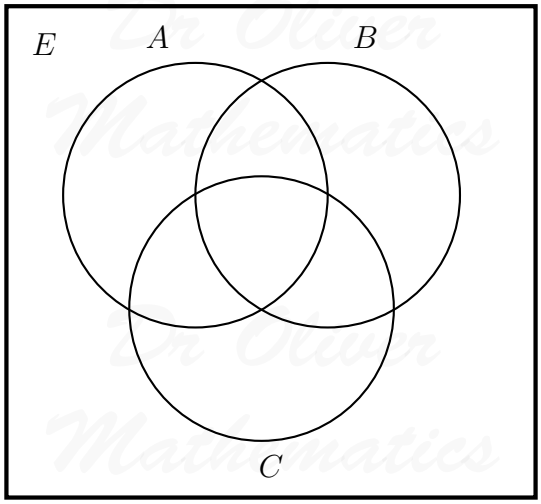
Dr Oliver Mathematics
GCSE Mathematics
2018 November Paper 2H: Calculator
1 hour 30 minutes

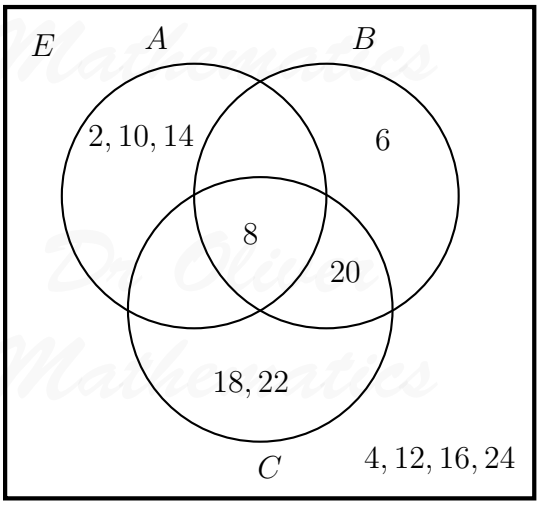
The total number of marks available is 80.
You must write down all the stages in your working.

1. $E = \{\text{even numbers between 1 and 25}\}$.
   $A = \{2, 8, 10, 14\}$.
   $B = \{6, 8, 20\}$.
   $C = \{8, 18, 20, 22\}$.

   (a) Complete the Venn diagram for this information. (4)



Solution



A number is chosen at random from E.

(b) Find the probability that the number is a member of $A \cap B$. (2)

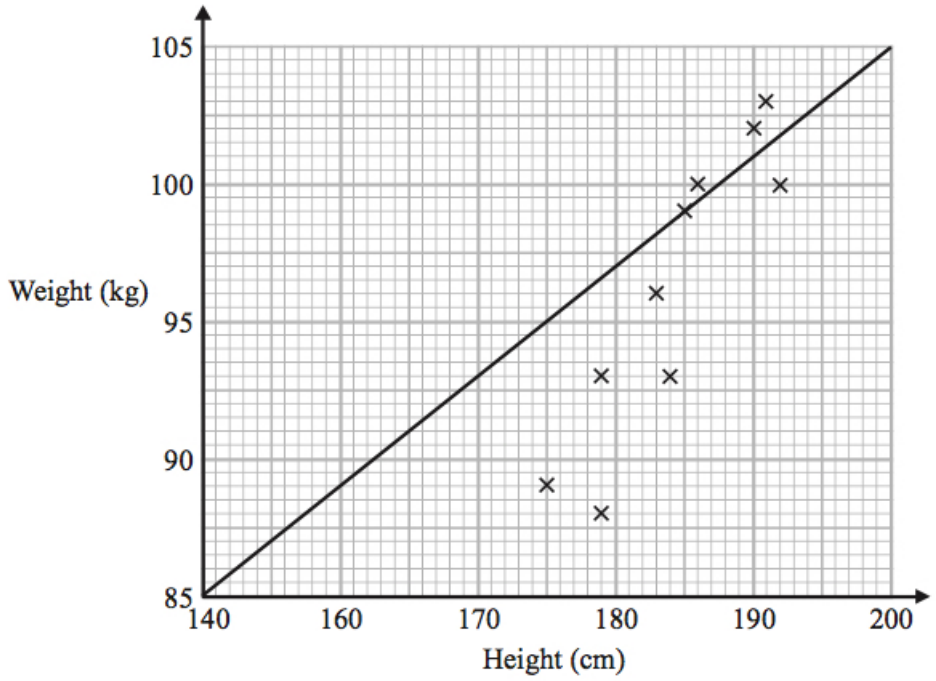
Solution

$$\mathrm{P}(A \cap B) = \underline{\underline{\frac{1}{12}}}.$$

2. Sean has information about the height, in cm, and the weight, in kg, of each of ten rugby players. (2)

He is asked to draw a scatter graph and a line of best fit for this information.

