Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2007 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 49.
You must write down all the stages in your working.

1. Alistair buys an antique chair for £600. (3)
   It is expected to increase in value at the rate of 4.5% each year.
   How much is it expected to be worth in 3 years?

Solution

$$\begin{aligned}\text{Chair} &= 600 \times 1.045^3 \\ &= 684.699\,675 \text{ (FCD)} \\ &= \underline{\underline{£684.70 \text{ (nearest penny)}}}.\end{aligned}$$

2. Solve the equation (4)

$$3x^2 - 2x - 10 = 0.$$

   Give your answer **correct to 2 significant figures**.

Solution

$a = 3$, $b = -2$, and $c = -10$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times (-10)}}{2 \times 3} \\ &= \frac{2 \pm \sqrt{124}}{6} \\ &= -1.522\,588\,121, 2.189\,254\,788 \text{ (FCD)} \\ &= \underline{\underline{-1.5, 2.2 \text{ (2 sf)}}}.\end{aligned}$$

3. (a) During his lunch hour, Luke records the number of birds that visit his bird-table. (4)
The numbers recorded last week were:

$$28, 32, 14, 19, 18, 26, 31.$$

Find the mean and standard deviation for this data.

Solution

x	x^2
28	784
32	1 024
14	196
19	361
18	324
26	676
31	961
$\sum x = 168$	$\sum x^2 = 4\,326$

$$\begin{aligned}\text{Mean} &= \frac{\sum x}{n} \\ &= \frac{168}{7} \\ &= \underline{\underline{24 \text{ birds}}}\end{aligned}$$

and

$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} \\ &= \sqrt{\frac{4\,326 - \frac{168^2}{7}}{6}} \\ &= \underline{\underline{7 \text{ birds}}}.\end{aligned}$$

(b) Over the same period, Luke's friend, Erin also recorded the number of birds visiting her bird-table. (2)
Erin's recordings have a mean of 25 and a standard deviation of 5.
Make two valid comparisons between the friends' recordings.

Solution

Average: Since the mean for Luke (24) is lower than the mean for Erin (25), Luke had less birds on average.

Spread: Since the standard deviation for Erin (5) is smaller than the range for Luke (7), the number of birds visiting Luke's table varies more.

4. Solve the inequality (2)

$$\frac{x}{4} - \frac{1}{2} < 5.$$

Solution

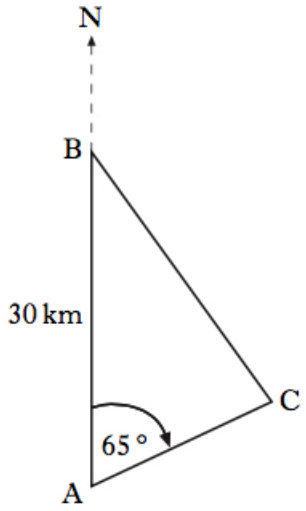
$$\frac{x}{4} - \frac{1}{2} < 5 \Rightarrow \frac{x}{4} < 5\frac{1}{2}$$
$$\Rightarrow \underline{\underline{x < 22}}.$$

5. Mark takes some friends out for a meal. (3)
The restaurant adds a 10% service charge to the price of the meal.
The total bill is £148.50.
What was the price of the meal?

Solution

$$\text{Meal} = \frac{148.50}{1.1}$$
$$= \underline{\underline{£135}}.$$

6. Brunton is 30 kilometres due north of Appleton. (4)
From Appleton, the bearing of Carlton is 065°.
From Brunton, the bearing of Carlton is 153°.



Calculate the distance between Brunton and Carlton.

