Dr Oliver Mathematics
GCSE Mathematics
2003 November Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Express 120 as the product of powers of its prime factors. (3)

Solution

$$\begin{array}{r|l} & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

Hence

$$\begin{aligned} 120 &= 2 \times 2 \times 2 \times 3 \times 5 \\ &= \underline{\underline{2^3 \times 3 \times 5}}. \end{aligned}$$

(b) Find the Lowest Common Multiple of 120 and 150. (2)

Solution

$$\begin{array}{r|l} & 150 \\ \hline 2 & 75 \\ \hline 3 & 25 \\ \hline 5 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

Hence

$$\begin{aligned} 150 &= 2 \times 3 \times 5 \times 5 \\ &= 2 \times 3 \times 5^2. \end{aligned}$$

Hence,

$$\text{LCM}(120,\ 150) = 2^3 \times 3 \times 5^2 = \underline{\underline{600}}.$$

2. Nassim thinks of a number. (4)
When he multiplies his number by 5 and subtracts 16 from the result, he gets the same answer as when he adds 10 to his number and multiplies the result by 3.
Find the number Nassim is thinking of.

Solution

Let x be the number Nassim is thinking of. Now,

$$5x - 16 = 3(x + 10) \Rightarrow 5x - 16 = 3x + 30$$
$$\Rightarrow 2x = 46$$
$$\Rightarrow \underline{\underline{x = 23}}.$$

3. The grouped frequency table shows information about the weights, in kilograms, of 20 students, chosen at random from Year 11. (3)

Weight (m kg)	Frequency
$50 \leqslant w < 60$	7
$60 \leqslant w < 70$	8
$70 \leqslant w < 80$	3
$80 \leqslant w < 90$	2

There are 300 students in Year 11.
Work out an estimate for the number of students in Year 11 whose weight is between 50 kg and 60 kg.

Solution

There are

$$7 + 8 + 3 + 2 = 20$$

students in the year and

$$\frac{7}{20} \times 300 \approx 7 \times 15 = \underline{\underline{105 \text{ students}}}.$$

4. (a) Simplify (3)

(i) $p^2 \times p^7$,

Solution

$$p^2 \times p^7 = \underline{\underline{p^9}}.$$

(ii) $x^8 \div x^3$,

Solution

$$x^8 \div x^3 = \underline{\underline{x^5}}.$$

(iii) $\dfrac{y^4 \times y^3}{y^5}$.

Solution

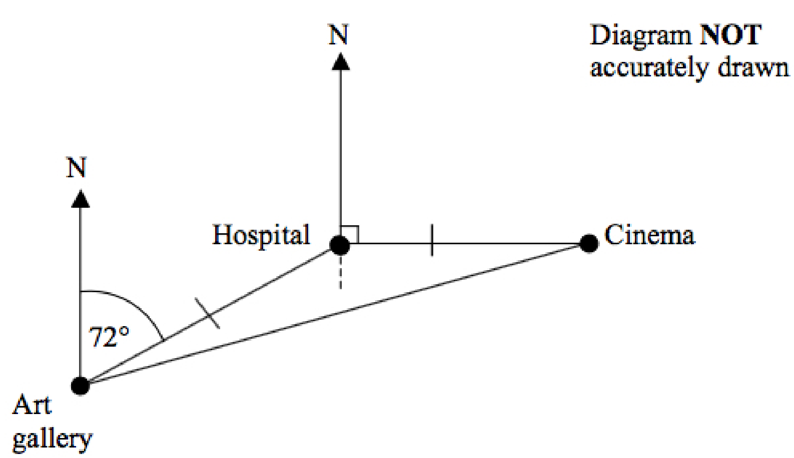
$$\frac{y^4 \times y^3}{y^5} = \frac{y^7}{y^5} = \underline{\underline{y^2}}.$$

(b) Expand $t(3t^2 + 4)$. (2)

Solution

$$t(3t^2 + 4) = \underline{\underline{3t^3 + 4t}}.$$

5. The diagram shows the position of each of three buildings in a town. (3)



The bearing of the Hospital from the Art gallery is 072°.
The Cinema is due East of the Hospital.
The distance from the Hospital to the Art gallery is equal to the distance from the Hospital to the Cinema.
Work out the bearing of the Cinema from the Art gallery.

