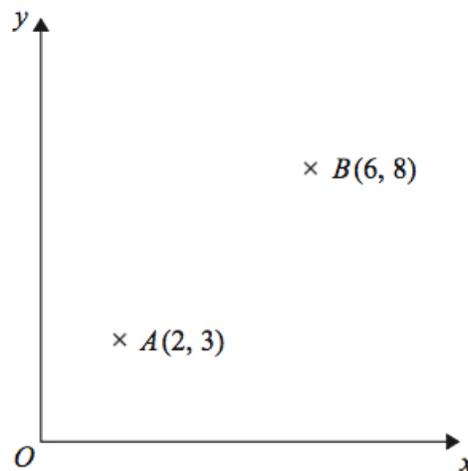# Dr Oliver Mathematics
GCSE Mathematics
2014 June Paper 2H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. The point A has coordinates $(2, 3)$.   (2)
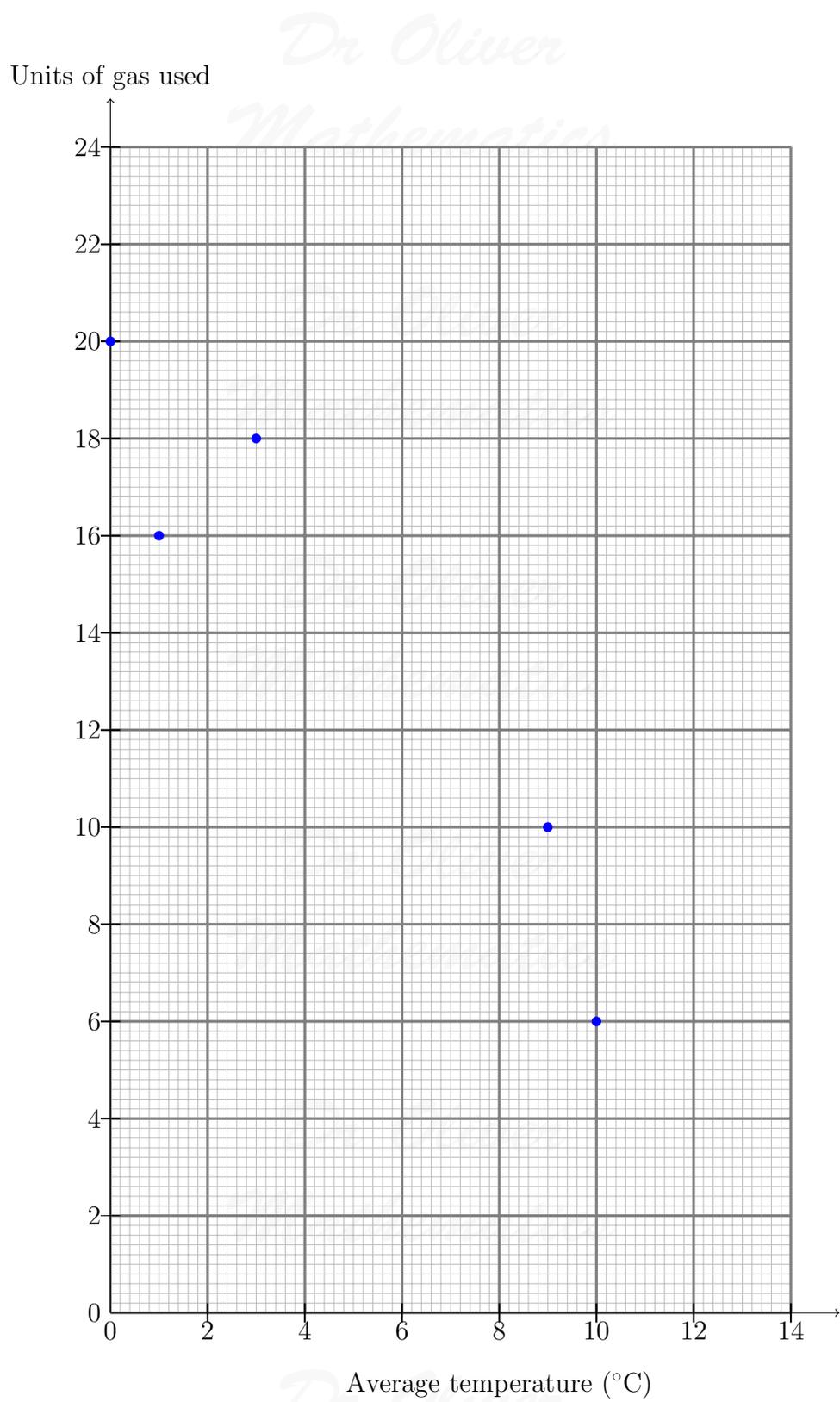   The point B has coordinates $(6, 8)$.

   M is the midpoint of the line AB.

   Find the coordinates of M.

   Solution

   $$\left(\frac{2+6}{2}, \frac{3+8}{2}\right) = (4, 5\tfrac{1}{2}).$$

2. The table shows the average temperature on each of seven days and the number of units of gas used to heat a house on these days.