Here is his answer.



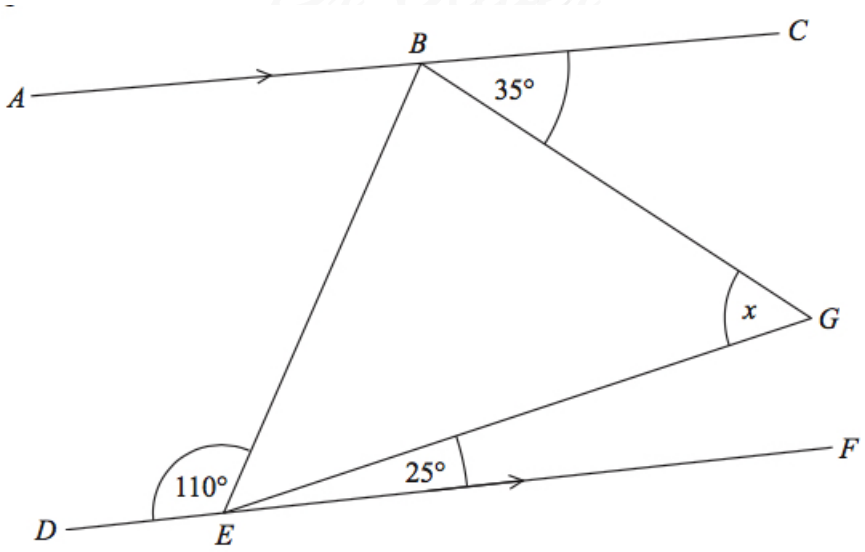
Sean has plotted the points accurately.

Write down two things that are wrong with his answer.

Solution

E.g., it does not go through the origin, the horizontal scale is incorrect, the "line of best fit" is not right.

3. BEG is a triangle. (4)



ABC and DEF are parallel lines.

Work out the size of angle x.
Give a reason for each stage of your working.

Solution

$\angle BEG = 180 - (110 + 25) = 55^\circ$ (supplementary angles)
$\angle ABE = 55 + 25 = 80^\circ$ (alternate angles)
$\angle EBG = 180 - (80 + 35) = 65^\circ$ (alternate angles)
$\angle x = 180 - (55 + 65) = \underline{\underline{60^\circ}}$ (competing the triangle)

4. Northern Bank has two types of account.
Both accounts pay compound interest.

Cash savings account
Interest
2.5% per annum

Shares account
Interest
3.5% per annum

Ali invests £2 000 in the cash savings account.
Ben invests £1 600 in the shares account.

(a) Work out who will get the most interest by the end of 3 years. (4)
You must show all your working.

Solution

Ali will get

$$2\,000 \times (1.025)^3 - 2\,000 = 153.781\,25$$

and Ben will get

$$1\,600 \times (1.035)^3 - 1\,600 = 173.948\,6.$$

Hence, Ben gets more.

In the 3rd year the rate of interest for the shares account is changed to 4% per annum.

(b) Does this affect who will get the most interest by the end of 3 years? (1)
Give a reason for your answer.

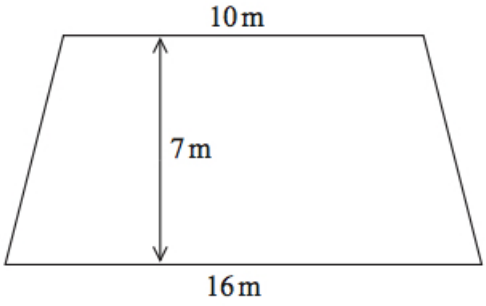
Solution

Ben will get

$$1\,600 \times (1.035)^2 \times (1.04) - 1\,600 = 182.518\,4.$$

Hence, Ben gets more.

