

Dr Oliver Mathematics
Advanced Subsidiary Paper 1: Pure Mathematics
November 2021: Calculator
2 hours

The total number of marks available is 100.

You must write down all the stages in your working.

Inexact answers should be given to three significant figures unless otherwise stated.

1. Using algebra, solve the inequality (3)

$$x^2 - x > 20,$$

writing your answer in set notation.

2. Given (3)

$$\frac{9^{x-1}}{3^{y+2}} = 81,$$

express y in terms of x , writing your answer in simplest form.

3. Find (4)

$$\int \frac{3x^4 - 4}{2x^3} dx,$$

writing your answer in simplest form.

4. (In this question the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively.)

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O .

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O .

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

- (a) prove that the stone passes through O , (2)

- (b) calculate the speed of the stone. (3)

5. Figure 1 shows part of the curve with equation

$$y = 3x^2 - 2.$$

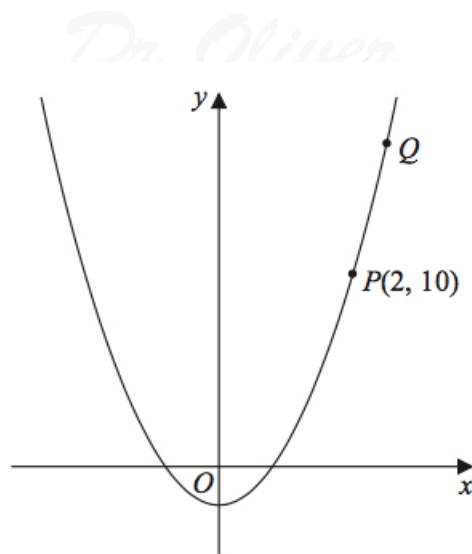


Figure 1: $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

- (a) Find the gradient of the tangent to the curve at P . (2)

The point Q with x -coordinate $2 + h$ also lies on the curve.

- (b) Find the gradient of the line PQ , giving your answer in terms of h in simplest form. (3)
 (c) Explain briefly the relationship between part (b) and the answer to part (a). (1)

6. (a) Using algebra, find all solutions of the equation (3)

$$3x^3 - 17x^2 - 6x = 0.$$

- (b) Hence find all real solutions of (3)

$$3(y - 2)^6 - 17(y - 2)^4 - 6(y - 2)^2 = 0.$$

7. A parallelogram $PQRS$ has area 50 cm^2 .

Given

- PQ has length 14 cm
- QR has length 7 cm, and
- angle SPQ is obtuse,

find

- (a) the size of angle SPQ , in degrees, to 2 decimal places, (3)

(b) the length of the diagonal SQ , in cm, to one decimal place. (2)

8.

$$g(x) = (2 + ax)^8, \text{ where } a \text{ is a constant.}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$,

(a) find the value of a . (4)

Using this value of a ,

(b) find the constant term in the expansion of (3)

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8.$$

9. Find the value of the constant k , $0 < k < 9$, such that (4)

$$\int_k^9 \frac{6}{\sqrt{x}} dx = 20.$$

10. A student is investigating the following statement about natural numbers.

“($n^3 - n$) is a multiple of 4.”

(a) Prove, using algebra, that the statement is true for all odd numbers. (4)

(b) Use a counterexample to show that the statement is not always true. (1)

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km², is modelled by the equation

$$A = 80 - 45e^{ct},$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started. (1)

On 1st January 2019 an area of 60 km² of the nature reserve was covered by trees.

- (b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures. (4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

- (c) State a reason why the model is not appropriate for this plan. (1)

12. (a) (i) Solve, for $0^\circ < \theta \leq 450^\circ$, the equation (5)

$$5 \cos^2 \theta = 6 \sin \theta,$$

giving your answers to one decimal place.

- (ii) A student's attempt to solve the question (2)

“Solve, for $-90^\circ < x < 90^\circ$, the equation $3 \tan x - 5 \sin x = 0$ ”

is set out below.

$$3 \tan x - 5 \sin x = 0$$

$$3 \frac{\sin x}{\cos x} - 5 \sin x = 0$$

$$3 \sin x - 5 \sin x \cos x = 0$$

$$3 - 5 \cos x = 0$$

$$\cos x = \frac{3}{5}$$

$$x = 53.1^\circ$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40^\circ) = \frac{3}{5}$$

are $\alpha_1, \alpha_2, \alpha_3$, and α_4 .