Solution

$$\text{Angle } AHC = 72 + 90 = 162^\circ,$$

$$\text{Angle } HAC = \tfrac{1}{2}(180 - 162) = 9^\circ,$$

and the bearing of of the Cinema from the Art gallery is

$$72 + 9 = \underline{\underline{81^\circ}}.$$

6. Here are some expressions. (3)

$\frac{1}{2}ac$	πc	$2b$	$2ab^2$	abc	$a(b+c)$	$\frac{ab}{c}$	πa^2

The letters a, b, and c represent lengths.
π, 2, and $\frac{1}{2}$ are numbers which have no dimensions.
Three of the expressions could represent areas.
Tick (✓) the boxes underneath the three expressions which could represent areas.

Solution

$\frac{1}{2}ac$	πc	$2b$	$2ab^2$	abc	$a(b+c)$	$\frac{ab}{c}$	πa^2
✓					✓		✓

7. Work out (3)

$$5\tfrac{2}{3} - 2\tfrac{3}{4}.$$

Solution

$$\begin{aligned} 5\tfrac{2}{3} - 2\tfrac{3}{4} &= (5-2) + \left(\tfrac{8}{12} - \tfrac{9}{12}\right) \\ &= 3 - \tfrac{1}{12} \\ &= \underline{\underline{2\tfrac{11}{12}}}. \end{aligned}$$

8. The table shows information about the heights of 40 bushes.

Height (h cm)	Frequency
$170 \leqslant w < 175$	5
$175 \leqslant w < 180$	18
$180 \leqslant w < 185$	12
$185 \leqslant w < 190$	4
$190 \leqslant w < 195$	1

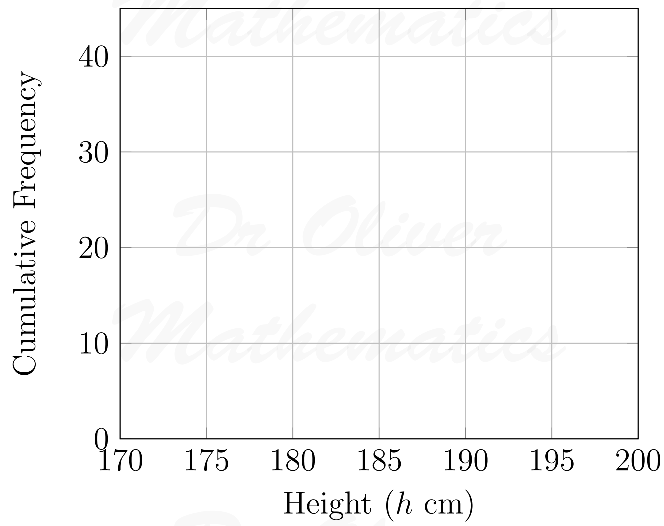
(a) Complete the cumulative frequency table (1)

Height (h cm)	Cumulative Frequency
$170 \leqslant w < 175$	
$170 \leqslant w < 180$	
$170 \leqslant w < 185$	
$170 \leqslant w < 190$	
$170 \leqslant w < 195$	

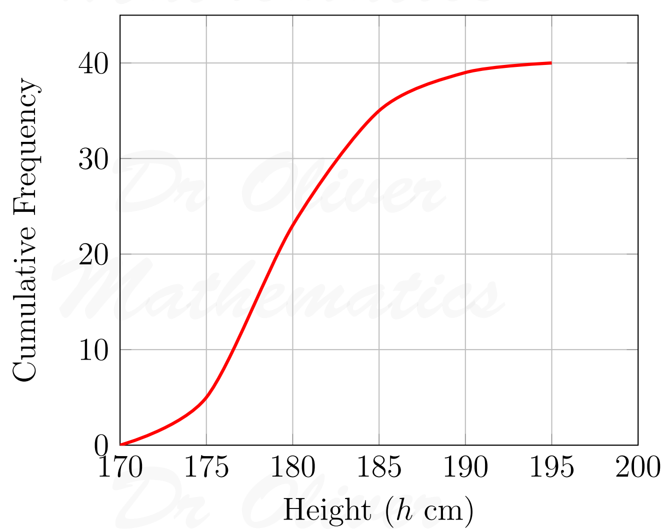
Solution

Height (h cm)	Cumulative Frequency
$170 \leqslant w < 175$	$\underline{\underline{5}}$
$170 \leqslant w < 180$	$5 + 18 = \underline{\underline{23}}$
$170 \leqslant w < 185$	$23 + 12 = \underline{\underline{35}}$
$170 \leqslant w < 190$	$35 + 4 = \underline{\underline{39}}$
$170 \leqslant w < 195$	$39 + 1 = \underline{\underline{40}}$

