Dr Oliver Mathematics
Mathematics: Advanced Higher
2026 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Differentiate (2)

$$f(x) = 3\sin^{-1} 7x.$$

Solution

Well,

$$\begin{aligned} f(x) = 3\sin^{-1} 7x &\Rightarrow f'(x) = 3 \times \frac{7}{\sqrt{1-(7x)^2}} \\ &\Rightarrow \underline{\underline{f'(x) = \frac{21}{\sqrt{1-49x^2}}}}. \end{aligned}$$

2. Write down the binomial expansion of (4)

$$\left(x^2 - \frac{5}{x}\right)^4$$

and simplify your answer.

Solution

Now,

$$\begin{aligned} &\left(x^2 - \frac{5}{x}\right)^4 \\ = \;& (x^2)^4 + \binom{4}{1}(x^2)^3\left(-\frac{5}{x}\right) + \binom{4}{2}(x^2)^2\left(-\frac{5}{x}\right)^2 + \binom{4}{3}(x^2)\left(-\frac{5}{x}\right)^3 + \left(-\frac{5}{x}\right)^4 \\ &= \underline{\underline{x^8 - 20x^5 + 150x^2 - \frac{500}{x} + \frac{625}{x^4}}}. \end{aligned}$$

3. (a) Find and simplify the Maclaurin expansion, up to and including the term in x^3, for:

(i) e^{3x}, (2)

Solution

Well,

$$\begin{aligned} e^{3x} &= 1 + (3x) + \tfrac{1}{2!}(3x)^2 + \tfrac{1}{3!}(3x)^3 + \dots \\ &= \underline{\underline{1 + 3x + \tfrac{9}{2}x^2 + \tfrac{9}{2}x^3 + \dots}}. \end{aligned}$$

(ii) $\ln(1 + x)$. (2)

Solution

$$\ln(1 + x) = \underline{\underline{x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \dots}}.$$

(b) Hence find and simplify the Maclaurin expansion, up to and including the term in x^3, for (2)

$$e^{3x} \ln\left(\frac{1}{1+x}\right).$$

Solution

Now,

$$\begin{aligned} e^{3x} \ln\left(\frac{1}{1+x}\right) &= e^{3x} \ln\left(1 + x\right))^{-1} \\ &= -e^{3x} \ln\left(1 + x\right)) \\ &= -(1 + 3x + \tfrac{9}{2}x^2 + \tfrac{9}{2}x^3 + \dots)(x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \dots) \end{aligned}$$

$\times$	1	$+3x$	$+\frac{9}{2}x^2$	$+\frac{9}{2}x^3$
x	x	$+3x^2$	$+\frac{9}{2}x^3$	$\dots$
$-\frac{1}{2}x^2$	$-\frac{1}{2}x^2$	$-\frac{3}{2}x^3$	$\dots$	$\dots$
$+\frac{1}{3}x^3$	$+\frac{1}{3}x^3$	$\dots$	$\dots$	$\dots$

$$\begin{aligned} &= -(x + \tfrac{5}{2}x^2 + \tfrac{10}{3}x^3 + \dots) \\ &= \underline{\underline{-x - \tfrac{5}{2}x^2 - \tfrac{10}{3}x^3 + \dots}}. \end{aligned}$$

4. (a) Use the Euclidean algorithm to find d, the greatest common divisor of 1 428 and 567. (1)

Solution

Well,

$$\begin{aligned} 1\,428 &= 2 \times 567 + 294 \\ 567 &= 1 \times 294 + 273 \\ 294 &= 1 \times 273 + 21 \\ 273 &= 13 \times 21 + 0, \end{aligned}$$

and so the greatest common divisor of 1 428 and 567 is $\underline{\underline{21}}$.

(b) Find integers a and b such that (2)

$$1\,428a + 567b = d.$$

Solution

Now,

$$\begin{aligned} 21 &= 294 - 273 \\ &= 294 - (567 - 294) \\ &= 2 \times 294 - 567 \\ &= 2(1\,428 - 2 \times 567) - 567 \\ &= \underline{\underline{2 \times 1\,428 - 5 \times 567}}; \end{aligned}$$

hence, $\underline{\underline{a = 2}}$ and $\underline{\underline{b = -5}}$.

