Dr Oliver Mathematics
GCSE Mathematics
2009 November Paper 3H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Using the information that

$$74 \times 234 = 17\,316,$$

write down the value of

(a) 740×234, (1)

Solution

$$740 \times 234 = \underline{\underline{173\,160}}.$$

(b) 74×2.34. (1)

Solution

$$74 \times 2.34 = \underline{\underline{173.16}}.$$

2. Work out an estimate for the value of (3)

$$\frac{31 \times 4.92}{0.21}.$$

Solution

Round to 1 significant figure:

$$\begin{aligned}\frac{31 \times 4.92}{0.21} &\approx \frac{30 \times 5}{0.2} \\ &= \frac{150}{0.2} \\ &= \underline{\underline{750}}.\end{aligned}$$

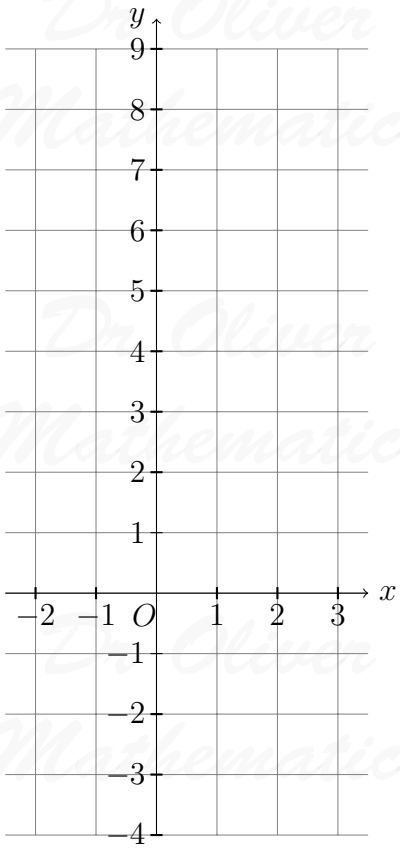
3. (a) Complete the table of values for $y = 2x + 2$. (2)

x	-2	-1	0	1	2	3
y			0	2		

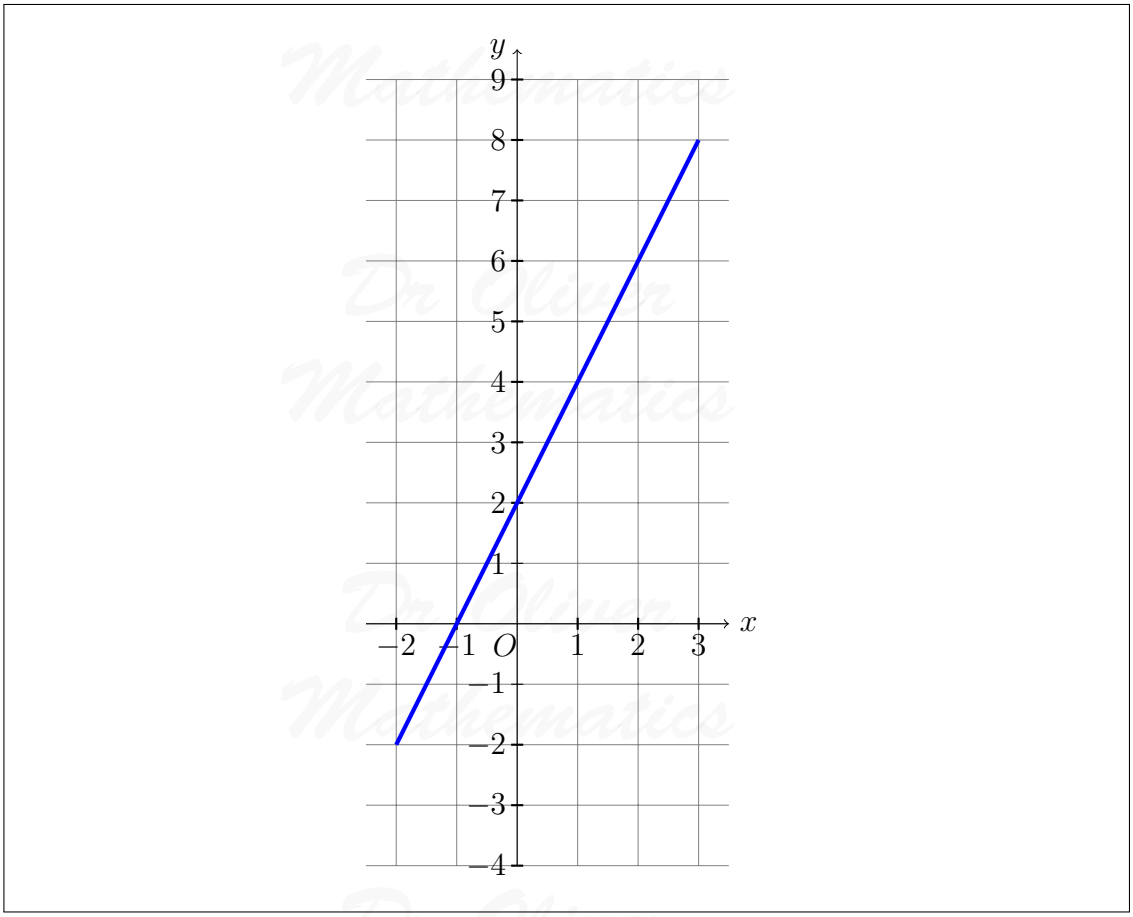
Solution

x	-2	-1	0	1	2	3
y	$\underline{\underline{-2}}$		0	2	$\underline{\underline{4}}$	$\underline{\underline{6}}$	$\underline{\underline{8}}$