Solution

Well,

$$\angle ABC = 180 - 153 = 27^\circ \text{ (supplementary angles)}$$

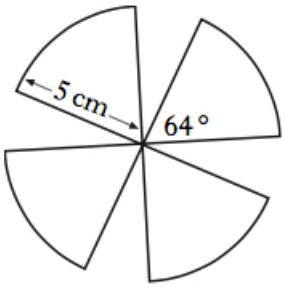
and

$$\angle ACB = 180 - 65 - 27 = 88^\circ.$$

Finally,

$$\frac{BC}{\sin BAC} = \frac{AB}{\sin ACB} \Rightarrow \frac{BC}{\sin 65^\circ} = \frac{30}{\sin 88^\circ}$$
$$\Rightarrow BC = \frac{30 \sin 65^\circ}{\sin 88^\circ}$$
$$\Rightarrow BC = 27.205\,806\,65 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{BC = 27.2 \text{ km (3 sf)}}}.$$

7. A fan has four identical plastic blades. (3)

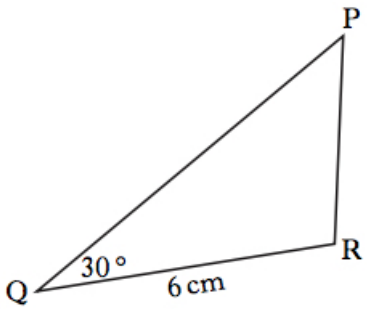


Each blade is a sector of a circle of radius 5 centimetres.
The angle at the centre of each sector is 64°.
Calculate the **total** area of plastic required to make the blades.

Solution

$$\begin{aligned}\text{Area} &= 4 \times \frac{64}{360} \times \pi \times 5^2 \\ &= 55.850\,536\,06 \text{ (FCD)} \\ &= \underline{\underline{55.9 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

8. In triangle PQR, (3)
$QR = 6$ centimetres,
angle $PQR = 30°$,
and the area of triangle $PQR = 15$ square centimetres.



Calculate the length of PQ.

Solution

$$\tfrac{1}{2} \times PQ \times 6 \times \sin 30^\circ = 15 \Rightarrow PQ = \frac{15}{\frac{1}{2} \times 6 \times \sin 30^\circ}$$
$$\Rightarrow \underline{\underline{PQ = 10 \text{ cm}}}.$$

9. To make "14 carat" gold, copper, and pure gold are mixed in the ratio 5 : 7. (3)
A jeweller has 160 grams of copper and 245 grams of pure gold.
What is the maximum weight of "14 carat" gold that the jeweller can make?

Solution

$$\frac{7}{5} \times 160 = 224;$$

hence, he can make

$$160 + 224 = \underline{\underline{384 \text{ g}}}$$

of "14 carat" gold.

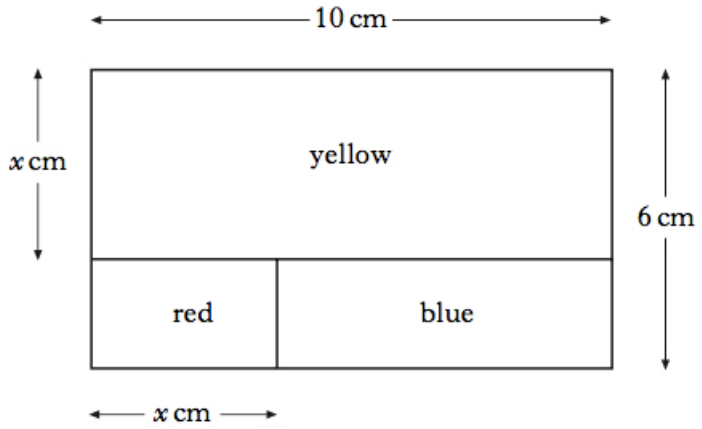
10. Solve **algebraically** the equation (3)

$$5 \cos x^\circ + 4 = 0, \qquad 0 \leqslant x < 360.$$

Solution

$$5 \cos x^\circ + 4 = 0 \Rightarrow 5 \cos x^\circ = -4$$
$$\Rightarrow \cos x^\circ = -\tfrac{4}{5}$$
$$\Rightarrow x = 143.130\,102\,4, 216.869\,897\,6 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 143.1, 216.7 \text{ (1 dp)}}}.$$

11. A decorator's logo is rectangular and measures 10 centimetres by 6 centimetres.
It consists of three rectangles: one red, one yellow, and one blue.



The yellow rectangle measures 10 centimetres by x centimetres.
The width of the red rectangle is x centimetres.