(b) On the grid, draw a cumulative frequency graph for your table. (2)

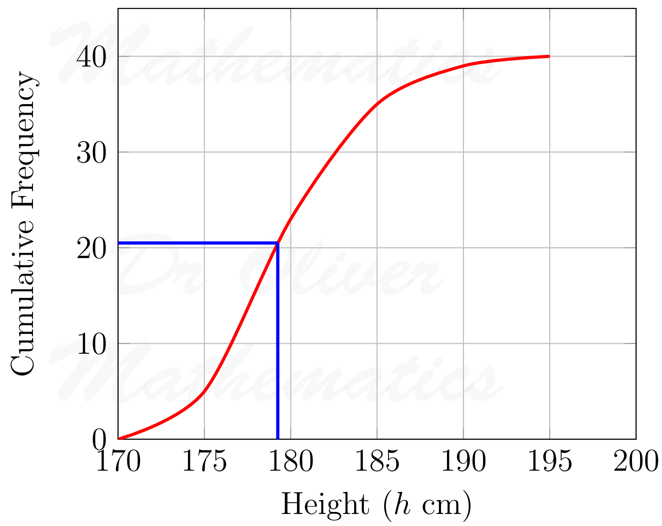


Solution



(c) Use the graph to find an estimate for the median height of the bushes. (1)

Solution



Use your graph to find a read-off from CF = 20 or CF = 20.5: approximately 179.

9. The diagram shows a trapezium.

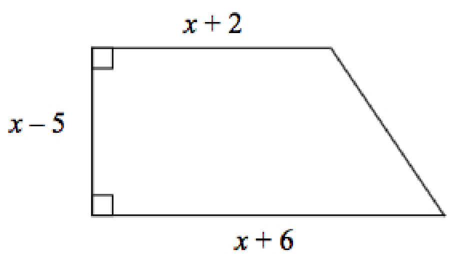


Diagram **NOT** accurately drawn

The lengths of three of the sides of the trapezium are $(x - 5)$, $(x + 2)$, and $(x + 6)$.
All measurements are given in centimetres.
The area of the trapezium is 36 cm^2.

(a) Show that (4)

$$x^2 - x - 56 = 0.$$

Solution

$$\tfrac{1}{2}(x-5)[(x+2)+(x+6)] = 36 \Rightarrow \tfrac{1}{2}(x-5)(2x+8) = 36$$
$$\Rightarrow (x-5)(x+4) = 36$$
$$\Rightarrow x^2 - x - 20 = 36$$
$$\Rightarrow \underline{\underline{x^2 - x - 56 = 0}},$$

as required.

(b) (i) Solve the equation (4)

$$x^2 - x - 56 = 0.$$

Solution

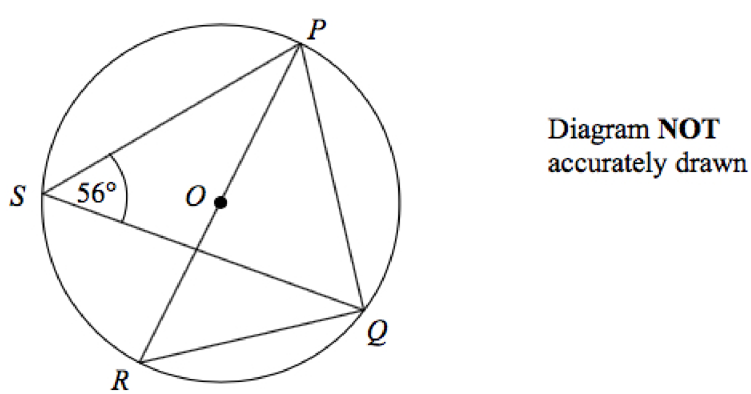
$$x^2 - x - 56 = 0 \Rightarrow (x-8)(x+7) = 0$$
$$\Rightarrow \underline{\underline{x = -7 \text{ or } x = 8}}.$$

(ii) Hence find the length of the shortest side of the trapezium.