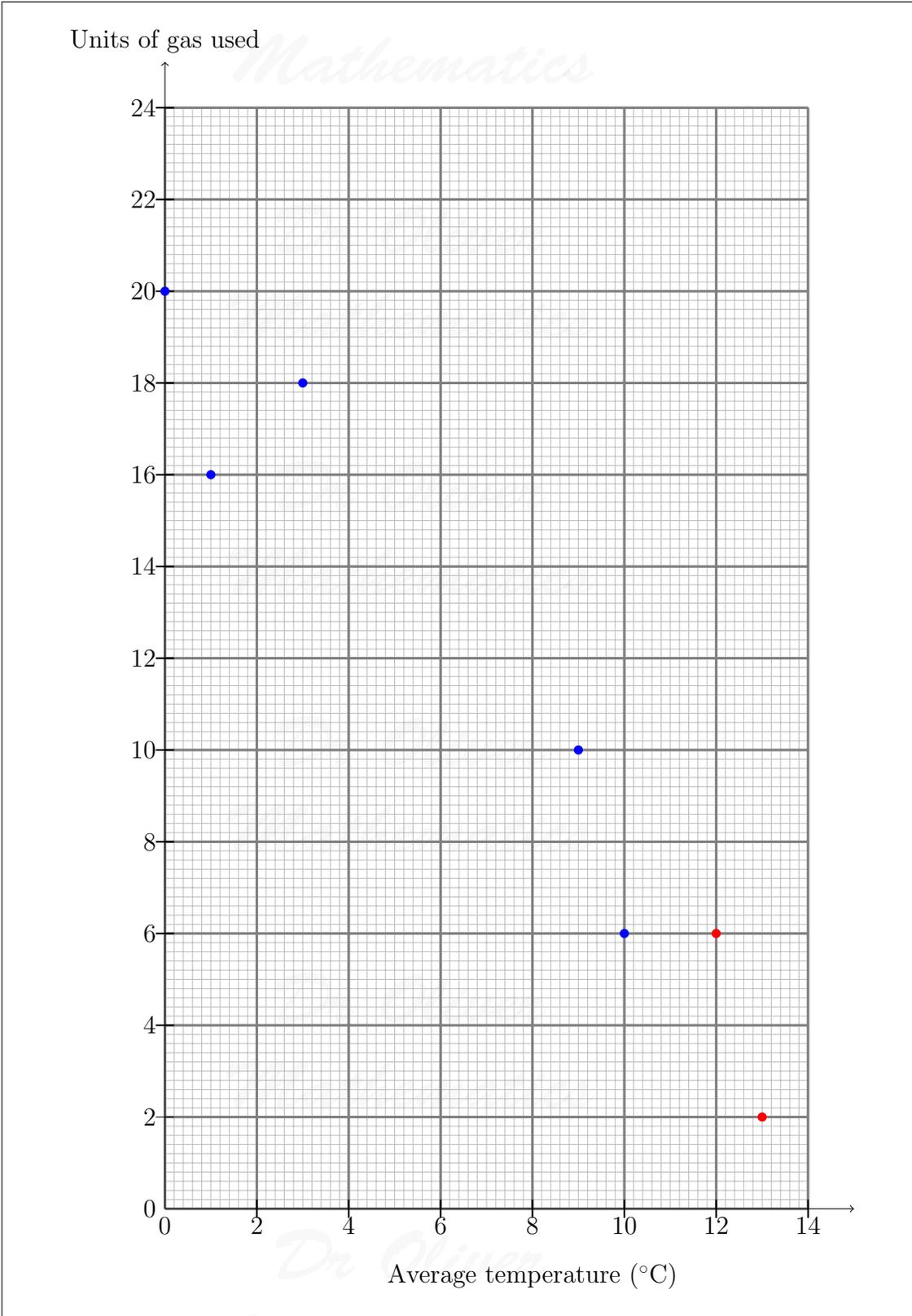| Average temperature (°C)	0	1	3	9	10	12	13
Units of gas used	20	16	18	10	6	6	2

Units of gas used

Average temperature (°C)

(a) Complete the scatter graph to show the information in the table. (1)

The first 5 points have been plotted for you.

(b) Describe the relationship between the average temperature and the number of units (1)
of gas used.

> **Solution**
> <u>Negative correlation</u>, i.e., as average temperature goes up, the number of units of gas goes down.