(b) On the grid, draw the graph of $y = 2x + 2$. (2)



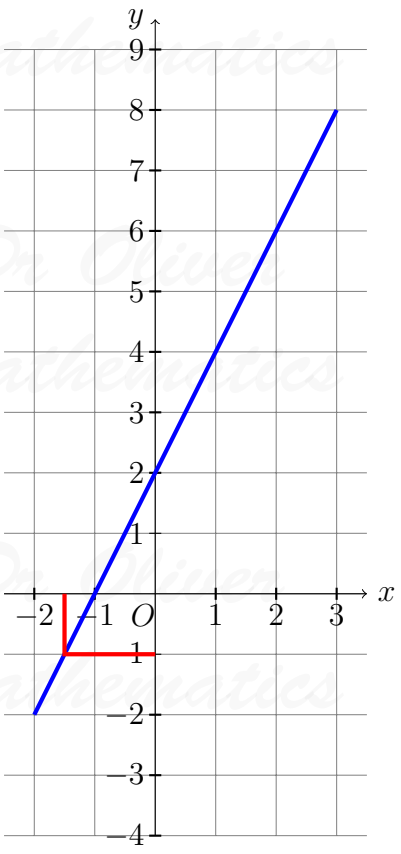
Solution



(c) Use your graph to find (2)

(i) the value of y when $x = -1.5$,

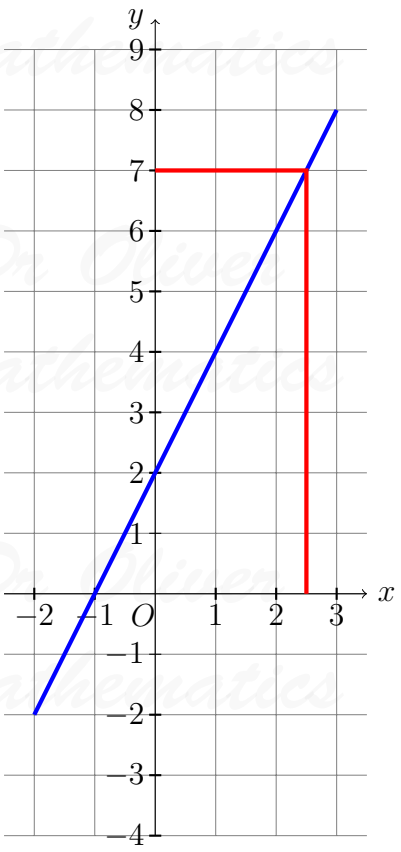
Solution



Correct read-off: approximately $\underline{\underline{-1}}$.

(ii) the value of x when $y = 7$.

