Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2013 Nov Paper 1 Variant 3: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. The coefficient of x^2 in the expansion of $(2 + px)^6$ is 60.
   (a) Find the value of the positive constant p. (3)
   (b) Using your value of p, find the coefficient of x^2 in the expansion of (3)
   $$(3 - x)(2 + px)^6.$$

2. Solve (5)
   $$2\log_{10} y - \log_{10}(5y + 60) = 1.$$

3. Show that (4)
   $$\tan^2\theta - \sin^2\theta \equiv \sin^4\theta \sec^2\theta.$$

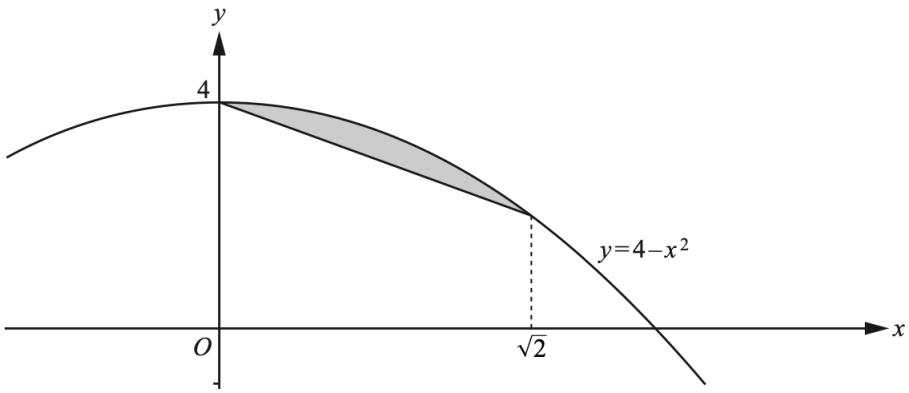
4. A curve has equation
   $$y = \frac{e^{2x}}{(x+3)^2}.$$
   (a) Show that (4)
   $$\frac{dy}{dx} = \frac{Ae^{2x}(x+2)}{(x+3)^3},$$
   where A is a constant to be found.
   (b) Find the exact coordinates of the point on the curve where $\frac{dy}{dx} = 0$. (2)

5. For $x \in \mathbb{R}$, the functions f and g are defined by
   $$f(x) = 2x^3,$$
   $$g(x) = 4x - 5x^2.$$
   (a) Express $f^2(\frac{1}{2})$ as a power of 2. (2)
   (b) Find the values of x for which f and g are increasing at the same rate with respect to x. (4)

6. **Do not use a calculator in this question.** (6)

The diagram shows part of the curve

$$y = 4 - x^2.$$



Show that the area of the shaded region can be written in the form

$$\frac{\sqrt{2}}{p},$$

where p is an integer to be found.

7. It is given that

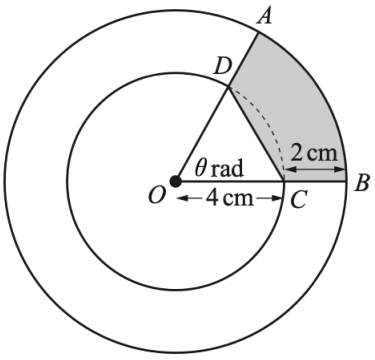
$$\mathbf{A} = \begin{pmatrix} 2t & 2 \\ t^2 - t + 1 & t \end{pmatrix}.$$

(a) Find the value of t for which $\det \mathbf{A} = 1$. (3)

(b) In the case when $t = 3$, find $\mathbf{A}^{-1}$ and hence solve (5)

$$3x + y = 5,$$
$$7x + 3y = 11.$$

8. The diagram shows two concentric circles, centre O, radii 4 cm and 6 cm.



The points A and B lie on the larger circle and the points C and D lie on the smaller circle such that ODA and OCB are straight lines.

(a) Given that the area of triangle OCD is 7.5 cm^2, show that $\theta = 1.215$ radians, to 3 decimal places. (2)

(b) Find the perimeter of the shaded region. (4)

(c) Find the area of the shaded region. (3)

9. (a) (i) Solve (4)

$$6\sin^2 x = 5 + \cos x \text{ for } 0^\circ < x < 180^\circ.$$

(ii) Hence, or otherwise, solve (3)

$$6\cos^2 y = 5 + \sin y \text{ for } 0^\circ < y < 180^\circ.$$

(b) Solve (4)

$$4\cot^2 z - 3\cot z = 0 \text{ for } 0 < z < \pi \text{ radians.}$$

10. The variables s and t are related by the equation

$$t = ks^n,$$

where k and n are constants.

The table below shows values of variables s and t.

s	2	4	6	8
t	25.00	6.25	2.78	1.56

(a) A straight line graph is to be drawn for this information with $\log_{10} t$ plotted on the vertical axis. (1)
State the variable which must be plotted on the horizontal axis.

(b) Draw this straight line graph on the grid below. (3)

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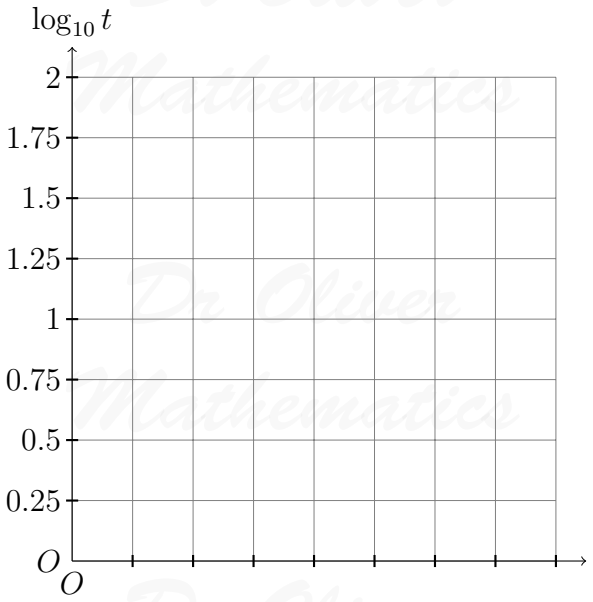

(c) Use your graph to find the value of k and of n. (4)

(d) Estimate the value of s when $t = 4$. (2)

11. (a) Given that (5)

$$\int_0^k \left(2\mathrm{e}^{2x} - \tfrac{5}{2}\mathrm{e}^{-2x}\right) \mathrm{d}x = \tfrac{3}{4},$$

where k is a constant, show that

$$4\mathrm{e}^{4k} - 12\mathrm{e}^{2k} + 5 = 0.$$

(b) Using a substitution of $y = \mathrm{e}^{2k}$, or otherwise, find the possible values of k. (4)

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