(c) Estimate the average temperature on a day when 12 units of gas are used. (2)
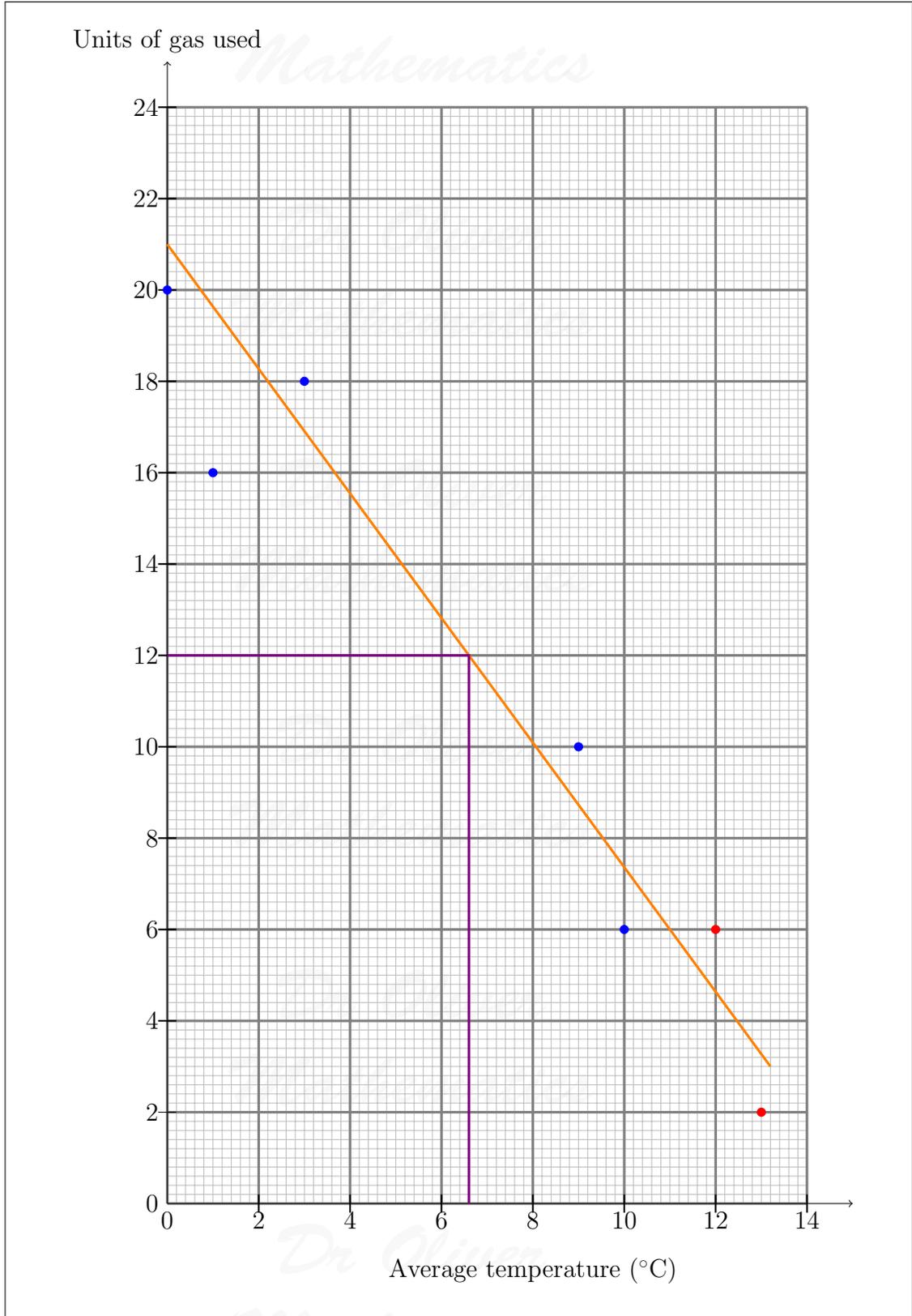
> **Solution**
> Draw a line of best fit:

Correct read-off: approximately 6.6°C.

3. $x = 0.7$. (2)

Work out the value of
$$\frac{(x + 1)^2}{2x}.$$
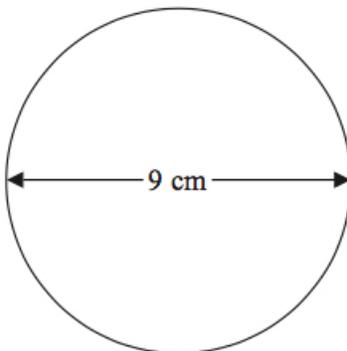
Write down all the figures on your calculator display.

Solution

$$\frac{(0.7 + 1)^2}{2 \times 0.7} = \frac{2.89}{1.4}$$
$$= 2.064\,285\,714 \text{ (FCD)}.$$

4. Here is a circle. (2)

The diameter of the circle is 9 cm.

Work out the circumference of this circle.
Give your answer correct to 3 significant figures.

Solution

Radius = 4.5 cm

and the

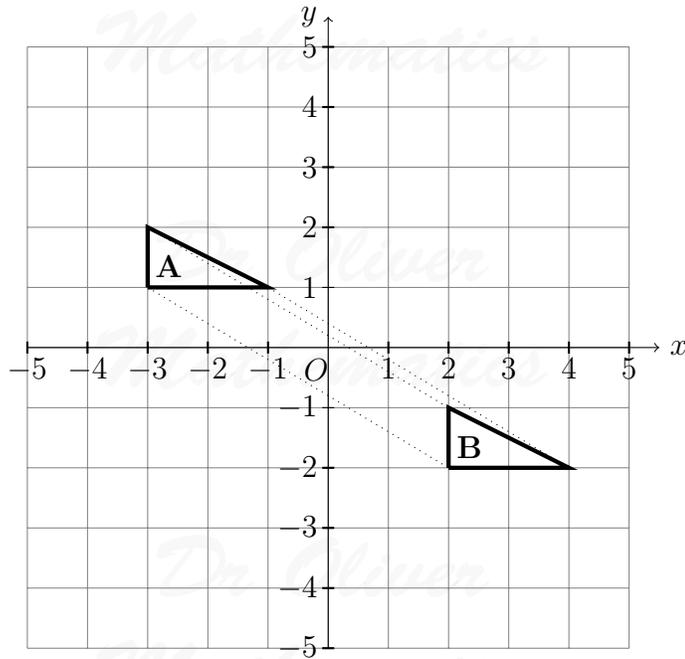$$\text{circumference} = 2 \times \pi \times 4.5$$
$$= 28.274\,333\,88 \ (\text{FCD})$$
$$= \underline{\underline{28.3 \text{ cm } (3 \text{ sf})}}.$$