(a) Show that the area, A, of the blue rectangle is given by the expression (2)

$$A = x^2 - 16x + 60.$$

Solution

$$(x \times 10) + [x \times (6 - x)] + A = 6 \times 10 \Rightarrow 10x + 6x - x^2 + A = 60$$
$$\Rightarrow \underline{\underline{A = x^2 - 16x + 60}},$$

as required.

The area of the blue rectangle is equal to $\frac{1}{5}$ of the total area of the logo.

(b) Calculate the value of x. (4)

Solution

$$\tfrac{1}{5} \times 60 = 12 \text{ cm}^2$$

and so

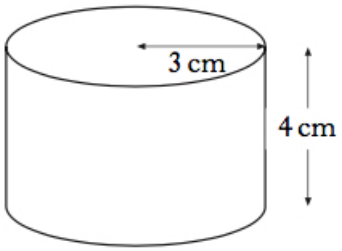
$$x^2 - 16x + 60 = 12 \Rightarrow x^2 - 16x + 48 = 0$$

add to: -16
multiply to: -48 $\Big\}$ $-4, -12$

$$\Rightarrow (x-4)(x-12) = 0$$
$$\Rightarrow x - 4 = 0 \text{ or } x - 12 = 0$$
$$\Rightarrow x = 4 \text{ or } x = 12;$$

but $x \neq 12$ and so $\underline{\underline{x = 4}}$.

12. A cylindrical paperweight of radius 3 centimetres and height 4 centimetres is filled with sand.

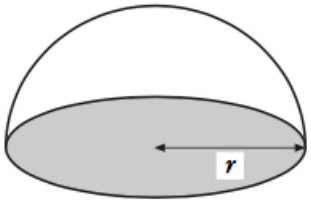


(a) Calculate the volume of sand in the paperweight. (2)

Solution

$$\begin{aligned}\text{Volume} &= \pi r^2 h \\ &= \pi \times 3^2 \times 4 \\ &= 113.097\,335\,5 \text{ (FCD)} \\ &= \underline{\underline{113 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

Another paperweight, in the shape of a hemisphere, is filled with sand.



It contains the same volume of sand as the first paperweight.

(b) Calculate the radius of the hemisphere. (3)

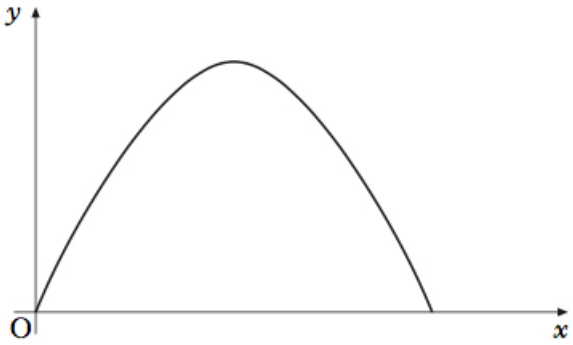
Solution

$$\tfrac{2}{3} \times \pi \times r^3 = 36\pi \Rightarrow r^3 = 54$$
$$\Rightarrow r = 3.779\,763\,15 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{r = 3.78 \text{ cm (3 sf)}}}.$$

13. The profit made by a publishing company of a magazine is calculated by the formula (4)

$$y = 4x(140 - x),$$

where y is the profit (in pounds) and x is the selling price (in pence) of the magazine. The graph below represents the profit y against the selling price x.



Find the maximum profit the company can make from the sale of the magazine.

Solution

$$\begin{aligned} y = 0 &\Rightarrow 4x(140 - x) = 0 \\ &\Rightarrow 4x = 0 \text{ or } 140 - x = 0 \\ &\Rightarrow x = 0 \text{ or } x = 140 \end{aligned}$$

and the vertex of the curve is

$$\frac{0 + 140}{2} = 70.$$

Finally,

$$\begin{aligned} x = 70 &\Rightarrow y = 4 \times 70 \times (140 - 70) \\ &\Rightarrow y = 4 \times 70 \times 70 \\ &\Rightarrow \underline{\underline{y = £19\,600}}. \end{aligned}$$