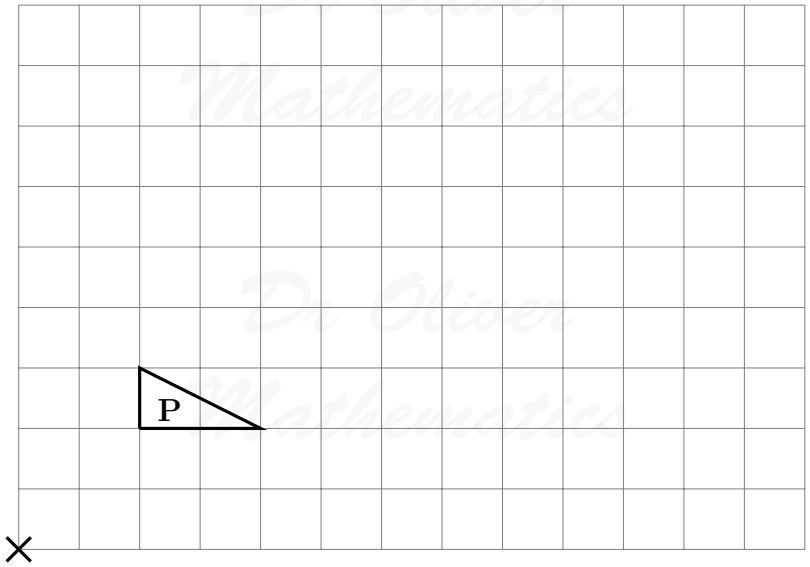
Solution



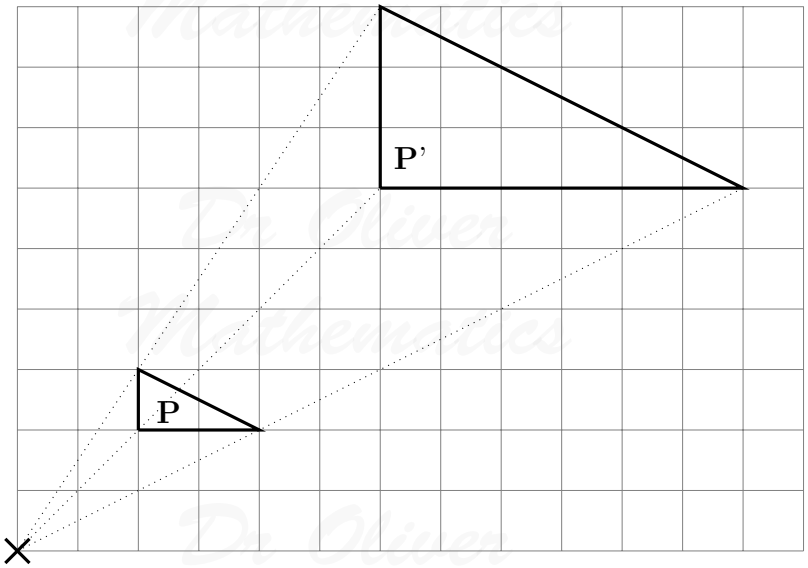
Correct read-off: approximately 2.5.

4. Triangle **P** has been drawn on a grid.

   (a) On the grid, draw an enlargement of the triangle **P** with scale factor 3. (2)

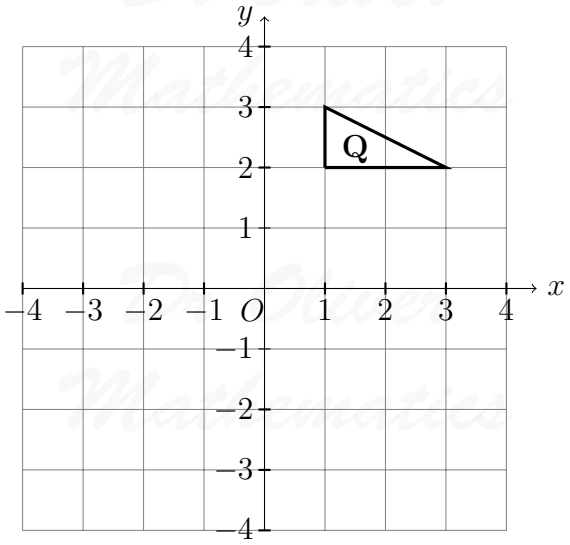


Solution

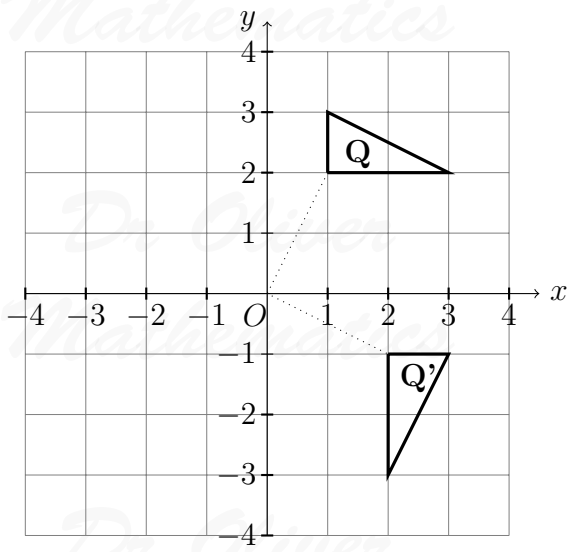


Triangle **Q** has been drawn on a grid.

(b) On the grid, rotate triangle **Q** 90° clockwise, centre O. (3)



Solution



5. Here are the weights in grams, to the nearest gram, of 15 eggs.

33	46	41	54	51
38	60	44	55	51
62	55	52	37	63

(a) Complete the ordered stem and leaf diagram to show this information. You must include a key. (3)

Solution

6	0	2	3			
5	1	1	2	4	5	5
4	1	4	6			
3	3	7	8			

Key: 6|3 means 63 grams.

Meg is going to pick at random one of the eggs.

(b) Work out the probability that this egg will have a weight of more than 45 grams. (2)