5. Describe the single transformation that maps triangle **A** onto triangle **B**. (2)
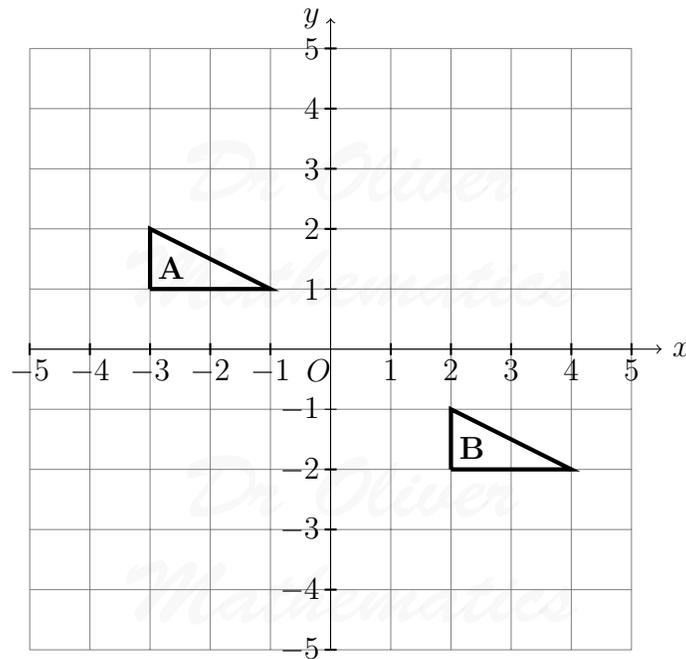
Solution

It is a <u>translation</u> by $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.

6. Sue is driving home from her friend's house. (3)

Sue drives 10 miles from her friend's house to the motorway, 240 miles on the motorway, and 5 miles from the motorway to her home.

Sue takes 20 minutes to drive from her friend's house to the motorway, drives at an average speed of 60 mph on the motorway, and takes 25 minutes to drive from the motorway to her home.

Sue stops for a 30 minute rest on her drive home.

Sue leaves her friend's house at 9.00 am.

What time does Sue get home?
You must show all your working.

Solution

She takes

$$\frac{20}{60} + \frac{240}{60} + \frac{25}{60} + \frac{30}{60} = 5.25 \text{ hours.}$$

Now,

$$\text{time} = 9 + 5.25 = 14:15 \text{ or } 2:15 \text{ pm.}$$

7. PRS and TWY are parallel straight lines. (3)
   $QRWZ$ is a straight line.
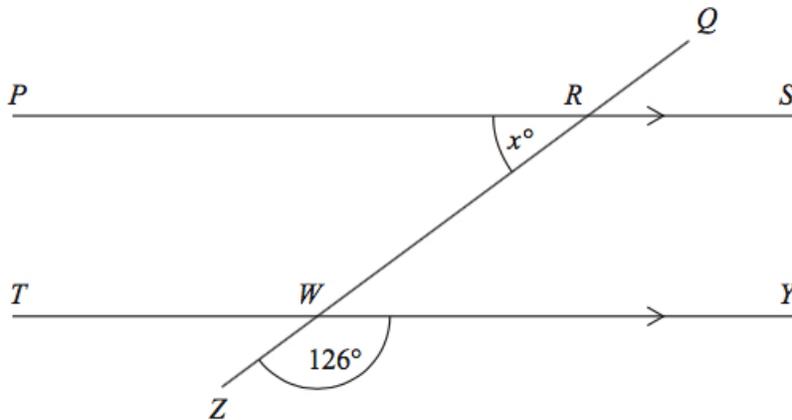
Work out the value of x.
Give reasons for your answer.

Solution

$\angle WRS = 126°$ (corresponding angles)
$x = 180 - 126 = \underline{54}$ (supplementary angles)

8. Lorna carries out a survey about the number of times customers go to a shop.

   She asks at random 100 customers how many times they went to the shop last month.

   The table shows Lorna's results.

Number of times	0	1	2	3	4	5	6	more than 6
Frequency	4	12	13	17	25	13	11	5

10

One of the 100 customers is chosen at random.

(a) What is the probability that this customer went to the shop 5 or more times? (2)

Solution

$$P(5 \text{ or more times}) = \frac{13 + 11 + 5}{100}$$

$$= \frac{29}{100}.$$

Last month the shop had a total of 1 500 customers.