5. Given (5)

$$y = x^{4x},$$

use logarithmic differentiation to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.
Write your answer in terms of x.

Solution

Logarithmic differentiation:

$$\begin{aligned} y = x^{4x} &\Rightarrow \ln y = \ln x^{4x} \\ &\Rightarrow \ln y = 4x \ln x \end{aligned}$$

quotient rule:

$$\begin{aligned} u = 4x &\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4 \\ v = \ln x &\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x} \end{aligned}$$

$$\begin{aligned} &\Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = (4x)\left(\frac{1}{x}\right) + (4)(\ln x) \\ &\Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + 4\ln x \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = (4 + 4\ln x)x^{4x}}}. \end{aligned}$$

6. (a) An arithmetic sequence has terms

$$u_3 = 6 \text{ and } u_{11} = 10.$$

For this sequence, find the:

(i) common difference, (1)

Solution
Let the first term be a and let the common difference be d. Then,

$$\begin{aligned} a + 2d = 6 \quad (1) \\ a + 10d = 10 \quad (2). \end{aligned}$$

Do (2) − (1):

$$8d = 4 \Rightarrow \underline{\underline{d = \tfrac{1}{2}}}.$$

(ii) first term, and (1)

Solution

Now,

$$\begin{aligned} d = \tfrac{1}{2} &\Rightarrow a + 2(\tfrac{1}{2}) = 6 \\ &\Rightarrow a + 1 = 6 \\ &\Rightarrow \underline{\underline{a = 5}}. \end{aligned}$$

(iii) sum of the first 109 terms. (1)

Solution
Next,

$$\begin{aligned} S_{109} &= \tfrac{1}{2}n[2a + (n-1)d] \\ &= \tfrac{1}{2} \times 109 \times [(2 \times 5) + (108 \times \tfrac{1}{2})] \\ &= \tfrac{109}{2} \times 64 \\ &= \underline{\underline{3\,488}}. \end{aligned}$$

(b) The terms

$$v_3 = 18 \text{ and } v_4 = 27$$

form part of a geometric sequence.

For this sequence, find:

(i) common ratio, (1)

Solution
Let the first term be a and let the common ratio be r. Then,

$$\begin{aligned} ar^2 &= 18 \quad (1) \\ ar^3 &= 27 \quad (2). \end{aligned}$$

Do (2) $\div$ (1):

$$r = \frac{ar^3}{ar^2} = \frac{27}{18} = \underline{\underline{\tfrac{3}{2}}}.$$

(ii) first term, and (1)

Solution

Next,

$$\begin{aligned} r = \tfrac{3}{2} &\Rightarrow a(\tfrac{3}{2})^2 = 18 \\ &\Rightarrow \tfrac{9}{4}a = 18 \\ &\Rightarrow \underline{\underline{a = 8}}. \end{aligned}$$

(iii) an expression, in terms of n, for the sum of the first n terms. (1)

Solution
Well,

$$\begin{aligned} S_n &= \frac{a(r^n - 1)}{r - 1} \\ &= \frac{8[(\frac{3}{2})^n - 1]}{\frac{1}{2}} \\ &= \underline{\underline{16[(\tfrac{3}{2})^n - 1]}}. \end{aligned}$$

(c) Find **algebraically** the least value of n such that the sum of the geometric series exceeds the sum of the first 109 terms in the arithmetic sequence (1)

Solution
Now,

$$\begin{aligned} S_n > 3\,488 &\Rightarrow 16[(\tfrac{3}{2})^n - 1] > 3\,488 \\ &\Rightarrow (\tfrac{3}{2})^n - 1 > 218 \\ &\Rightarrow (\tfrac{3}{2})^n > 219 \\ &\Rightarrow \log_{10}(\tfrac{3}{2})^n > \log_{10} 219 \\ &\Rightarrow n \log_{10} \tfrac{3}{2} > \log_{10} 219 \\ &\Rightarrow n > \frac{\log_{10} 219}{\log_{10} \frac{3}{2}} \\ &\Rightarrow n > 13.29\ldots; \end{aligned}$$

hence, $\underline{\underline{n = 14}}$.

