

Dr Oliver Mathematics
AQA Further Maths Level 2
June 2023 Paper 1
1 hour 45 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

You are **not** permitted to use a scientific or graphical calculator in this paper.

1. The function f is given by

$$f(x) = 2x + 1.$$

- (a) Work out x when

$$f(x) = -5.$$

(2)

The function g is given by

$$g(x) = x^2.$$

- (b) Work out

$$f \circ g(3).$$

(2)

2. Factorise fully

$$6x^2y + 21xy.$$

(2)

3. (a) Circle the transformation matrix that represents a reflection in the line $y = -x$.

(1)

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

- (b) Show that

$$\begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = k\mathbf{I},$$

(2)

where k is an integer.

4. $S(7, 2)$ and $T(5, -4)$ are points on a straight line.

- (a) Work out the gradient of the line.

(2)

- (b) Work out the distance between S and T .

(3)

Give your answer in the form $a\sqrt{b}$ where a and b are both integers greater than 1.

5. X_n and Y_n are the n th terms of two sequences. (3)

$$X_n = (n - 1)(n + 1)$$

$$Y_n = (n + 1)(n + 2).$$

Prove that every term of the sequence with n th term

$$Y_n - X_n$$

is a multiple of 3.

6. The equation of a curve is (4)

$$y = x^6 + 4x^2 - 7.$$

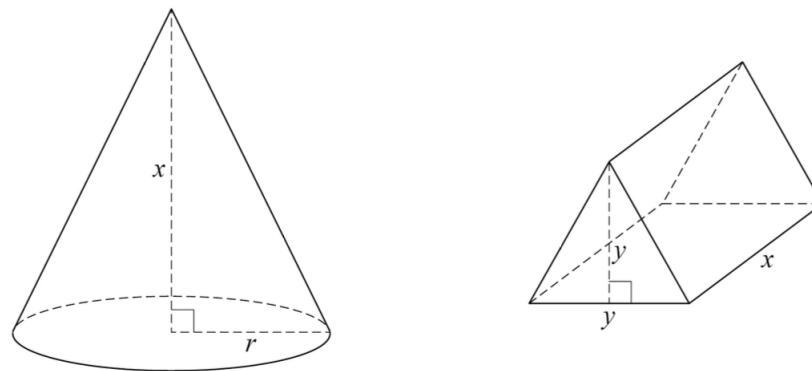
Work out the equation of the tangent to the curve at the point $(1, -2)$.

Give your answer in the form $y = mx + c$.

7. The diagram below shows a cone and a prism. (4)

All measurements are in cm.

- The cone has base radius r and perpendicular height x .
- The prism has a triangular cross section with base y and perpendicular height y .
- The length of the prism is x .



$\text{Volume of a cone} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$

The volume of the cone is **four** times the volume of the prism.

Express r in terms of y .

8. A circle has centre $(0, 0)$ and radius 5. (6)

A straight line has equation

$$2y = x + 5.$$

Work out the coordinates of the **two** points where the circle and straight line intersect.

Do **not** use trial and improvement.

You **must** show your working.

9. Rearrange (4)

$$w = \frac{y^2 + 5}{y^2 - 2}$$

to make y the subject.

10. Rationalise the denominator and simplify fully (4)

$$\frac{1 + \sqrt{5}}{3 - \sqrt{5}}.$$

11. (3)

$$y = \frac{1}{12}x^4 + 3x^2 + 4.$$

Work out the **positive** value of x for which

$$\frac{d^2y}{dx^2} = 55.$$

12. (a) Write down the value of x for $0^\circ \leq x \leq 360^\circ$ when (1)

$$\sin x = -1.$$

- (b) Work out the values of y for $0^\circ \leq y \leq 360^\circ$ when (3)

$$\sqrt{3} \tan y = 1.$$

13. Write (3)

$$\frac{2x - 3}{x} - \frac{1}{3x} + 1$$

as a single fraction.

Give your answer in its simplest form.

14. Solve (4)

$$\frac{8}{x} + 3x \leq 10,$$

where x is positive.

15. Solve (4)

$$\left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)^2 = x^2 + x.$$

16. The expansions of (4)

$$(1 + 12x)^4$$

and

$$(a + 4x)^3$$

have the same coefficient of x^2 .

Work out the value of a .

17. The curve (5)

$$y = ax^3 + bx^2 + 7$$

has a stationary point at $(-2, 11)$.

Work out the values of a and b .

18. Solve the simultaneous equations (5)

$$2x + y = 13$$

$$x + 3z = 2$$

$$z - 2y = -7.$$

Do **not** use trial and improvement.

You **must** show your working.

19. (3)

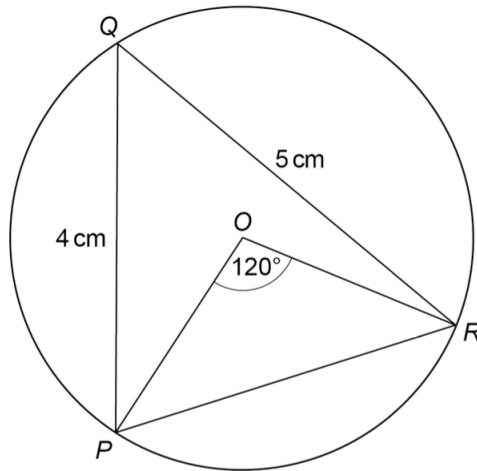
$$8x^2 + 20x + n \equiv c(x + d)^2 + 3,$$

where c , d , and n are constants.

Work out the values of c , d , and n .

20. P , Q , and R are points on a circle, centre O . (6)

- Angle $POR = 120^\circ$.
- $PQ = 4$ cm.
- $QR = 5$ cm.



Not drawn
accurately

Work out the radius of the circle.
Give your answer in the form $\sqrt{k}$ where k is an integer.