x+6}{x+5} = \frac{x+7}{x+6} - \frac{x+7}{x+6}$$

$$\Rightarrow \quad \frac{(x+5)^2 - (x+4)(x+6)}{(x+4)(x+5)} = \frac{(x+7)^2 - (x+6)(x+8)}{(x+6)(x+7)}$$

$\times$	x	$+4$
x	x^2	$+4x$
$+6$	$+6x$	$+24$

$\times$	x	$+6$
x	x^2	$+6x$
$+8$	$+8x$	$+48$

$$\Rightarrow \quad \frac{(x^2+10x+25) - (x^2+10x+24)}{(x+4)(x+5)} = \frac{(x^2+14x+49) - (x^2+14x+48)}{(x+6)(x+7)}$$

$$\Rightarrow \quad \frac{1}{(x+4)(x+5)} = \frac{1}{(x+6)(x+7)}$$

$$\Rightarrow \quad (x+4)(x+5) = (x+6)(x+7)$$

$\times$	x	$+4$
x	x^2	$+4x$
$+5$	$+5x$	$+20$

$\times$	x	$+6$
x	x^2	$+6x$
$+7$	$+7x$	$+42$

$\Rightarrow \quad x^2 + 9x + 20 = x^2 + 13x + 42$

$\Rightarrow \quad 4x = -22$

$\Rightarrow \quad \underline{\underline{x = -5\frac{1}{2}}}.$