7. A curve is defined parametrically by

$$x = 3\ln(2t + 1),\ y = t - \tfrac{1}{2}t^2,\ \text{where } t > 0.$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (2)
Simplify your answer.

Solution

Well,

$$x = 3\ln(2t+1) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{6}{2t+1}$$

and

$$y = t - \tfrac{1}{2}t^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 1 - t.$$

Now,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} \\ &= \frac{1-t}{\frac{6}{2t+1}} \\ &= \underline{\underline{\tfrac{1}{6}(2t+1)(1-t)}}.\end{aligned}$$

(b) Find the coordinates of the stationary point on the curve. (2)

Solution

Next,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow \tfrac{1}{6}(2t+1)(1-2t) = 0 \\ &\Rightarrow 2t+1 = 0 \text{ or } 1-t = 0 \\ &\Rightarrow t = -\tfrac{1}{2} \text{ or } t = 1.\end{aligned}$$

Then,

$$t = -\tfrac{1}{2} \Rightarrow x = \text{undefined}$$

and

$$t = 1 \Rightarrow x = 3\ln 3,\ y = \tfrac{1}{2};$$

hence, the coordinates of the stationary point on the curve are $\underline{\underline{(3\ln 3, \tfrac{1}{2})}}$.

8. The volume, V cubic metres, of water held in a reservoir is given by (4)

$$V = 18(2+\sqrt{h})^6 - 1\,152,$$

where h metres is the depth of water.

Water is pumped out of the reservoir at a constant rate of $0.6 \text{ m}^3\,\text{s}^{-1}$.

Find the rate of change of the depth of water in the reservoir when the depth is 9 metres.

Solution

Well,

$$\begin{aligned} V = 18(2+\sqrt{h})^6 - 1\,152 &\Rightarrow V = 18(2+h^{\frac{1}{2}})^6 - 1\,152 \\ &\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = 108(2+h^{\frac{1}{2}})^5 \times \tfrac{1}{2}h^{-\frac{1}{2}} \\ &\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = 54h^{-\frac{1}{2}}(2+h^{\frac{1}{2}})^5 \end{aligned}$$

and

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -0.6.$$

Now,

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} &\Rightarrow -0.6 = 54h^{-\frac{1}{2}}(2+h^{\frac{1}{2}})^5 \times \frac{\mathrm{d}h}{\mathrm{d}t} \\ &\Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-0.6}{54h^{-\frac{1}{2}}(2+h^{\frac{1}{2}})^5}. \end{aligned}$$

Next,

$$\begin{aligned} h = 9 &\Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{0.6}{54(\frac{1}{3})(2+3)^5} \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}h}{\mathrm{d}t} = -\tfrac{1}{93\,750} \text{ m}\,\text{s}^{-1}}}. \end{aligned}$$

9. Express (3)

$$3\,442_5$$

in base 9.

Solution

Well,

$$\begin{aligned} 3\,442_5 &= (3 \times 5^3) + (4 \times 5^2) + (4 \times 5) + 2 \\ &= 375 + 100 + 20 + 2 \\ &= 497_{10}, \end{aligned}$$

and

$$\begin{aligned} 497_{10} &= (6 \times 9^2) + 11 \\ &= (6 \times 9^2) + (1 \times 9) + 2 \\ &= \underline{\underline{612_9}}. \end{aligned}$$

10. A curve is defined by the equation

$$x^2 e^{6y} + x^2 + y^5 = 50.$$

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. (4)