(b) Work out an estimate for the number of customers who went to the shop exactly 2 times last month. (2)

Solution

$$\frac{13}{100} \times 1\,500 = \underline{\underline{195}}.$$

The owner of a different shop is carrying out a survey on the ages of his customers.
He records the ages of the first 10 customers in his shop after 9 am one morning.

(c) This may **not** be a suitable sample. (2)
Give **two** reasons why.

Solution

E.g., it is biased, sample too small, customers at this time may not be representative of ages of all customers.

9. The diagram shows a trapezium. (4)

Diagram **NOT** accurately drawn
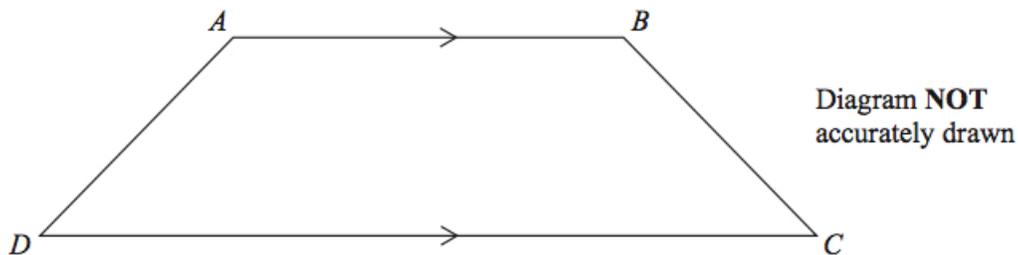
$AD = x$ cm.
BC is the same length as AD.

AB is twice the length of AD.
DC is 4 cm longer than AB.

The perimeter of the trapezium is 38 cm.

Work out the length of AD.

Solution

$$x + x + 2x + (2x + 4) = 38 \Rightarrow 6x + 4 = 38$$
$$\Rightarrow 6x = 34$$
$$\Rightarrow x = 5\tfrac{2}{3}.$$

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10. (a) Simplify (1)
$$(p^3)^2.$$

Solution

$$(p^3)^2 = \underline{\underline{p^6}}.$$

(b) Simplify (1)
$$\frac{t^8}{t^3}.$$

Solution

$$\frac{t^8}{t^3} = t^{8-3} = \underline{\underline{t^5}}.$$

$2^3 \times 2^n = 2^9.$

(c) Work out the value of n. (1)

Solution

$$2^3 \times 2^n = 2^9 \Rightarrow 2^{3+n} = 2^9$$
$$\Rightarrow 3 + n = 9$$
$$\Rightarrow \underline{\underline{n = 6}}.$$

$2x^3 = 128.$

(d) Work out the value of x. (1)

Solution

$$2x^3 = 128 \Rightarrow x^3 = 64$$
$$\Rightarrow \underline{\underline{x = 4}}.$$

11. Here is a plan of Martin's driveway. (5)

4 m

11 m
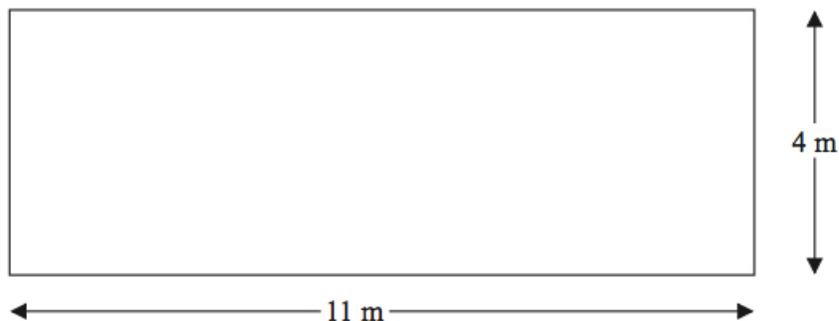
Martin is going to cover his driveway with gravel.
The gravel will be 6 cm deep.

Gravel is sold in bags.
There are 0.4 m^3 of gravel in each bag.
Each bag of gravel costs £38.

Martin gets a discount of 30% off the cost of the gravel.

Work out the total amount of money Martin pays for the gravel.

Solution

Now,

$$\text{volume} = 11 \times 4 \times 0.06$$
$$= 2.64 \text{ m}^3$$

and he needs

$$\frac{2.64}{0.4} = 6.6 = 7 \text{ bags}.$$

Finally,

$$\text{cost} = 38 \times 7 \times 0.7 = \underline{\underline{£186.20}}.$$

12. Here are the first five terms of an arithmetic sequence.

$$4 \qquad 9 \qquad 14 \qquad 19 \qquad 24$$

(a) Find, in terms of n, an expression for the nth term of this sequence. (2)

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{ccccccc} 4 & & 9 & & 14 & & 19 \\ & 5 & & 5 & & 5 & \\ a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 5$$

and

$$a + b = 10 \Rightarrow 5 + b = 4$$
$$\Rightarrow b = -1.$$

Hence,

$$n\text{th term} = \underline{\underline{5n - 1}}.$$

Here are the first five terms of a different sequence.

$$2 \qquad 2 \qquad 0 \qquad -4 \qquad -10$$

An expression for the nth term of this sequence is $3n - n^2$.