5. The diagram shows a floor in the shape of a trapezium. (5)



£16.99

John is going to paint the floor.

Each 5 litre tin of paint costs £16.99.
1 litre of paint covers an area of 2 m^2.

John has £160 to spend on paint.

Has John got enough money to buy all the paint he needs?
You must show how you get your answer.

Solution

John needs to paint

$$\begin{aligned}\text{area} &= \tfrac{1}{2}h(a+b)\\ &= \tfrac{1}{2}(7)(10+16)\\ &= 91\ \text{m}^2\end{aligned}$$

and each 5 litre tin of paint will do

$$5 \times 2 = 10\ \text{m}^2.$$

Now, he needs

$$\frac{91}{5\times 2} = 9.1\ 5 \text{ litres tins.}$$

Ugh: 9.1 is not enough — so buy 10! Next,

$$16.99 \times 10 = £169.90;$$

hence, John has not got enough money.

6. A is the point with coordinates $(5, 9)$. (3)
B is the point with coordinates $(d, 15)$.

The gradient of the line AB is 3

Work out the value of d.

Solution

$$\begin{aligned}\frac{15-9}{d-5} = 3 &\Rightarrow \frac{6}{d-5} = 3\\ &\Rightarrow \frac{6}{3} = d-5\\ &\Rightarrow 2 = d-5\\ &\Rightarrow d = 7.\end{aligned}$$

7. (a) Write the number (1)

$$0.000\,086\,23$$

in standard form.

Solution

$$0.000\,086\,23 = \underline{\underline{8.623 \times 10^{-5}}}.$$

(b) Work out (2)

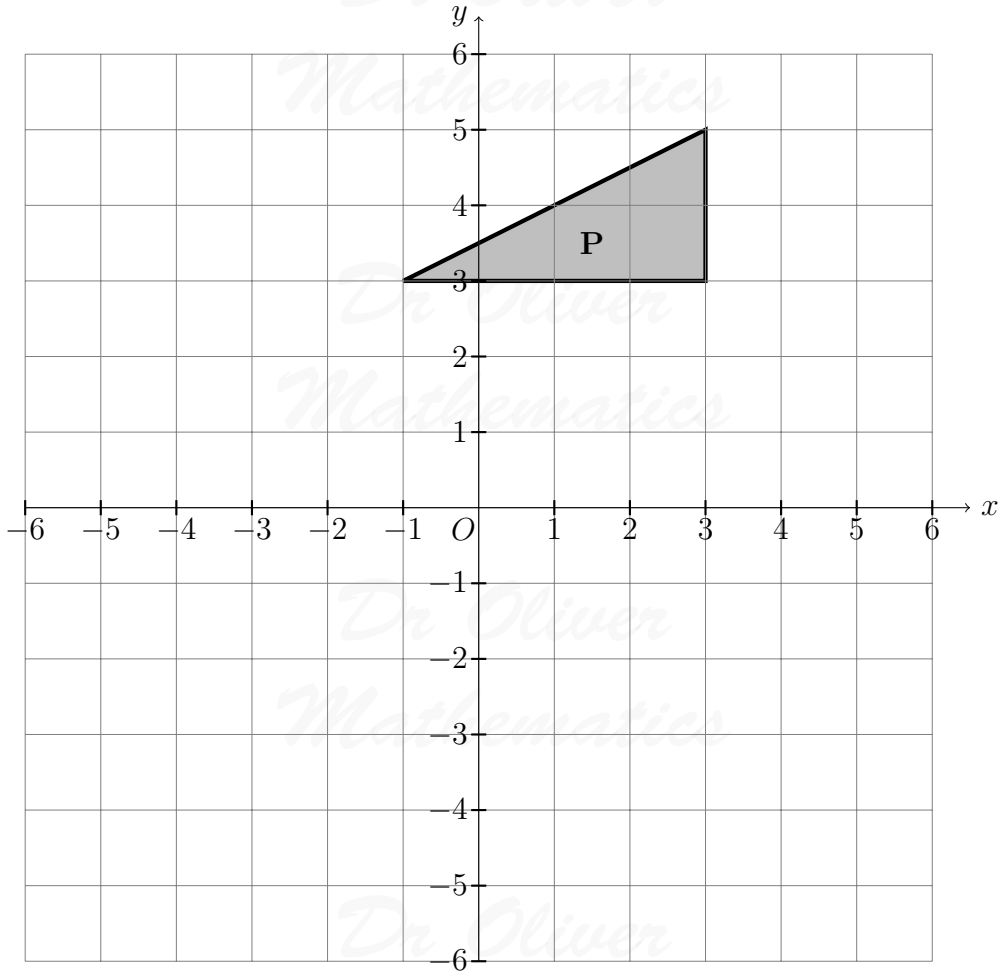
$$\frac{3.2 \times 10^3 + 5.1 \times 10^{-2}}{4.3 \times 10^{-4}}.$$