Solution
Implicit differeniation:

$$\begin{aligned} & x^2 e^{6y} + x^2 + y^5 = 50 \\ \Rightarrow\quad & (2x)(e^{6y}) + (x^2)\left(6e^{6y}\frac{dy}{dx}\right) + 2x + 5y^4\frac{dy}{dx} = 0 \\ \Rightarrow\quad & 6x^2 e^{6y}\frac{dy}{dx} + 5y^4\frac{dy}{dx} = -2x - 2xe^{6y} \\ \Rightarrow\quad & (6x^2 e^{6y} + 5y^4)\frac{dy}{dx} = -2x(1 + e^{6y}) \\ \Rightarrow\quad & \underline{\underline{\frac{dy}{dx} = \frac{-2x(1 + e^{6y})}{6x^2 e^{6y} + 5y^4}}}. \end{aligned}$$

(b) Given $x > 0$, explain why the derivative is never zero. (1)

Solution
Well, $e^{6y} > 0$ and $5y^4 \geqslant 0$ so the denominator is always positive.
Hence, the derivative is $\underline{\underline{\text{never zero}}}$.

11. Find (4)

$$\int \left(\frac{2x+4}{x^2+4}\right)\,\mathrm{d}x.$$

Solution

Well,

$$\begin{aligned}\int \left(\frac{2x+4}{x^2+4}\right)\mathrm{d}x &= \int \left(\frac{2x}{x^2+4}\right)\mathrm{d}x + \int \left(\frac{4}{x^2+4}\right)\mathrm{d}x\\ &= \int \left(\frac{2x}{x^2+4}\right)\mathrm{d}x + 4\int \left(\frac{1}{x^2+2^2}\right)\mathrm{d}x\\ &= \underline{\underline{\ln|x^2+4| + 2\tan^{-1}(\tfrac{1}{2}x) + c.}}\end{aligned}$$

12. Prove by induction that (5)

$$7^n + 2$$

is divisible by 3 for all $n \in \mathbb{N}$.

Solution

$\underline{n = 1}$:

$$\begin{aligned}7^1 + 2 &= 7 + 2\\ &= 9\\ &= 3 \times 3,\end{aligned}$$

so the result is true of $n = 1$.

Suppose the result is true for some $n = k$, i.e.,

$$7^k + 2 = 3m,$$

for some integer m. Now,

$$7^k + 2 = 3m \Rightarrow 7^k = 3m - 2$$

and

$$\begin{aligned} 7^{k+1} + 2 &= 7 \times 7^k + 2 \\ &= 7(3m - 2) + 2 \\ &= 21m - 14 + 2 \\ &= 21m - 12 \\ &= 3(7m - 4), \end{aligned}$$

and $7^{k+1} + 2$ is divisible by 3.

Hence, by mathematical induction, $7^n + 2$ is divisible by 3 for all $n \in \mathbb{N}$.

13. (a) Express using partial fractions (2)

$$\frac{x+3}{(x+7)(x+5)}.$$

Solution
Well,

$$\begin{aligned} \frac{A}{(x+7)} + \frac{B}{(x+5)} &\equiv \frac{A(x+5)}{(x+7)(x+5)} + \frac{B(x+7)}{(x+7)(x+5)} \\ &\equiv \frac{Ax + 5A + Bx + 7B}{(x+7)(x+5)} \\ &\equiv \frac{(A+B)x + (5A+7B)}{(x+7)(x+5)}, \end{aligned}$$

so

$$\begin{aligned} A + B = 1 &\quad (1) \\ 5A + 7B = 3 &\quad (2). \end{aligned}$$

Now,

$$A + B = 1 \Rightarrow B = 1 - A \quad (3)$$

and insert in to (2):

$$\begin{aligned} 5A + 7B = 3 &\Rightarrow 5A + 7(1 - A) = 3 \\ &\Rightarrow 5A + 7 - 7A = 3 \\ &\Rightarrow -2A = -4 \\ &\Rightarrow A = 2 \\ &\Rightarrow B = -1; \end{aligned}$$

hence,

$$\frac{x+3}{(x+7)(x+5)} = \underline{\underline{\frac{2}{(x+7)} - \frac{1}{(x+5)}}}.$$

(b) Hence find the particular solution of the differential equation (7)

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{2}{x+3}\right) y = \frac{1}{(x+7)(x+5)(x+3)}, \text{ where } x \geqslant 0,$$

given that $y = 0$ when $x = 3$.