- (b) Find, to the nearest degree, the value of α_4 . (2)

13. The resting heart rate, h , of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q,$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$.

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235 .

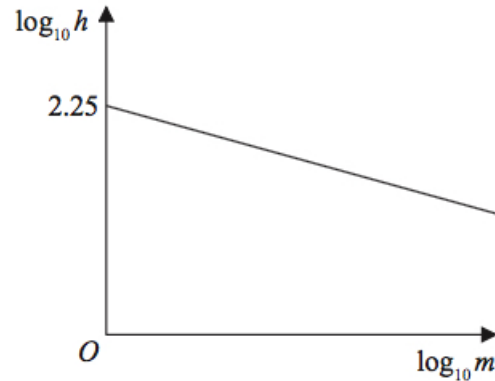


Figure 2: $\log_{10} h$ and $\log_{10} m$

- (a) Find, to 3 significant figures, the value of p and the value of q . (3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

- (b) Comment on the suitability of the model for this mammal. (3)
(c) With reference to the model, interpret the value of the constant p . (1)

14. A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8.$$

- (a) Write $f(x)$ in the form (3)

$$a(x + b)^2 + c,$$

where a , b , and c are constants to be found.

The curve C has a maximum turning point at M .

- (b) Find the coordinates of M . (2)

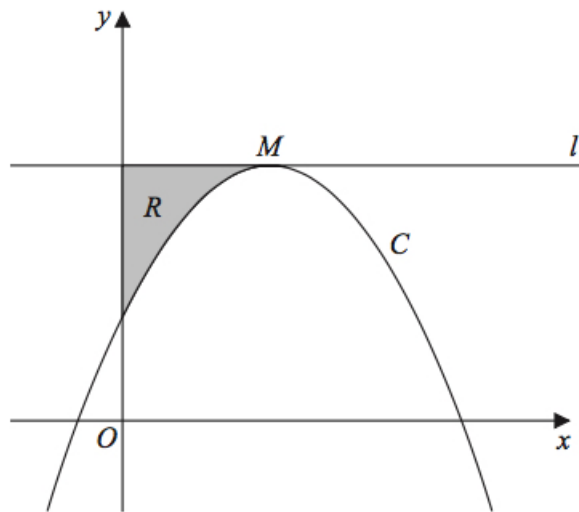


Figure 3: the curve C

Figure 3 shows a sketch of the curve C .

The line l passes through M and is parallel to the x -axis.

The region R , shown shaded in Figure 3, is bounded by C , l , and the y -axis.

(c) Using algebraic integration, find the area of R .

(5)

15. Figure 4 shows a sketch of a circle C with centre $N(7, 4)$.

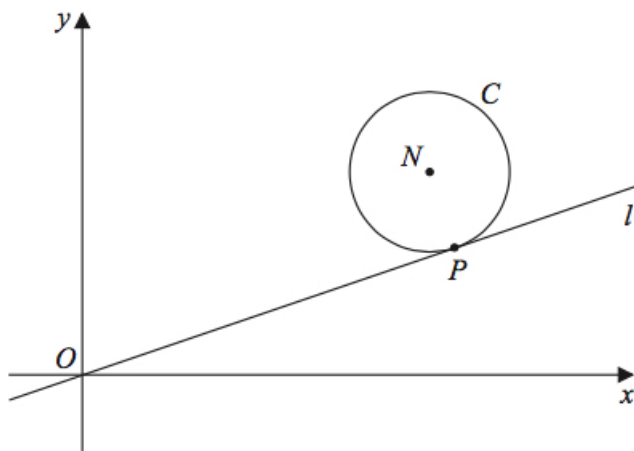


Figure 4: a sketch of a circle C

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P .

Find

- (a) the equation of line PN in the form $y = mx + c$, where m and c are constants, (2)
- (b) an equation for C . (4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C .

- (c) Find the value of k . (3)

16. The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b,$$

and a and b are constants.

Given

- the point $(2, 10)$ lies on C and
 - the gradient of the curve at $(2, 10)$ is -3 ,
- (a) (i) show that the value of a is -2 , (4)
 - (ii) find the value of b .
 - (b) Hence show that C has no stationary points. (3)
 - (c) Write $f(x)$ in the form (2)

$$(x - 4)Q(x),$$

where $Q(x)$ is a quadratic expression to be found.

- (d) Hence deduce the coordinates of the points of intersection of the curve with equation (2)

$$y = f(0.2x)$$

and the coordinate axes.