Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2010 Paper 1: Non-Calculator
55 minutes

The total number of marks available is 38.
You must write down all the stages in your working.

1. Evaluate (2)

$$40\% \text{ of } £11.50 - £1.81.$$

Solution

$$10\% \text{ of } £11.50 = 1.15$$
$$40\% \text{ of } £11.50 = 4.60$$

and

$$\begin{aligned} 40\% \text{ of } £11.50 - £1.81 &= 4.60 - 1.81 \\ &= \underline{\underline{£2.79}} \end{aligned}$$

2. Evaluate (2)

$$\tfrac{2}{5} \div 1\tfrac{1}{10}.$$

Solution

$$\begin{aligned} \tfrac{2}{5} \div 1\tfrac{1}{10} &= \tfrac{2}{5} \div \tfrac{11}{10} \\ &= \tfrac{2}{5} \times \tfrac{10}{11} \\ &= \tfrac{2}{1} \times \tfrac{2}{11} \\ &= \underline{\underline{\tfrac{4}{11}}}. \end{aligned}$$

3. Change the subject of the formula to s: (3)

$$t = \frac{7s + 4}{2}.$$

Solution

$$\begin{aligned} t = \frac{7s+4}{2} &\Rightarrow 7s + 4 = 2t \\ &\Rightarrow 7s = 2t - 4 \\ &\Rightarrow \underline{\underline{s = \frac{2t-4}{7}}}. \end{aligned}$$

4. Two functions are given below.

$$\begin{aligned} \mathrm{f}(x) &= x^2 - 4x, \\ \mathrm{g}(x) &= 2x + 7. \end{aligned}$$

(a) If $\mathrm{f}(x) = \mathrm{g}(x)$, show that (2)

$$x^2 - 6x - 7 = 0.$$

Solution

$$\begin{aligned} \mathrm{f}(x) = \mathrm{g}(x) &\Rightarrow x^2 - 4x = 2x + 7 \\ &\Rightarrow \underline{\underline{x^2 - 6x - 7 = 0}}, \end{aligned}$$

as required.

(b) Hence find **algebraically** the values of x for which $\mathrm{f}(x) = \mathrm{g}(x)$. (2)

Solution

$$\left.\begin{array}{ll} \text{add to:} & -6 \\ \text{multiply to:} & -7 \end{array}\right\} -7, +1$$

$$\begin{aligned} x^2 - 6x - 7 = 0 &\Rightarrow (x-7)(x+1) = 0 \\ &\Rightarrow x - 7 = 0 \text{ or } x + 1 = 0 \\ &\Rightarrow \underline{\underline{x = 7 \text{ or } x = -1}}. \end{aligned}$$

5. A bag contains 27 marbles.
Some are black and some are white.
The probability that a marble chosen at random is black is $\frac{4}{9}$.

(a) What is the probability that a marble chosen at random is white? (1)

Solution

$$1 - \tfrac{4}{9} = \underline{\underline{\tfrac{5}{9}}}.$$

(b) How many white marbles are in the bag? (1)

Solution

$$\tfrac{5}{9} \times 27 = \underline{\underline{15 \text{ marbles}}}.$$

6. Cleano washing powder is on special offer. (3)

Each box on special offer contains 20% more powder than the standard box.
A box on special offer contains 900 grams of powder.
How many grams of powder does the standard box contain?

Solution

$$\begin{aligned}\text{Standard box} &= \frac{900}{1.2} \\ &= \frac{900}{\frac{6}{5}} \\ &= 900 \times \frac{5}{6} \\ &= 150 \times 5 \\ &= \underline{\underline{750 \text{ g}}}.\end{aligned}$$

7. A straight line has equation $y = mx + c$, where m and c are constants.

   (a) The point $(2, 7)$ lies on this line. (1)
   Write down an equation in m and c to illustrate this information.

   Solution
   $\underline{\underline{7 = 2m + c}}$ (1).

   (b) A second point $(4, 17)$ also lies on this line. (1)
   Write down another equation in m and c to illustrate this information.