Solution

Integrating factor:

$$\begin{aligned} \text{IF} &= \mathrm{e}^{\int \frac{2}{x+3}\,\mathrm{d}x} \\ &= \mathrm{e}^{2\ln(x+3)} \\ &= \mathrm{e}^{\ln(x+3)^2} \\ &= (x+3)^2. \end{aligned}$$

Now,

$$\begin{aligned} & \frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{2}{x+3}\right) y = \frac{1}{(x+7)(x+5)(x+3)} \\ \Rightarrow\quad & (x+3)^2 \left[\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{2}{x+3}\right) y\right] = \frac{x+3}{(x+7)(x+5)} \\ \Rightarrow\quad & \frac{\mathrm{d}}{\mathrm{d}x}\left[y(x+3)^2\right] = \frac{x+3}{(x+7)(x+5)} \\ \Rightarrow\quad & \frac{\mathrm{d}}{\mathrm{d}x}\left[y(x+3)^2\right] = \frac{2}{(x+7)} - \frac{1}{(x+5)} \\ \Rightarrow\quad & y(x+3)^2 = \int \left(\frac{2}{(x+7)} - \frac{1}{(x+5)}\right) \mathrm{d}x \\ \Rightarrow\quad & y(x+3)^2 = 2\ln(x+7) - \ln(x+5) + c. \end{aligned}$$

Next,

$$\begin{aligned} x = 3, y = 0 \Rightarrow 0 &= 2\ln 10 - \ln 8 + c \\ &\Rightarrow -\ln 100 + \ln 8 = c \\ &\Rightarrow \ln \tfrac{2}{25} = c. \end{aligned}$$

Hence,

$$\begin{aligned} y(x+3)^2 &= 2\ln(x+7) - \ln(x+5) + \ln \tfrac{2}{25} \\ \Rightarrow \quad y &= \underline{\underline{\frac{2\ln(x+7) - \ln(x+5) + \ln \frac{2}{25}}{(x+3)^2}}}. \end{aligned}$$

14. The plane π contains the points $P(2, 3, -4)$, $Q(3, 5, 1)$, and $R(6, 0, -6)$.

(a) Determine the Cartesian equation of π. (4)

Solution

Well,

$$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \text{ and } \overrightarrow{PR} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

Now,

$$\begin{aligned} \overrightarrow{PQ} \times \overrightarrow{PR} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 5 \\ 4 & -3 & -2 \end{vmatrix} \\ &= 11\mathbf{i} + 22\mathbf{j} - 11\mathbf{k} \\ &= 11(\mathbf{i} + 2\mathbf{j} - \mathbf{k}), \end{aligned}$$

so, $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ is a normal vector.

Next,

$$\begin{aligned} x + 2y - z &= 2 + 2(3) - (-4) \\ &= 12; \end{aligned}$$

hence, the Cartesian equation of π is

$$\underline{\underline{x + 2y - z = 12}}.$$

The line L has symmetric equations

$$\frac{x-8}{2} = \frac{y+2}{-1} = \frac{z+2}{3}.$$

L intersects π at the point S.

(b) Find the coordinates of S. (3)

Solution
Well,

$$\frac{x-8}{2} = \frac{y+2}{-1} = \frac{z+2}{3} = \lambda$$

and that gives us

$$\begin{aligned}\frac{x-8}{2} = \lambda &\Rightarrow x - 8 = 2\lambda \\ &\Rightarrow x = 2\lambda + 8, \\ \frac{y+2}{-1} = \lambda &\Rightarrow y + 2 = -\lambda \\ &\Rightarrow y = -\lambda - 2, \\ \frac{z+2}{3} = \lambda &\Rightarrow z + 2 = 3\lambda \\ &\Rightarrow z = 3\lambda - 2.\end{aligned}$$

Substitute into π:

$$\begin{aligned}x + 2y - z = 12 &\Rightarrow (2\lambda + 8) + 2(-\lambda - 2) - (3\lambda - 2) = 12 \\ &\Rightarrow 2\lambda + 8 - 2\lambda - 4 - 3\lambda + 2 = 12 \\ &\Rightarrow -3\lambda = 6 \\ &\Rightarrow \lambda = -2.\end{aligned}$$

Hence, the coordinates of $\underline{\underline{S(4, 0, -8)}}$.