Give your answer in standard form, correct to 3 significant figures.

Solution

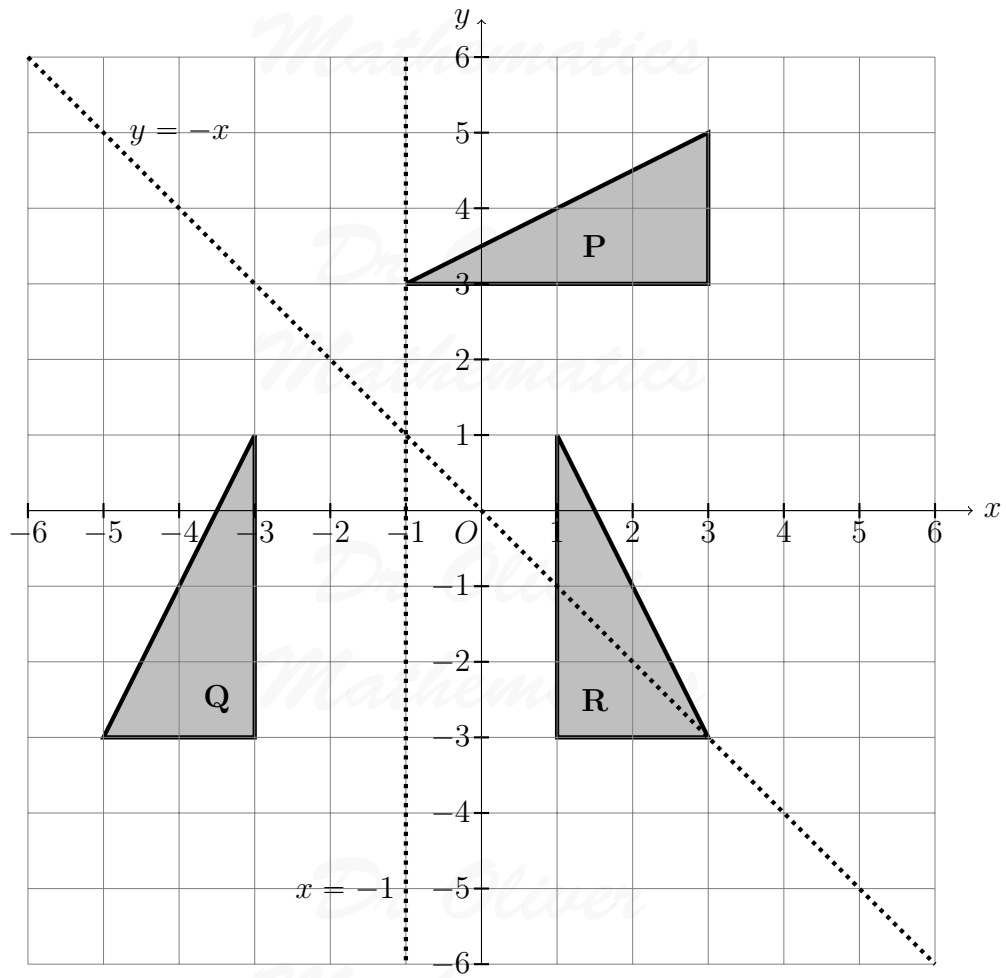
$$\begin{aligned}\frac{3.2 \times 10^3 + 5.1 \times 10^{-2}}{4.3 \times 10^{-4}} &= \frac{3.200\,051 \times 10^3}{4.3 \times 10^{-4}} \\ &= 7.441\,979\,07 \times 10^6 \text{ (FCD)} \\ &= \underline{\underline{7.44 \times 10^6 \text{ (3 sf)}}}\end{aligned}$$

8. Triangle **P** is reflected in the line $y = -x$ to give triangle **Q**. (3)
Triangle **Q** is reflected in the line $x = -1$ to give triangle **R**.