Solution
The shortest side of the trapezium is

$$8 - 5 = \underline{\underline{3 \text{ cm}}}.$$

10. P, Q, R, and S are points on the circumference of a circle, centre O.



PR is a diameter of the circle.

Angle $PSQ = 56°$.

(a) Find the size of angle PQR. (2)
Give a reason for your answer.

> **Solution**
> Angle $PQR = \underline{\underline{90°}}$ (angle in a semi-circle is a right angle).

(b) Find the size of angle PRQ. (2)
Give a reason for your answer.

> **Solution**
> Angle $PRQ = \underline{\underline{56°}}$ (angles in the same segment are equal).

(c) Find the size of angle POQ. (2)
Give a reason for your answer.

> **Solution**
> Angle $POQ = \underline{\underline{112°}}$ (the angle at the centre is twice the angle at the circumference).

11. The fraction, p, of an adult's dose of medicine which should be given to a child who weighs w kg is given by the formula

$$p = \frac{3w + 20}{200}.$$

(a) Use the formula (3)

$$p = \frac{3w + 20}{200}$$

to find the weight of a child whose dose is the same as an adult's dose.

> **Solution**
>
> $$1 = \frac{3w + 20}{200} \Rightarrow 3w + 20 = 200$$
> $$\Rightarrow 3w = 180$$
> $$\Rightarrow \underline{\underline{w = 60}}.$$

(b) Make w the subject of the formula (3)

$$p = \frac{3w + 20}{200}.$$

Solution

$$p = \frac{3w+20}{200} \Rightarrow 200p = 3w + 20$$
$$\Rightarrow 200p - 20 = 3w$$
$$\Rightarrow \underline{\underline{w = \frac{200p-20}{3}}}.$$

$$\frac{3w+20}{200} = \frac{A}{A+12}.$$

(c) Express A in terms of w. (4)

Solution

$$\frac{3w+20}{200} = \frac{A}{A+12} \Rightarrow (A+12)(3w+20) = 200A$$
$$\Rightarrow 3Aw + 20A + 36w + 240 = 200A$$
$$\Rightarrow 36w + 240 = 180A - 3Aw$$
$$\Rightarrow 36w + 240 = A(180 - 3w)$$
$$\Rightarrow A = \frac{36w+240}{180-3w}$$
$$\Rightarrow A = \frac{3(12w+80)}{3(60-w)}$$
$$\Rightarrow \underline{\underline{A = \frac{12w+80}{60-w}}}.$$

12. Mathstown College has 500 students, all of them in the age range 16 to 19.
The incomplete table shows information about the students.

Age (years)	Number of male students	Number of female students
16	50	30
17	60	40
18	76	54
19		

A newspaper reporter is carrying out a survey into students' part-time jobs.
She takes a sample, stratified both by age and by gender, of 50 of the 500 students

(a) Calculate the number of 18 year old male students to be sampled. (3)

Solution

$$\begin{aligned}\frac{50}{500} \times 76 &= \frac{1}{10} \times 76 \\ &= 7.6\end{aligned}$$

so 8 students.

In the sample, there are 9 female students whose age is 19 years.

(b) Work out the least number of 19 year old female students in the college. (2)

Solution

Let x be the number of 19 year old female students in the college. Then

$$\begin{aligned}\frac{50}{500} \times x = 9 \Rightarrow x &= 9 \times 10 \\ &\Rightarrow 90 \text{ students;}\end{aligned}$$

hence, there must be at least number of 85 students 19 year old female students in the college

A newspaper photographer is going to take photographs of two students from Mathstown College.
He chooses one student at random from all of the 16 year old students and one student at random from all of the 17 year old students.