(c) Calculate the size of the acute angle between L and π. (3)

Solution
The direction vector is

$$\mathbf{d} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}.$$

Now,

$$\begin{aligned}\mathbf{n}.\mathbf{d} &= 2 - 2 - 3 \\ &= -3,\end{aligned}$$

$$\begin{aligned}|\mathbf{n}| &= \sqrt{1^2 + 2^2 + (-1)^2} \\ &= \sqrt{6},\end{aligned}$$

and

$$\begin{aligned}|\mathbf{d}| &= \sqrt{2^2 + (-1)^2 + 3^2}\\ &= \sqrt{14}.\end{aligned}$$

Finally,

$$\cos\theta = \frac{-3}{\sqrt{6} \times \sqrt{14}} \Rightarrow \theta = 109.106\,605\,4 \text{ (FCD)}.$$

But it says "the size of the acute angle":

$$\begin{aligned}109.106\ldots - 90 &= 19.106\,605\,4 \text{ (FCD)}\\ &= \underline{\underline{19.1^\circ \text{ (3 sf)}}}.\end{aligned}$$

15. Let r be a positive real number and consider the following statement:

If r is irrational, then $\sqrt{r}$ is irrational.

(a) Write down the contrapositive of the statement. (1)

Solution
<u>If $\sqrt{r}$ is rational, the r is rational</u>.

(b) Hence prove that the statement is true. (3)

Solution
Suppose that $\sqrt{r}$ is rational. Then,

$$\sqrt{r} = \tfrac{p}{q},$$

where p, $q \in \mathbb{Z}$. Now,

$$r = \left(\tfrac{p}{q}\right)^2 = \tfrac{p^2}{q^2},$$

so r is rational.

Hence,

<u>if r is irrational, then $\sqrt{r}$ is irrational</u>.

16. (a) Given (2)

$$y = \ln(\cos x),\ 0 \leqslant x < \tfrac{1}{2}\pi,$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\tan x.$$

Solution
Well,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{\cos x} \times (-\sin x) \\ &= -\frac{\sin x}{\cos x} \\ &= \underline{\underline{-\tan x}},\end{aligned}$$

as required.

For a function $\mathrm{g}(x)$, it is known that

$$\int x\,\mathrm{g}(x)\,\mathrm{d}x = 2x\tan 2x - \int 2\tan 2x\,\mathrm{d}x.$$

(b) (i) Determine the exact value of (2)

$$\int_0^{\frac{1}{6}\pi} x\,\mathrm{g}(x)\,\mathrm{d}x.$$

Solution
Well,

$$\begin{aligned}\int_0^{\frac{1}{6}\pi} x\,\mathrm{g}(x)\,\mathrm{d}x &= [2x\tan 2x]_{x=0}^{\frac{1}{6}\pi} - \int_0^{\frac{1}{6}\pi} 2\tan 2x\,\mathrm{d}x \\ &= \left(\tfrac{1}{3}\pi \times \sqrt{3} - 0\right) - [-\ln(\cos 2x)]_{x=0}^{\frac{1}{6}\pi} \\ &= \tfrac{1}{\sqrt{3}}\pi - \left(-\ln\tfrac{1}{2} + 0\right) \\ &= \underline{\underline{\tfrac{1}{\sqrt{3}}\pi + \ln\tfrac{1}{2}}}.\end{aligned}$$

(ii) Find an expression for $\mathrm{g}(x)$ in terms of x. (1)

Solution
Think about

$$\int uv' = uv - \int u'v.$$

So

$$u = x \text{ and } v = 2\tan 2x.$$

Finally,

$$v = 2\tan 2x \Rightarrow v' = 4\sec^2 2x;$$

hence,

$$\underline{\underline{\mathrm{g}(x) = 4\sec^2 2x}}.$$