Solution

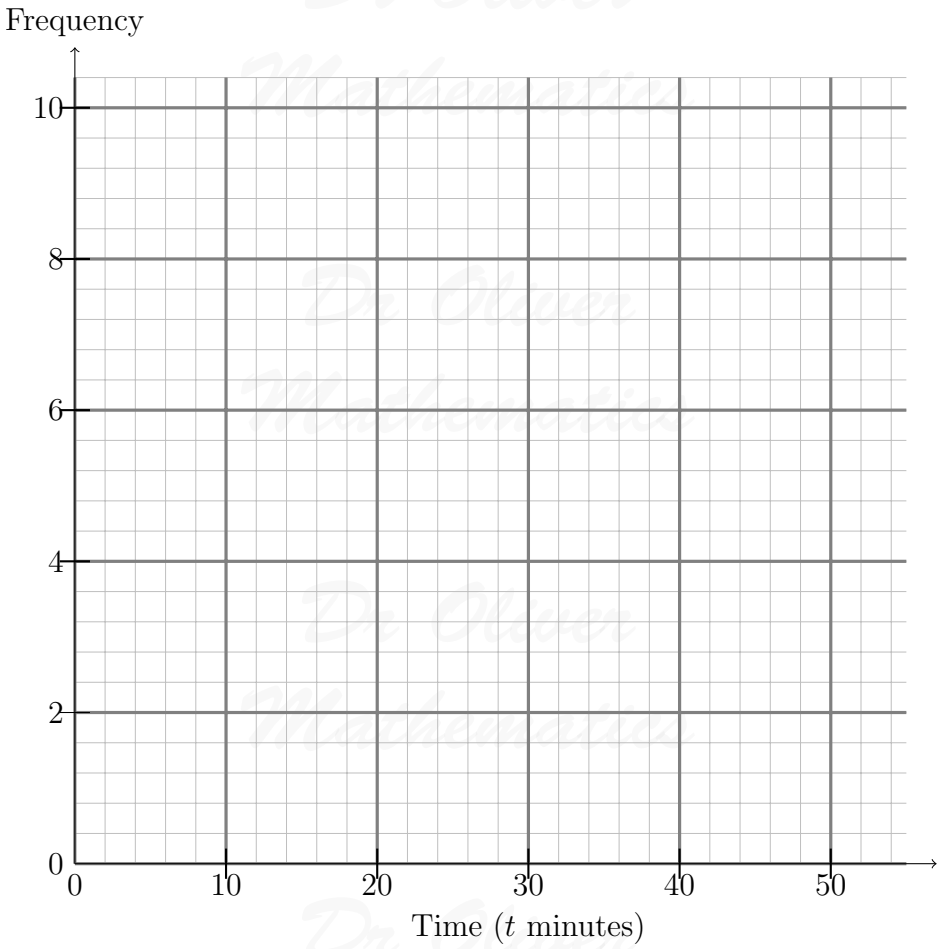
There are 10 eggs whose weight is more than 45 grams and so the probability is

$$\frac{10}{15} = \underline{\underline{\frac{2}{3}}}.$$

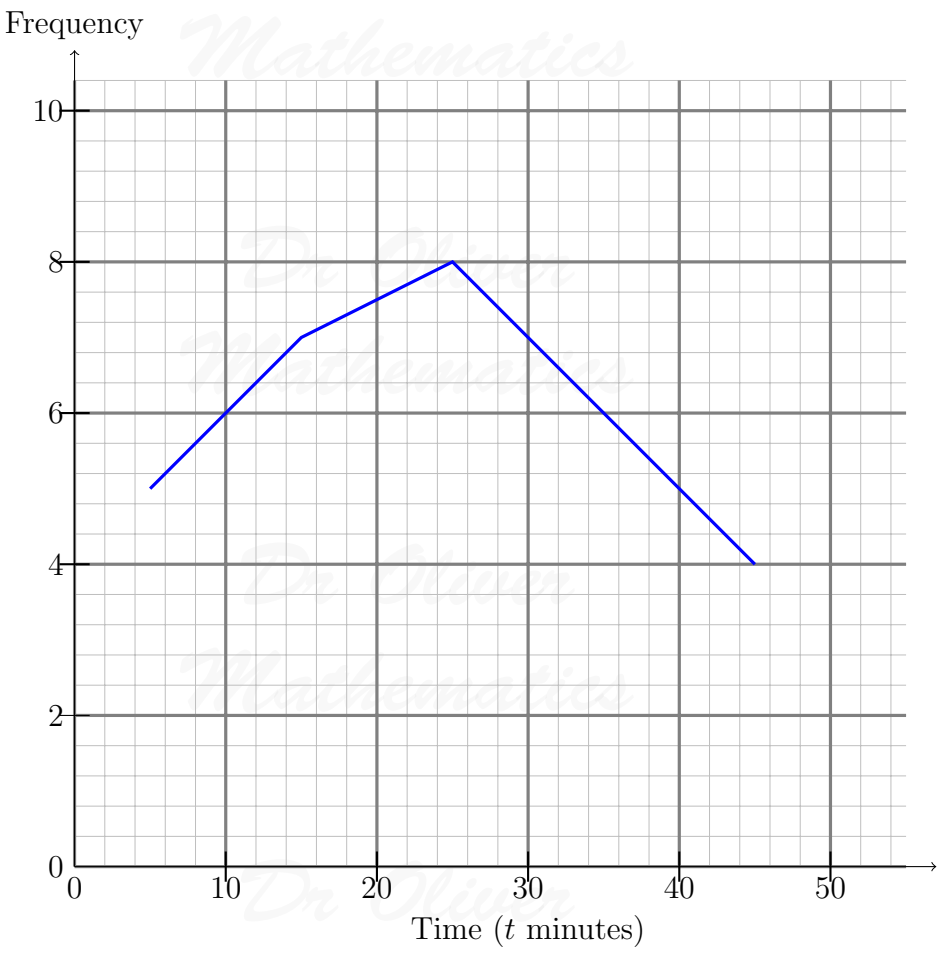
6. 30 students took a test.
The table shows information about how long it took them to complete the test.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	5
$10 < t \leqslant 20$	7
$20 < t \leqslant 30$	8
$30 < t \leqslant 40$	6
$40 < t \leqslant 50$	4

(a) On the grid, draw a frequency polygon for this information. (2)



Solution



(b) Write down the modal class interval. (1)

Solution

$20 < t \leqslant 30$.

