

Dr Oliver Mathematics
Mathematics
Binomial Expansion
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics.
The total number of marks available is 128.

For $n \in \mathbb{N}$,

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n.$$

1. Find the first three terms, in ascending powers of x , of the binomial expansion of $(3+2x)^5$, giving each term in its simplest form. (4)
2. (a) Write down the first three terms, in ascending powers of x , of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant. (2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

- (b) find the value of p and the find the value of q . (4)
3. (a) Find the first three terms, in ascending powers of x , of the binomial expansion of $(1 + px)^9$, (2)

where p is a constant.

The first 3 terms are 1, $36x$, and qx^2 , where q is a constant.

- (b) Find the values of p and the the values of q . (4)
4. Find the first three terms, in ascending powers of x , of the binomial expansion of $(2+x)^6$, giving each term in its simplest form. (4)
5. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(1 - 2x)^5$. Give each term in its simplest form. (2)
- (b) If x is small, so that x^2 and higher powers can be ignored, show that (4)

$$(1 + x)(1 - 2x)^5 \approx 1 - 9x.$$

6. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of $(1 + kx)^6$, (3)

where k is a non-zero constant.

Given that, in this expansion, the coefficients of x and x^2 are equal,

- (b) the value of k , (2)
- (c) the coefficient of x^3 . (1)
7. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(1 + \frac{1}{2}x)^{10}$, giving each term in its simplest form. (4)
- (b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places. (3)
8. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form. (4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2 ,

- (b) find the value of a . (2)
9. Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form. (4)
10. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(2 + kx)^7$, where k is a constant. Give each term in its simplest form. (4)

Given that, in this expansion, the coefficient of x^2 is 6 times the coefficient of x ,

- (b) find the value of k . (2)
11. Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$(3 - x)^6,$$

and simplify each term.

12. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form. (4)

Given that the coefficient of x^2 is 525,

- (b) find the possible values of a . (2)
13. Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b . (1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$. (3)

14. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(1 + \frac{1}{4}x)^8$, giving each term in its simplest form. (4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places. (3)

15. Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(2 - 3x)^5$, giving each term in its simplest form. (4)

16. Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(2 - 3x)^5$. Give each term in its simplest form. (4)

17. (a) Use the binomial expansion to find all the terms of the expansion of $(2 + 3x)^4$. Give each term in its simplest form. (4)

(b) Write down the expansion of $(2 - 3x)^4$ (1)

in ascending powers of x , giving each term in its simplest form.

18. Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(2 - 3x)^5$. Give each term in its simplest form. (4)

19. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of (4)

$$(2 - 3x)^6,$$

giving each term in its simplest form.

(b) Hence, or otherwise, find the first 3 terms, in ascending powers of x , of the binomial expansion of (3)

$$(1 + \frac{1}{2})(2 - 3x)^6.$$

20. Find the first 4 terms, in ascending powers of x , of the binomial expansion of (4)

$$(1 + \frac{3}{2}x)^8,$$

giving each term in its simplest form.

21. Find the first 3 terms, in ascending powers of x , of the binomial expansion of (4)

$$(2 - \frac{1}{4}x)^{10},$$

giving each term in its simplest form.

22. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of (4)

$$(2 - 9x)^{10},$$

giving each term in its simplest form.

$$f(x) = (1 + kx)(2 - 9x)^{10}, \text{ where } k \text{ is a constant.}$$

The expansion, in ascending powers of x , of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2,$$

where A and B are constants.

- (b) Write down the value of A . (1)
- (c) Find the value of k . (2)
- (d) Hence find the value of B . (2)
23. In the binomial expansion of $(2k + x)^n$, where k is a constant and n is a positive integer, the coefficient of x^3 is equal to the coefficient of x^2 .
- (a) Prove that $n = 6k + 2$. (6)
- (b) Given also that $k = \frac{2}{3}$, expand $(2k + x)^n$ in ascending powers of x up to the term in x^3 , giving each term as an exact fraction in its simplest form. (5)
24. Find the first 4 terms, in ascending powers of x , of the binomial expansion of (4)

$$\left(3 - \frac{1}{3}x\right)^5,$$

giving each term in its simplest form.