(c) Calculate the probability that he will choose two female students. (3)

Solution

$$\begin{aligned}\tfrac{30}{50+30} \times \tfrac{40}{60+40} &= \tfrac{30}{80} \times \tfrac{40}{100} \\ &= \tfrac{3}{8} \times \tfrac{4}{10} \\ &= \tfrac{3}{2} \times \tfrac{1}{10} \\ &= \tfrac{3}{20}.\end{aligned}$$

13. Convert the recurring decimal $0.\dot{2}\dot{9}$ to a fraction. (2)

Solution

Let $x = 0.\dot{2}\dot{9}$. Then

$$\begin{aligned} 100x &= 29.\dot{2}\dot{9} \quad (1) \\ x &= 0.\dot{2}\dot{9} \quad (2). \end{aligned}$$

Subtract (1) – (2):

$$99x = 29 \Rightarrow \underline{\underline{x = \tfrac{29}{99}}}.$$

14. $ABCD$ and $DEFG$ are squares. (3)

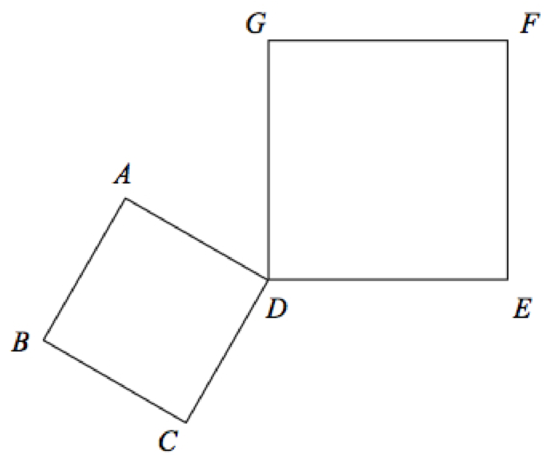


Diagram **NOT** accurately drawn

Prove that triangle CDG and triangle ADE are congruent.

Solution

DG and DE are same length as they are squares.
AD and DC are same length as they are squares.
$\angle CDG = \angle CDA + \angle ADG = \angle ADG + \angle GDE = \angle ADE$.
So, triangles CDG and ADE are congruent (SAS).

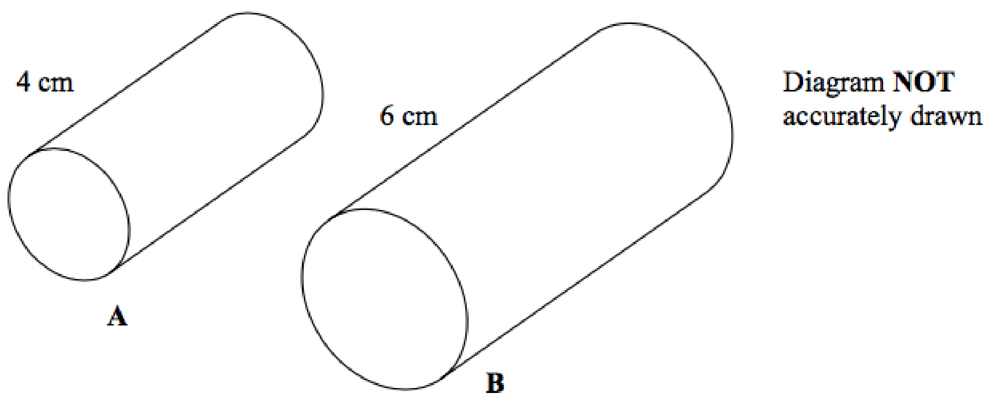
15. A straight line, **L**, passes through the point with coordinates $(4, 7)$ and is perpendicular to the line with equation $y = 2x + 3$. (3)
Find an equation of the straight line **L**.

Solution

The gradient is $-\frac{1}{2}$ and the equation is

$$\begin{aligned} y - 7 = -\tfrac{1}{2}(x-4) &\Rightarrow y - 7 = -\tfrac{1}{2}x + 2 \\ &\Rightarrow \underline{\underline{y = -\tfrac{1}{2}x + 9}}. \end{aligned}$$

16. Cylinder **A** and cylinder **B** are mathematically similar. (3)



The length of cylinder **A** is 4 cm and the length of cylinder **B** is 6 cm.
The volume of cylinder **A** is 80 cm^3.
Calculate the volume of cylinder **B**.