7. (a) Work out (2)

$$\tfrac{3}{8} + \tfrac{1}{4}.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} \tfrac{3}{8} + \tfrac{1}{4} &= \tfrac{3}{8} + \tfrac{2}{8} \\ &= \tfrac{5}{8}. \end{aligned}$$

(b) Work out (2)

$$\frac{2}{3} \times \frac{4}{5}.$$

Solution

$$\frac{2}{3} \times \frac{4}{5} = \underline{\underline{\frac{8}{15}}}.$$

(c) Work out (3)

$$423 \times 12.$$

You **must** show **all** your working.

Solution

×	400	20	3
10	4 000	200	30
2	800	40	6

$$423 \times 12 = 4\,000 + 200 + 800 + 30 + 40 + 6$$
$$= \underline{\underline{5\,076}}.$$

8. Simon wants to find out how much people spend using their mobile phone.
He uses this question on a questionnaire.

How much do you spend using your mobile phone?

☐ £1–£5 ☐ £5–£10 ☐ £10–£15

(a) Write down **two** things that are wrong with this question. (2)

Solution

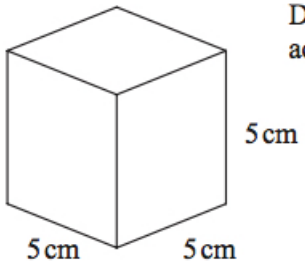
E.g., no time frame, no space for someone who spends nothing, overlap between £5, overlap between £10, no space for someone who spends £100, etc.

(b) Design a better question for his questionnaire to find out how much people spend using their mobile phone. (2)
You should include some response boxes.

Solution

A suitable question with a time frame, e.g., "How much do you spend using your mobile today/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, $£0 \leqslant x < £5$, $£5 \leqslant x < £15$, $£15 \leqslant x < £20$, $x \geqslant £20$.

9. (a) A solid cube has sides of length 5 cm. (4)



Work out the total surface area of the cube.
State the units of your answer.

Solution

$$6 \times 5 \times 5 = 6 \times 25$$
$$= \underline{\underline{150 \text{ cm}^2}}.$$

The volume of the cube is 125 cm^3.

(b) Change 125 cm^3 into mm^3. (2)

Solution

$$125 \text{ cm}^3 = 125 \times 1 \text{ cm}^3$$
$$= 125 \times 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$$
$$= 125 \times 1\,000 \text{ mm}^3$$
$$= \underline{\underline{125\,000 \text{ mm}^3}}.$$

The weight of the cube is 87 grams, correct to the nearest gram.

(c) (i) What is the minimum the weight could be? (2)

Solution
86.5 grams.

(ii) What is the maximum the weight could be?

Solution
87.5 grams.

10. (a) Simplify (2)

$$3a + 4c - a + 3c.$$

Solution

$$3a + 4c - a + 3c = 2a + 7c.$$

(b) Expand (1)

$$y(2y - 3).$$

Solution

$$y(2y - 3) = 2y^2 - 3y.$$

(c) Factorise (2)

$$x^2 - 4x.$$

Solution

$$x^2 - 4x = x(x - 4).$$

(d) Expand and simplify (2)

$$2(x + 3) + 3(2x - 1).$$

Solution

$$2(x+3)+3(2x-1) = 2x+6+6x-3$$
$$= \underline{\underline{8x+3}}.$$

(e) Solve (2)

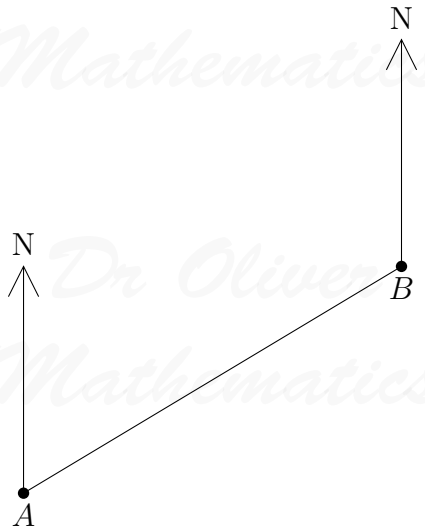
$$3(x+2) = 8.$$

Solution

$$3(x+2) = 8 \Rightarrow x+2 = 2\tfrac{2}{3}$$
$$\Rightarrow \underline{\underline{x = \tfrac{2}{3}}}.$$

11. The diagram shows the positions of two telephone masts, A and B, on a map.

(a) Measure the bearing of B from A. (1)