Describe fully the single transformation that maps triangle **R** to triangle **P**.

Solution



The point is at the centre of the two dotted lines: it is a rotation, 90° anticlockwise, about centre (-1,1).

9. Martin truncates the number N to 1 digit. (2)
The result is 7.

Write down the error interval for N.

Solution

$7 \leqslant N < 8$.

10. Robert makes 50 litres of green paint by mixing litres of yellow paint and litres of blue (5)

paint in the ratio 2 : 3.

Yellow paint is sold in 5 litre tins.
Each tin of yellow paint costs £26.

Blue paint is sold in 10 litre tins.
Each tin of blue paint costs £48.

Robert sells all the green paint he makes in 10 litre tins.
He sells each tin of green paint for £66.96.

Work out Robert's percentage profit on each tin of green paint he sells.

Solution

He needs 4 yellow paint (sold in 5 litre tins) ($4 \times 26 = £104$) and he needs 3 blue paint (sold in 10 litre tins) ($3 \times 48 = £144$) and he spends

$$104 + 144 = £248.$$

Now, he makes 50 litres of green paint (5×10 litres) so he makes

$$5 \times 66.96 = £334.80.$$

Hence,

$$\text{percentage profit} = \left(\frac{334.80 - 248}{248}\right) \times 100\%$$
$$= \underline{\underline{35\%}}.$$

11. In a restaurant there are (3)

- 9 starter dishes,
- 15 main dishes, and
- 8 dessert dishes.

Janet is going to choose one of the following combinations for her meal.

- a starter dish and a main dish,
- or a main dish and a dessert dish,
- or a starter dish, a main dish, and a dessert dish

Show that there are 1 335 different ways to choose the meal.

Solution

There are

$$\begin{aligned} & \text{starter, main + main, dessert + starter, main, dessert} \\ = \; & (9 \times 15) + (15 \times 8) + (9 \times 15 \times 8) \\ = \; & 135 + 120 + 1\,080 \\ = \; & \underline{\underline{1\,335}}, \end{aligned}$$

as required.

12. (a) Write (3)

$$\frac{4x^2 - 9}{6x + 9} \times \frac{2x}{x^2 - 3x}$$

in the form

$$\frac{ax + b}{cx + d},$$

where a, b, c, and d are integers.

Solution

Difference of two squares:

$$4x^2 - 9 = (2x)^2 - 3^2 = (2x - 3)(2x + 3).$$

Now,

$$\begin{aligned} \frac{4x^2 - 9}{6x + 9} \times \frac{2x}{x^2 - 3x} &= \frac{(2x - 3)(2x + 3)}{3(2x + 3)} \times \frac{2x}{x(x - 3)} \\ &= \frac{(2x - 3)\cancel{(2x + 3)}}{3\cancel{(2x + 3)}} \times \frac{\cancel{2x}}{\cancel{x}(x - 3)} \\ &= \frac{(2x - 3)}{3} \times \frac{2}{(x - 3)} \\ &= \underline{\underline{\frac{4x - 6}{3x - 9}}}; \end{aligned}$$

hence, $\underline{\underline{a = 4}}$, $\underline{\underline{b = -6}}$, $\underline{\underline{c = 3}}$, and $\underline{\underline{d = -9}}$.

(b) Express (3)

$$\frac{3}{x+1}+\frac{1}{x-2}-\frac{4}{x}$$

as a single fraction in its simplest form.

Solution

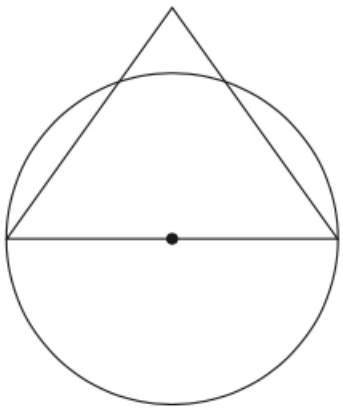
$$\frac{3}{x+1}+\frac{1}{x-2}-\frac{4}{x}=\frac{3x(x-2)}{x(x+1)(x-2)}+\frac{x(x+1)}{x(x+1)(x-2)}-\frac{4(x+1)(x-2)}{x(x+1)(x-2)}$$

×	x	-2
x	x^2	$-2x$
$+1$	$+x$	-2

$$=\frac{(3x^2-6x)+(x^2+x)-(4x^2-4x-8)}{x(x+1)(x-2)}$$

$$=\frac{-x+8}{x(x+1)(x-2)}.$$

13. The diagram shows a circle and an equilateral triangle. (3)



One side of the equilateral triangle is a diameter of the circle.
The circle has a circumference of 44 cm.