(b) Write down, in terms of n, an expression for the nth term of a sequence whose first five terms are (1)

$$4 \qquad 4 \qquad 0 \qquad -8 \qquad -20$$

> **Solution**
>
> $\underline{\underline{2(3n - n^2)}}$ or $\underline{\underline{6n - 2n^2}}$.

13. $-5 < y \leqslant 0$.
    y is an integer.

    (a) Write down all the possible values of y. (2)

    > **Solution**
    >
    > $\underline{\underline{-4, -3, -2, -1, 0}}$.

    (b) Solve (2)

    $$6(x - 2) > 15.$$

    > **Solution**
    >
    > $$6(x - 2) > 15 \Rightarrow x - 2 > 2\tfrac{1}{2}$$
    > $$\Rightarrow \underline{\underline{x > 4\tfrac{1}{2}}}.$$

14. Ali is planning a party. (5)

    He wants to buy some cakes and some sausage rolls.

    The cakes are sold in boxes.
    There are 12 cakes in each box.
    Each box of cakes costs £2.50.

    The sausage rolls are sold in packs.

15

There are 8 sausage rolls in each pack.
Each pack of sausage rolls costs £1.20.

Ali wants to buy more than 60 cakes and more than 60 sausage rolls.
He wants to buy exactly the same number of cakes as sausage rolls.

What is the least amount of money Ali will have to pay?

Solution

Cakes:
$$12 = 2^2 \times 3.$$

Sausage rolls:
$$8 = 2^3.$$

Now,
$$\text{LCM}(12, 8) = 2^3 \times 3 = 24$$
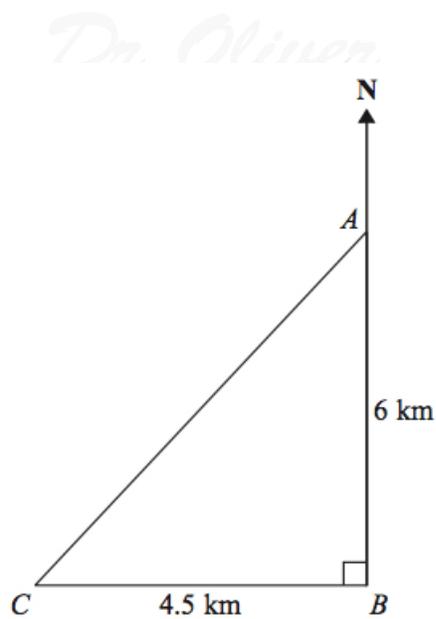
and he will have to buy
$$\frac{60}{24} = 2.5 \to 3 \quad (72 \text{ items}).$$

Finally, the total cost is

$$\left(\frac{72}{12} \times 2.50\right) + \left(\frac{72}{8} \times 1.20\right) = (6 \times 2.50) + (9 \times 1.20)$$
$$= 15 + 10.80$$
$$= \underline{\underline{£25.80}}.$$

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15. The diagram shows the positions of three turbines A, B, and C.

A is 6 km due north of turbine B.
C is 4.5 km due west of turbine B.

(a) Calculate the distance AC. (3)

Solution

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{6^2 + 4.5^2}$$
$$= \underline{7.5 \text{ km}}.$$

(b) Calculate the bearing of C from A. (4)
Give your answer correct to the nearest degree.

Solution

$$\text{Bearing} = 180 + \tan^{-1}\left(\frac{4.5}{6}\right)$$
$$= 216.869\,897\,6 \text{ (FCD)}$$
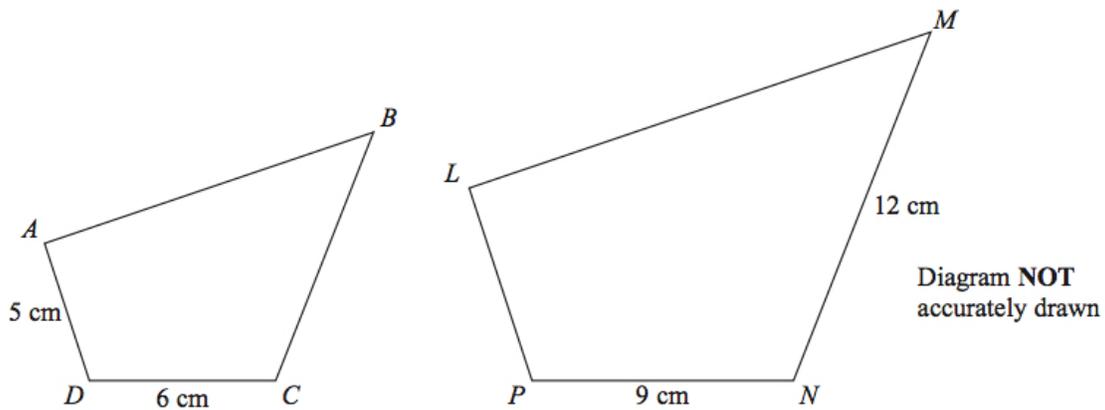$$= \underline{\underline{217° \text{ (3 sf)}}}.$$

16. Work out the value of (2)

$$(7.5 \times 10^4) \times (2.5 \times 10^3).$$

17

Give your answer in standard form.