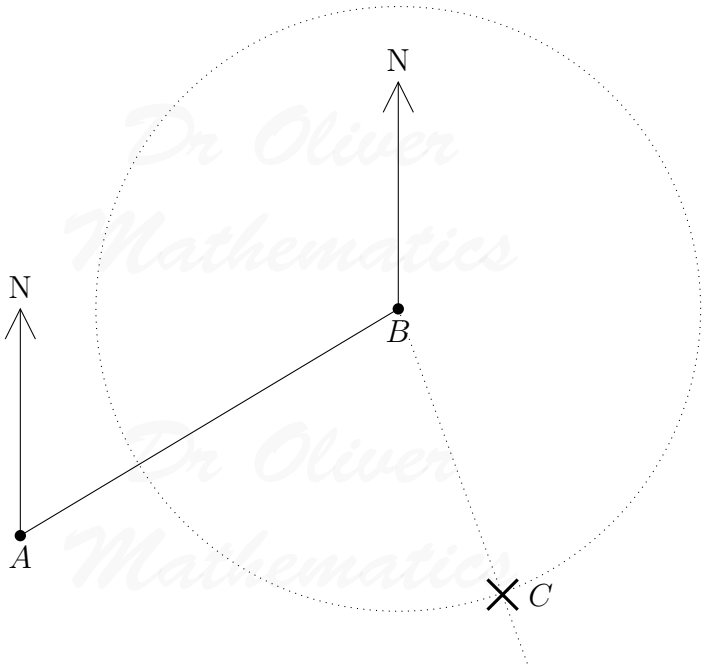
Solution

Correct read-off: approximately $\underline{\underline{59^\circ}}$.

Another mast C is on a bearing of 160° from B.
On the map, C is 4 cm from B.

(b) Mark the position of C with a cross ($\times$) and label it C. (2)

Solution



12. Batteries are sold in packets and boxes. (3)
Each packet contains 4 batteries.
Each box contains 20 batteries.
Bill buys p packets of batteries and b boxes of batteries.
Bill buys a total of N batteries.
Write down a formula for N in terms of p and b.

Solution

$$\underline{\underline{N = 4p + 20b}}.$$

13. (a) Write in standard form 213 000. (1)

Solution

$$213\,000 = \underline{\underline{2.13 \times 10^5}}.$$

(b) Write in standard form 0.001 23. (1)

Solution

$$0.001\,23 = \underline{\underline{1.23 \times 10^{-3}}}.$$

14. (a) Write down the value of 5^0. (1)

Solution

$$5^0 = \underline{\underline{1}}.$$

(b) Write down the value of 2^{-1}. (1)

Solution

$$2^{-1} = \underline{\underline{\tfrac{1}{2}}}.$$

15. k is an integer such that $-1 \leqslant k < 3$.

(a) List all the possible values of k. (2)

Solution

$\underline{\underline{-1, 0, 1, 2}}$.

(b) Solve the inequality (2)

$$6y \geqslant y + 10.$$

Solution

$$6y \geqslant y + 10 \Rightarrow 5y \geqslant 10$$
$$\Rightarrow \underline{\underline{y \geqslant 2}}.$$

16. Make q the subject of the formula (3)

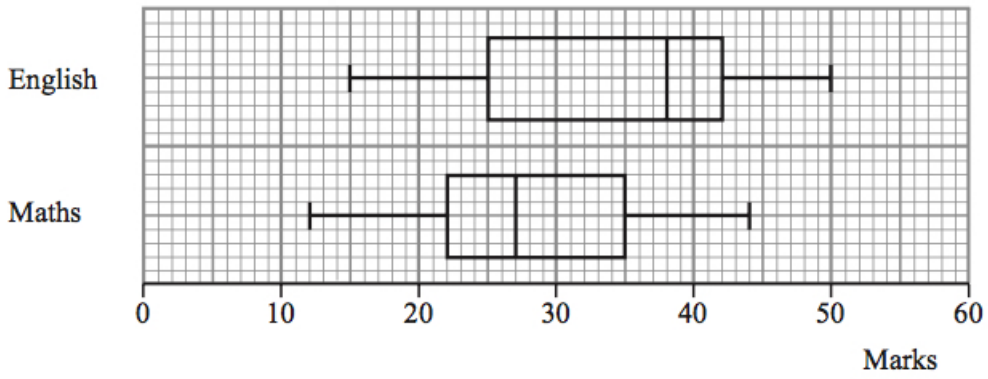
$$5(q+p) = 4 + 8p.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} 5(q+p) = 4 + 8p &\Rightarrow 5q + 5p = 4 + 8p \\ &\Rightarrow 5q = 4 + 3p \\ &\Rightarrow \underline{\underline{q = \tfrac{1}{5}(4+3p)}}. \end{aligned}$$

17. The box plots show the distribution of marks in an English test and in a Maths test for a group of students.



(a) What is the highest mark in the English test? (1)

Solution

<u>50 marks</u>.