Work out the area of the triangle.
Give your answer correct to 3 significant figures.

Solution

$$2\pi r = 44 \Rightarrow 2r = \frac{44}{\pi}$$

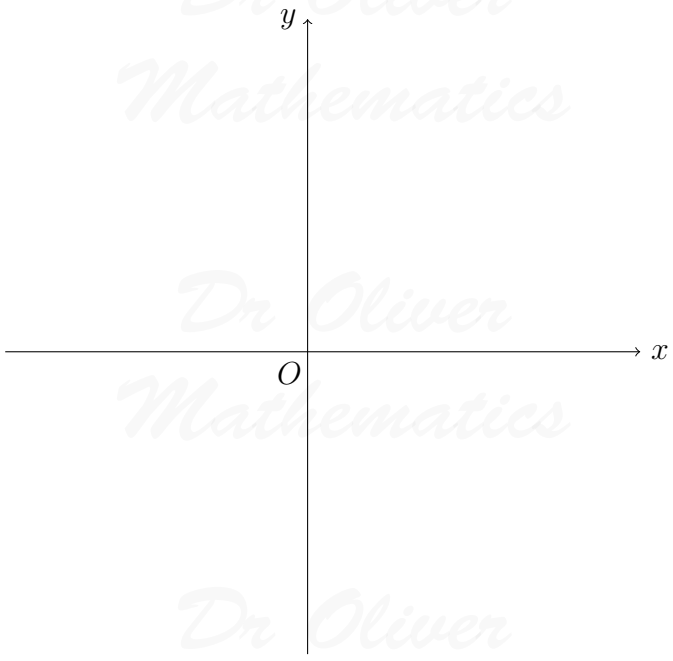
which is the length of each side of the triangle. In particular, it is an equilateral triangle so each angle is 60°. Hence,

$$\begin{aligned}\text{area} &= \tfrac{1}{2}ab\sin C \\ &= \tfrac{1}{2}\left(\frac{44}{\pi}\right)\left(\frac{44}{\pi}\right)(\sin 60^\circ) \\ &= 84.938\,823\,97 \text{ (FCD)} \\ &= \underline{\underline{89.9 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

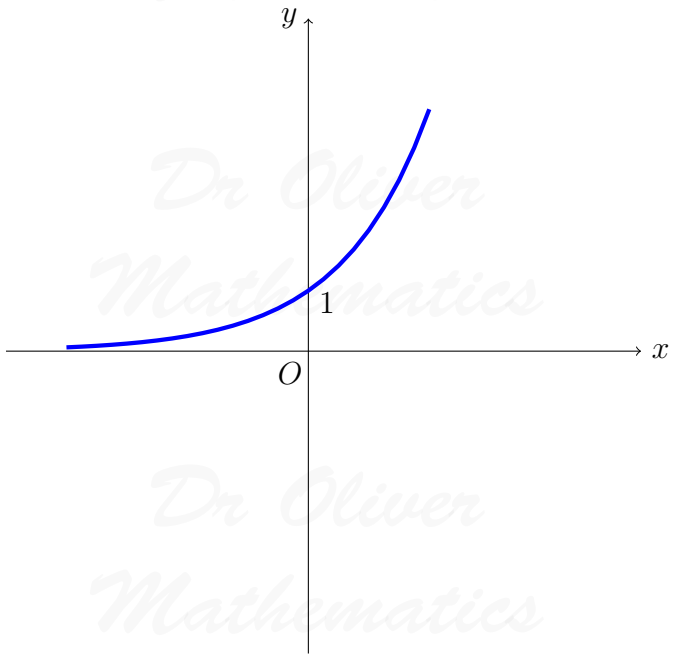
14. On the grid, sketch the curve with equation (2)

$$y = 2^x.$$

Give the coordinates of any points of intersection with the axes.