Solution

$$(7.5 \times 10^4) \times (2.5 \times 10^3) = (7.5 \times 2.5) \times (10^4 \times 10^3)$$
$$= 18.75 \times 10^7$$
$$= \underline{\underline{1.875 \times 10^8}}.$$

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17. Quadrilaterals $ABCD$ and $LMNP$ are mathematically similar.

Diagram **NOT** accurately drawn

Angle A = angle L.
Angle B = angle M.
Angle C = angle N.
Angle D = angle P.

(a) Work out the length of LP. (2)

Solution

$$LP = \frac{9}{6} \times 5$$
$$= \underline{\underline{7.5 \text{ cm}}}.$$

(b) Work out the length of BC. (2)

18

Solution

$$BC = \frac{6}{9} \times 12$$
$$= \underline{8 \text{ cm}}.$$

18. Katie invests £200 in a savings account for 2 years.

    The account pays compound interest at an annual rate of 3.3% for the first year and 1.5% for the second year.

    (a) Work out the total amount of money in Katie's account at the end of 2 years. (3)

    Solution

    $$200 \times 1.033 \times 1.015 = 209.699$$
    $$= \underline{\underline{£209.69 \text{ (2 dp)}}}.$$

    Katie travels to work by train.
    The cost of her weekly train ticket increases by 12.5% to £225.

    Katie's weekly pay increases by 5% to £535.50.

    (b) Compare the increase in the amount of money Katie has to pay for her weekly train ticket with the increase in her weekly pay. (3)

    Solution
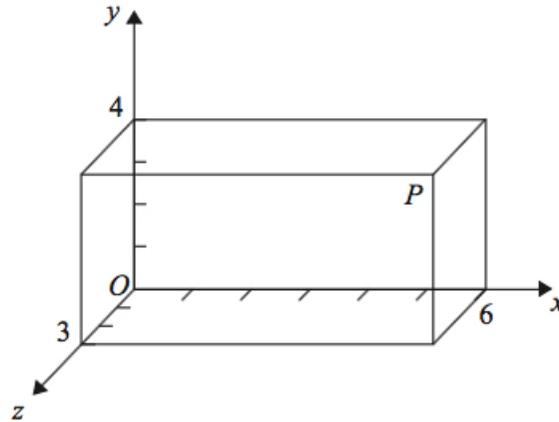   Train ticket:
    $$\frac{225}{1.125} = £200.$$
    Pay:
    $$\frac{535.5}{1.05} = £510.$$
    The pay has gone up <u>more</u> (£25.50) than the train ticket has (£25).

19. Here is a cuboid drawn on a 3-D grid. (2)

P is a vertex of the cuboid.

T divides the line OP in the ratio $1 : 2$.

Find the coordinates of T.

Solution

$P(6, 4, 3)$ and this makes $\underline{\underline{T(2, 1\frac{1}{3}, 1)}}$.

---

20. 25 students in class A did a science exam.   (3)
    30 students in class B did the same science exam.

    The mean mark for the 25 students in class A is 67.8
    The mean mark for all the 55 students is 72.0.

    Work out the mean mark for the students in class B.

Solution

Let the mean mark for the students in class B be x marks. Now,

$$\frac{(25 \times 67.8) + (30 \times x)}{25 + 30} = 72.0 \Rightarrow 1\,695 + 30x = 3\,960$$

$$\Rightarrow 30x = 2\,265$$

$$\Rightarrow \underline{\underline{x = 75.5}}$$

20

21. (a) Expand and simplify (2)
$$(y-2)(y-5).$$

Solution

×	y	-2
y	y^2	$-2y$
-5	$-5y$	$+10$

$$(y-2)(y-5) = \underline{\underline{y^2 - 7y + 10}}.$$

(b) Prove algebraically that (3)

$$(2n+1)^2 - (2n+1) \text{ is an even number}$$

for all positive integer values of n.

Solution

×	$2n$	$+1$
$2n$	$4n^2$	$+2n$
$+1$	$+2n$	$+1$

$$\begin{aligned}(2n+1)^2 - (2n+1) &= (4n^2 + 4n + 1) - (2n+1) \\ &= 4n^2 + 2n \\ &= 2(2n^2 + n)\end{aligned}$$

and, hence, it is $\underline{\underline{\text{always}}}$ an even number.

22. Shabeen has a biased coin. (3)
The probability that the coin will land on heads is 0.6.

Shabeen is going to throw the coin 3 times.

She says the probability that the coin will land on tails 3 times is less than 0.1.

Is Shabeen correct?
You must show all your working.

> **Solution**
>
> $$(1 - 0.6)^3 = 0.4^3 = 0.064;$$
>
> so, <u>yes</u>, Shabeen is correct.

23. (a) Explain what is meant by a stratified sample. (1)

> **Solution**
>
> The population is divided up into categories (strata) and then a random sample is chosen from each category. The size of each sample is in proportion to the size of each category within the population.