(b) Compare the distributions of the marks in the English test and marks in the Maths test. (2)

Solution

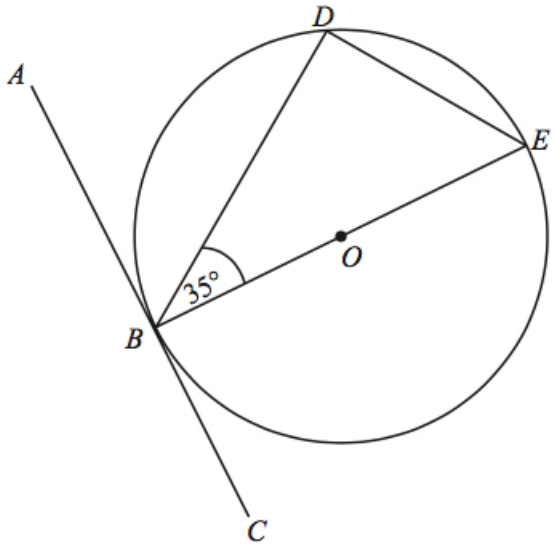
Average

Since the median for English (38) is higher than the median for Maths (27), the students scored more marks in English on average.

Spread
Since the range for Maths $(44 - 12 = 32)$ is smaller than the range for English $(50 - 15 = 35)$, the marks were more consistent in Maths.
OR
Since the IQR for Maths $(35 - 22 = 13)$ is smaller than the range for English $(42 - 25 = 17)$, the marks were more consistent in Maths.

Skewness
The English marks are negatively skewed whereas the Maths marks are positively skewed.

18. B, D, and E are points on a circle centre O.



ABC is a tangent to the circle.
BE is a diameter of the circle.
Angle $DBE = 35°$.

(a) Find the size of angle ABD. (2)
Give a reason for your answer.

Solution
Angle $ABD = 90 - 35 = \underline{\underline{55°}}$. (Complementary angle)

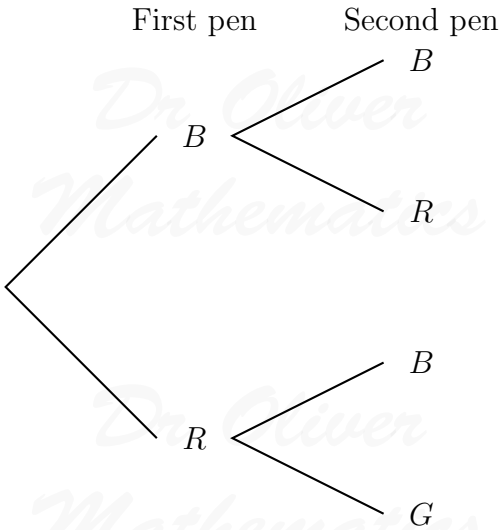
(b) Find the size of angle DEB. (2)
Give a reason for your answer.

Solution

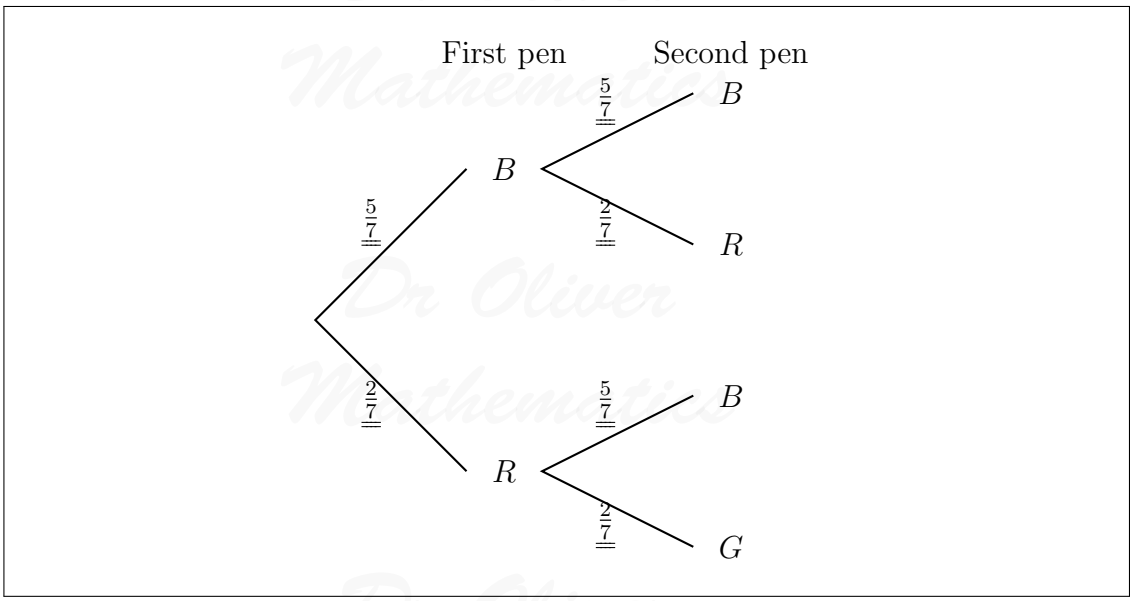
Angle $DEB = \underline{\underline{90^\circ}}$. (Angle in a semicircle)

19. Emma has 7 pens in a box.
5 of the pens are blue.
2 of the pens are red.
Emma takes at random a pen from the box and writes down its colour.
Emma puts the pen back in the box.
Then Emma takes at random a second pen from the box, and writes down its colour.