Solution



15. The equation of a circle is (1)

$$x^2 + y^2 = 42.25.$$

Find the radius of the circle.

Solution

$$\sqrt{42.25} = \underline{\underline{6.5}}.$$

16. There are only red counters and blue counters in a bag.

Joe takes at random a counter from the bag.
The probability that the counter is red is 0.65.
Joe puts the counter back into the bag.

Mary takes at random a counter from the bag.
She puts the counter back into the bag.

(a) What is the probability that Joe and Mary take counters of different colours? (2)

Solution

The probability that the counter is blue is $1 - 0.65 = 0.35$. Hence,

$$\begin{aligned} \text{P(different colours)} &= \text{P}(RB) + \text{P}(BR) \\ &= (0.65 \times 0.35) + (0.35 \times 0.65) \\ &= 0.227\,5 + 0.227\,5 \\ &= \underline{\underline{0.455}}. \end{aligned}$$

There are 78 red counters in the bag.

(b) How many blue counters are there in the bag? (2)

Solution

$$\frac{0.35}{0.65} \times 78 = \underline{\underline{42}}.$$

17. p and q are two numbers such that $p > q$. (5)

When you subtract 5 from p and subtract 5 from q the answers are in the ratio 5 : 1.
When you add 20 to p and add 20 to q the answers are in the ratio 5 : 2.

Find the ratio $p : q$.
Give your answer in its simplest form.

Solution

$$\begin{aligned} (p-5) : (q-5) = 5 : 1 \Rightarrow \frac{p-5}{q-5} &= \frac{5}{1} \\ \Rightarrow p - 5 &= 5(q-5) \\ \Rightarrow p - 5 &= 5q - 25 \\ \Rightarrow p &= 5q - 20 \quad (1) \end{aligned}$$

and

$$\begin{aligned} (p-5) : (q-5) = 5 : 1 \Rightarrow \frac{p+20}{q+20} &= \frac{5}{2} \\ \Rightarrow 2(p+20) &= 5(q+20) \\ \Rightarrow 2p + 40 &= 5q + 100 \\ \Rightarrow 2p &= 5q + 60 \quad (2). \end{aligned}$$

Do (2) − (1):

$$\begin{aligned} p = 80 &\Rightarrow 80 = 5q - 20 \\ &\Rightarrow 5q = 100 \\ &\Rightarrow q = 20. \end{aligned}$$

Hence,

$$p : q = 80 : 20 = \underline{\underline{4 : 1}}.$$

18. The straight line $\mathbf{L}_1$ passes through the points with coordinates $(4, 6)$ and $(12, 2)$. (4)
The straight line $\mathbf{L}_2$ passes through the origin and has gradient -3.

The lines $\mathbf{L}_1$ and $\mathbf{L}_2$ intersect at point P.

Find the coordinates of P.

Solution

$\underline{\mathbf{L}_1}$:

$$\begin{aligned} \text{Gradient} &= \frac{6-2}{4-12} \\ &= -\tfrac{1}{2} \end{aligned}$$

and so the equation of $\mathbf{L}_1$ is

$$y = -\tfrac{1}{2}x + c,$$

for some constant c. Now,

$$x = 4, y = 6 \Rightarrow 6 = -\tfrac{1}{2}(4) + c \Rightarrow c = 8.$$

Hence,

$$y = \tfrac{1}{2}x + 8 \quad (1).$$

$\underline{\mathbf{L}_2}$:

$$y = -3x \quad (2).$$

Do (1) = (2):

$$\begin{aligned} -\tfrac{1}{2}x + 8 = -3x &\Rightarrow \tfrac{5}{2}x = -8 \\ &\Rightarrow x = -\tfrac{16}{5} \\ &\Rightarrow y = \tfrac{48}{5}. \end{aligned}$$

Hence, $\underline{\underline{P(-3.2, 9.6)}}$.

19. Solve (5)

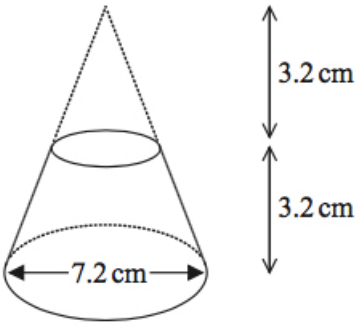
$$22 < \frac{m^2 + 7}{4} < 32.$$

Show all your working.