Solution

The length scale factor (LSF) is

$$\tfrac{6}{4} = \tfrac{3}{2}$$

and the volume scale factor (VSF) is

$$\left(\tfrac{3}{2}\right)^3 = \tfrac{27}{8}.$$

Finally, the volume of cylinder **B** is

$$\tfrac{27}{8} \times 80 = \underline{\underline{270 \text{ cm}^3}}.$$

17. (a) Evaluate (5)

(i) 3^{-2},

Solution

$$3^{-2} = \frac{1}{3^2} = \underline{\underline{\tfrac{1}{9}}}.$$

(ii) $36^{\frac{1}{2}}$,

Solution

$$36^{\frac{1}{2}} = \sqrt{36} = \underline{\underline{6}}.$$

(iii) $27^{\frac{2}{3}}$,

Solution

$$\begin{aligned} 27^{\frac{2}{3}} &= (27^{\frac{1}{3}})^2 \\ &= (\sqrt[3]{27})^2 \\ &= 3^2 \\ &= \underline{\underline{9}}. \end{aligned}$$

(iv) $(\frac{16}{81})^{-\frac{3}{4}}$.

Solution

$$\begin{aligned} (\tfrac{16}{81})^{-\frac{3}{4}} &= (\tfrac{81}{16})^{\frac{3}{4}} \\ &= \left(\frac{\sqrt[4]{81}}{\sqrt[4]{16}}\right)^3 \\ &= \left(\frac{3}{2}\right)^3 \\ &= \underline{\underline{\tfrac{27}{8}}}. \end{aligned}$$

(b) (i) Rationalise the denominator of (4)

$$\frac{21}{\sqrt{7}}$$

and simplify your answer.

Solution

$$\begin{aligned}\frac{21}{\sqrt{7}} &= \frac{21}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{21\sqrt{7}}{7} \\ &= \underline{\underline{3\sqrt{7}}}.\end{aligned}$$

(ii) Expand

$$(\sqrt{5} + 2\sqrt{3})(\sqrt{5} - 2\sqrt{3}).$$

Express your answer as simply as possible.

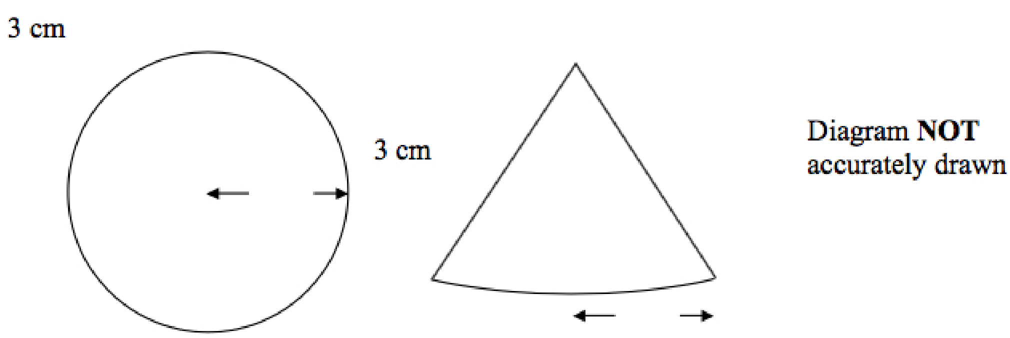
Solution

$\times$	$\sqrt{5}$	$+2\sqrt{3}$
$\sqrt{5}$	5	$+2\sqrt{15}$
$-2\sqrt{3}$	$-2\sqrt{15}$	-12

Hence,

$$(\sqrt{5} + 2\sqrt{3})(\sqrt{5} - 2\sqrt{3}) = 5 - 12 = \underline{\underline{-7}}.$$

18. The radius of a sphere is 3 cm. (7)
The radius of the base of a cone is also 3 cm.



The volume of the sphere is three times the volume of the cone.
Work out the curved surface area of the cone.
Give your answer as a multiple of π.