(a) Complete the probability tree diagram. (2)



Solution



(b) Work out the probability that Emma takes exactly one pen of each colour from the box. (3)

Solution

$$\begin{aligned} \text{P(one pen of each colour)} &= \text{P}(BR) + \text{P}(RB) \\ &= 2 \times \tfrac{5}{7} \times \tfrac{2}{7} \\ &= \tfrac{20}{49} \end{aligned}$$

20. Solve the simultaneous equations: (3)

$$\begin{aligned} 4x + y &= -1 \\ 4x - 3y &= 7. \end{aligned}$$

Solution

Subtract the two equations:

$$4y = -8 \Rightarrow \underline{\underline{y = -2}}$$
$$\Rightarrow 4x - 2 = -1$$
$$\Rightarrow 4x = 1$$
$$\Rightarrow \underline{\underline{x = \tfrac{1}{4}}}.$$

21. Work out (2)

$$(2 + \sqrt{3})(2 - \sqrt{3}).$$

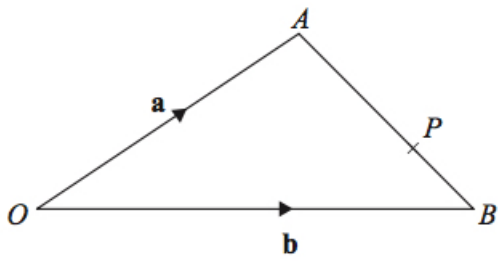
Give your answer in its simplest form.

Solution

$\times$	2	$+\sqrt{3}$
2	4	$+2\sqrt{3}$
$-\sqrt{3}$	$-2\sqrt{3}$	-3

$$(2 + \sqrt{3})(2 - \sqrt{3}) = \underline{\underline{1}}.$$

22. OAB is a triangle.



$\overrightarrow{OA} = \mathbf{a}$.
$\overrightarrow{OB} = \mathbf{b}$.

(a) Find the vector $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1)

Solution

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= \underline{\underline{-\mathbf{a} + \mathbf{b}}}.\end{aligned}$$

P is the point on AB so that $AP : PB = 2 : 1$.

(b) Find the vector $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (3)
Give your answer in its simplest form.

Solution

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP} \\ &= \overrightarrow{OA} + \tfrac{2}{3}\overrightarrow{AB} \\ &= \mathbf{a} + \tfrac{2}{3}(-\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} - \tfrac{2}{3}\mathbf{a} + \tfrac{2}{3}\mathbf{b} \\ &= \underline{\underline{\tfrac{1}{3}\mathbf{a} + \tfrac{2}{3}\mathbf{b}}}.\end{aligned}$$

23. Prove that the recurring decimal (3)

$$0.\dot{3}\dot{6} = \tfrac{4}{11}.$$

Solution

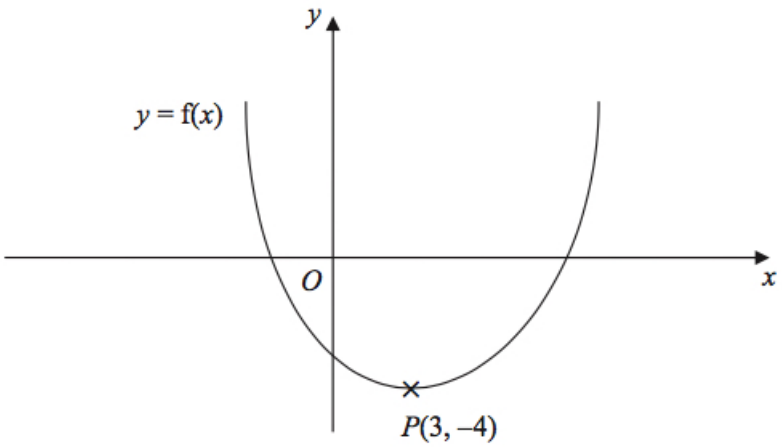
$$\begin{aligned}100x &= 36.\dot{3}\dot{6} \quad (1) \\ x &= 0.\dot{3}\dot{6} \quad (2)\end{aligned}$$

Subtract:

$$\begin{aligned}99x = 36 &\Rightarrow x = \tfrac{36}{99} \\ &\Rightarrow x = \tfrac{4\times 9}{11\times 9} \\ &\Rightarrow \underline{\underline{x = \tfrac{4}{11}}},\end{aligned}$$

as required.

24. This is a sketch of the curve with the equation $y = \mathrm{f}(x)$.



The only minimum point of the curve is at $P(3, -4)$.

(a) Write down the coordinates of the minimum point of the curve with the equation $y = \mathrm{f}(x - 2)$. (2)

Solution

$(5, -4)$.

(b) Write down the coordinates of the minimum point of the curve with the equation $y = \mathrm{f}(x + 5) + 6$. (2)

Solution

$(-2, 2)$.

25. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number. (3)

Solution

Let the two numbers be n and $(n + 1)$ for some $n \in \mathbb{N}$. Then

$$n + (n + 1) = 2n + 1,$$

and it is an odd number.