Solution

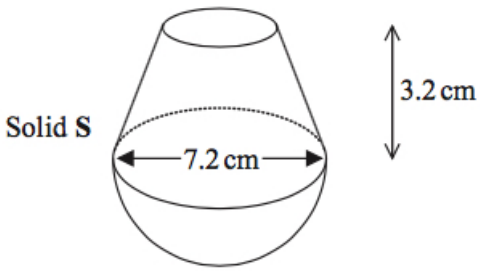
$$22 < \frac{m^2 + 7}{4} < 32 \Rightarrow 88 < m^2 + 7 < 128$$
$$\Rightarrow 81 < m^2 < 121$$
$$\Rightarrow \underline{\underline{9 < m < 11 \text{ or } -11 < m < -9}}.$$

20. Here is a frustum of a cone. (5)



The diagram shows that the frustum is made by removing a cone with height 3.2 cm from a solid cone with height 6.4 cm and base diameter 7.2 cm.

The frustum is joined to a solid hemisphere of diameter 7.2 cm to form the solid **S** shown below.



The density of the frustum is 2.4 g/cm^3.
The density of the hemisphere is 4.8 g/cm^3.

Calculate the average density of solid **S**.

Solution

Frustum:
We will cut the cone in half: radii are 1.8 cm (top) and 3.6 cm (bottom). Now,

$$\begin{aligned}\text{volume of the frustum} &= \tfrac{1}{3}\pi(3.6^2)(6.4) - \tfrac{1}{3}\pi(1.8^2)(3.2)\\ &= 24.192\pi\end{aligned}$$

and

$$\begin{aligned}\text{mass} &= \text{density} \times \text{volume}\\ &= 2.4 \times 24.192\pi\\ &= 58.060\,8\pi.\end{aligned}$$

Hemisphere:

$$\begin{aligned}\text{volume of the hemisphere} &= \tfrac{2}{3}\pi(3.6^3)\\ &= 31.104\pi\end{aligned}$$

and

$$\begin{aligned}\text{mass} &= \text{density} \times \text{volume}\\ &= 4.8 \times 31.104\pi\\ &= 149.299\,2\pi.\end{aligned}$$

In total:
The overall mass is

$$58.060\,8\pi + 149.299\,2\pi = 207.36\pi$$

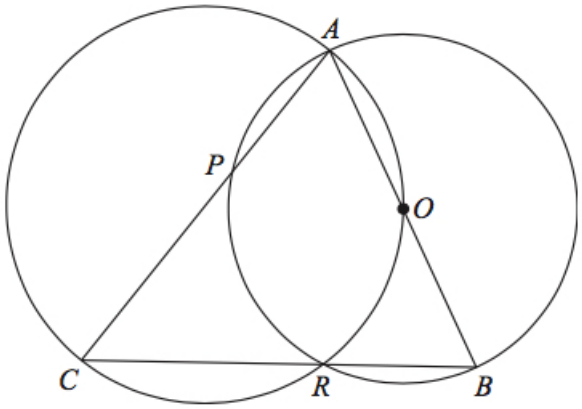
and the overall volume is

$$24.192\pi + 31.104\pi = 55.296\pi.$$

Hence,

$$\begin{aligned}\text{average density} &= \frac{207.36\pi}{55.296\pi}\\ &= \underline{\underline{3.75 \text{ g/cm}^3}}.\end{aligned}$$

21. A, B, R, and P are four points on a circle with centre O. (4)
A, O, R, and C are four points on a different circle.
The two circles intersect at the points A and R.



CPA, CRB, and AOB are straight lines.

Prove that angle CAB = angle ABC.

Solution

Let us call $x = \angle CAB$.
$OA = OP$ (radii) and $\angle OPA = x$.
$\angle ACB = 180 - x$ (supplementary angles)

Let us call $y = \angle CBA$.
$OB = OR$ (radii) and $\angle ORB = y$.
$\angle ACB = 180 - y$ (supplementary angles)

Now,

$$180 - x = 180 - y \Rightarrow x = y.$$

Hence,

$$\underline{\underline{\text{angle } CAB = \text{angle } ABC}},$$

as required.