Solution

$$\begin{aligned} & \tfrac{4}{3}\pi \times 3^3 = 3 \times \tfrac{1}{3}\pi \times 3^2 \times h \\ \Rightarrow\quad & 4 \times 3^3 = 3^3 h \\ \Rightarrow\quad & h = 4. \end{aligned}$$

Next, the slant height of the cone is

$$\begin{aligned} l &= \sqrt{3^2 + 4^2} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

Finally, the curved surface area of the cone is

$$\pi \times 3 \times 5 = \underline{\underline{15\pi}}.$$

19. OPQ is a triangle.

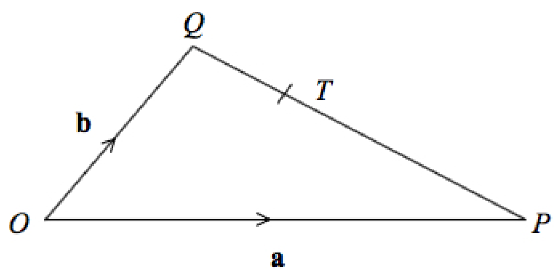


Diagram **NOT** accurately drawn

T is the point on PQ for which $PT : TQ = 2 : 1$.
$\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$.

(a) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, and expression for $\overrightarrow{PQ}$. (1)

Solution

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = \underline{\underline{\mathbf{b} - \mathbf{a}}}.$$

(b) Express $\overrightarrow{OT}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (2)
Give your answer in its simplest form.

Solution

$$\begin{aligned}\overrightarrow{OT} &= \overrightarrow{OP} + \overrightarrow{PT} \\ &= \overrightarrow{OP} + \tfrac{2}{3}\overrightarrow{PQ} \\ &= \mathbf{a} + \tfrac{2}{3}(\mathbf{b} - \mathbf{a}) \\ &= \mathbf{a} + \tfrac{2}{3}\mathbf{b} - \tfrac{2}{3}\mathbf{a} \\ &= \underline{\underline{\tfrac{1}{3}\mathbf{a} + \tfrac{2}{3}\mathbf{b}}}.\end{aligned}$$

20. The expression

$$x^2 - 6x + 14$$

can be written in the form

$$(x - p)^2 + q,$$

for all values of x.

(a) Find the value of (3)

(i) p,

Solution

$$\begin{aligned}x^2 - 6x + 14 &= (x^2 - 6x + 9) + 5 \\ &= (x - 3)^2 + 5;\end{aligned}$$

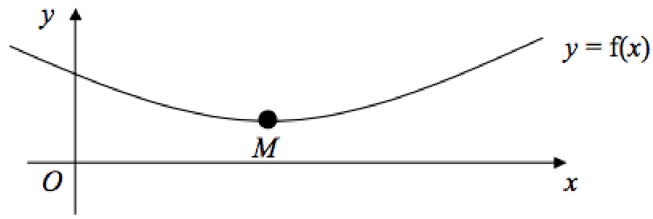
thus, $\underline{\underline{p = 3}}$ ...

(ii) q.

Solution

... and $\underline{\underline{q = 5}}$.

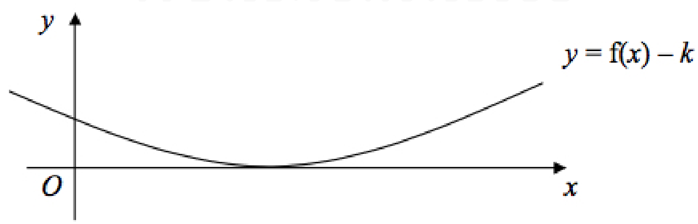
The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^2 - 6x + 14$.
Here is a sketch of the graph of $y = \mathrm{f}(x)$.



(b) Write down the coordinates of the minimum point, M, of the curve. (1)

Solution
$(3, 5)$.

Here is a sketch of the graph of $y = \mathrm{f}(x) - k$, where k is a positive constant.
The graph touches the x-axis.



(c) Find the value of k. (1)

Solution
$k = 5$.

(d) For the graph of $y = \mathrm{f}(x - 1)$, (3)

(i) write down the coordinates of the minimum point,

Solution
$(4, 5)$.

(ii) find the coordinates of the point where the curve crosses the y-axis.

Solution

$$\begin{aligned}(x-4)^2+5 &= (x^2-8x+16)+5 \\ &= x^2-8x+21;\end{aligned}$$

hence, the coordinates of the point where the curve crosses the y-axis is $\underline{\underline{(0, 21)}}$.