The table shows information about the ages of the people living in a village.

Age group	Number of people
Under 21	72
21 − 40	90
41 − 60	123
Over 60	314

Mrs Parrish carries out a survey of these people.
She uses a sample size of 50 people stratified by age group.

(b) Work out the number of people over 60 years of age in the sample. (2)

> **Solution**
>
> $$72 + 90 + 123 + 314 = 599$$
>
> and so the number of people over 60 years of age in the sample is
>
> $$\frac{314}{599} \times 50 = 26.210\ldots$$
> $$= \underline{\underline{26 \text{ persons.}}}$$

24. p is inversely proportional to t. (3)

When $t = 4$, $p = 12$.

Find the value of p when $t = 6$.

<div>

Solution

$$p \propto \frac{1}{t} \Rightarrow p = \frac{k}{t}$$

for some constant k. Now,

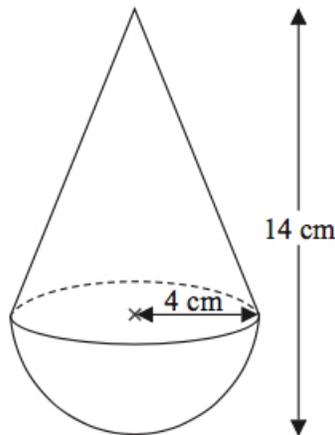$$12 = \frac{k}{4} \Rightarrow k = 48$$

and so

$$p = \frac{48}{t}.$$

Finally,

$$p = \frac{48}{6} = \underline{\underline{8}}.$$

</div>

25. The diagram shows a solid made from a hemisphere and a cone. (3)

The radius of the hemisphere is 4 cm.
The radius of the base of the cone is 4 cm.

Calculate the volume of the solid.
Give your answer correct to 3 significant figures.

Solution

The vertical height of the cone is

$$14 - 4 = 10 \text{ cm}$$

and the

$$\text{volume} = (\tfrac{1}{3} \times \pi \times 4^2 \times 10) + (\tfrac{2}{3} \times \pi \times 4^3)$$
$$= 301.592\,894\,7 \text{ (FCD)}$$
$$= \underline{\underline{302 \text{ cm}^3 \text{ (3 sf)}}}.$$

26. Solve the equations (5)

$$x^2 + y^2 = 36$$
$$x = 2y + 6.$$

Solution

Substitute the linear one into the non-linear one:

$\times$	$2y$	$+6$
$2y$	$4y^2$	$+12y$
$+6$	$+12y$	$+36$

$$(2y+6)^2 + y^2 = 36 \Rightarrow (4y^2 + 24y + 36) + y^2 = 36$$
$$\Rightarrow 5y^2 + 24y = 0$$
$$\Rightarrow y(5y + 24) = 0$$
$$\Rightarrow y = 0 \text{ or } 5y + 24 = 0$$
$$\Rightarrow y = 0 \text{ or } y = -4\tfrac{4}{5}$$
$$\Rightarrow x = 6 \text{ or } x = -3\tfrac{3}{5};$$

hence,

$$\underline{\underline{x = 6, y = 0 \text{ or } x = -3\tfrac{3}{5}, y = -4\tfrac{4}{5}}}.$$

27. *ABCD* is a parallelogram. (5)

Diagram **NOT** accurately drawn
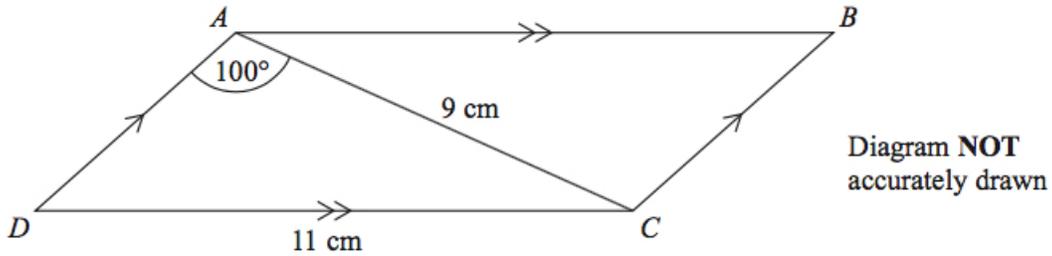
$AC = 9$ cm.
$DC = 11$ cm.
Angle $DAC = 100°$.

Calculate the area of the parallelogram.
Give your answer correct to 3 significant figures.

Solution

$$\frac{\sin ADC}{9} = \frac{\sin 100°}{11} \Rightarrow \sin ADC = \frac{9\sin 100°}{11}$$
$$\Rightarrow \angle ADC = 53.682\,923\,09 \text{ (FCD)}$$
$$\Rightarrow \angle ACD = 26.317\,076\,91 \text{ (FCD)}.$$

Finally,

$$\text{area} = 2 \times \tfrac{1}{2} \times 11 \times 9 \times \sin 26.317\ldots$$
$$= 43.890\,498\,37 \text{ (FCD)}$$
$$= \underline{\underline{43.9 \text{ cm}^2 \text{ (3 sf)}}}.$$