   Solution
   $\underline{\underline{17 = 4m + c}}$ (2).

   (c) Hence calculate the values of m and c. (3)

   Solution
   Now, do (2) − (1):

   $$\begin{aligned}2m = 10 &\Rightarrow \underline{\underline{m = 5}} \\ &\Rightarrow 7 = 10 + c \\ &\Rightarrow \underline{\underline{c = -3}}.\end{aligned}$$

   (d) Write down the gradient of this line. (1)

   Solution
   $\underline{\underline{5}}$.

8. (a) Simplify (1)

$$\sqrt{2} \times \sqrt{18}.$$

Solution

$$\begin{aligned}\sqrt{2} \times \sqrt{18} &= \sqrt{2} \times \sqrt{9 \times 2} \\ &= \sqrt{2} \times \sqrt{9} \times \sqrt{2} \\ &= \sqrt{2} \times 3\sqrt{2} \\ &= \underline{\underline{6}}.\end{aligned}$$

(b) Simplify (1)

$$\sqrt{2} + \sqrt{18}.$$

Solution

$$\begin{aligned}\sqrt{2} + \sqrt{18} &= \sqrt{2} + 3\sqrt{2} \\ &= \underline{\underline{4\sqrt{2}}}.\end{aligned}$$

(c) Hence show that (2)

$$\frac{\sqrt{2} \times \sqrt{18}}{\sqrt{2} + \sqrt{18}} = \frac{3\sqrt{2}}{4}.$$

Solution

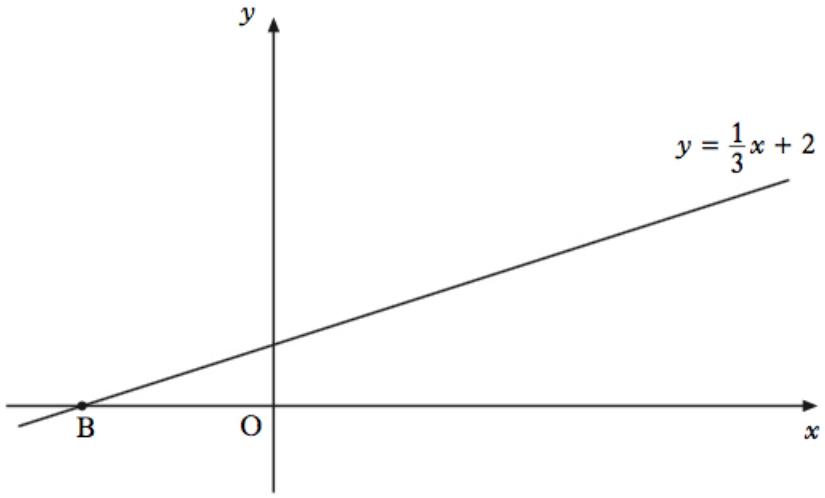
$$\begin{aligned}\frac{\sqrt{2} \times \sqrt{18}}{\sqrt{2} + \sqrt{18}} &= \frac{6}{4\sqrt{2}} \\ &= \frac{6}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{6\sqrt{2}}{8} \\ &= \underline{\underline{\frac{3\sqrt{2}}{4}}},\end{aligned}$$

as required.

9. Part of the graph of the straight line with equation

$$y = \tfrac{1}{3}x + 2,$$

is shown below.



(a) Find the coordinates of the point B. (2)

Solution

$$\begin{aligned} y = 0 &\Rightarrow \tfrac{1}{3}x + 2 = 0 \\ &\Rightarrow \tfrac{1}{3}x = -2 \\ &\Rightarrow x = -6, \end{aligned}$$

and the coordinates of the point $\underline{\underline{B(-6, 0)}}$.

(b) For what values of x is $y < 0$? (1)

Solution

$\underline{\underline{x < -6}}$.

10. A number pattern is shown below.

$$\begin{aligned} 1^3 &= \frac{1^2 \times 2^2}{4}, \\ 1^3 + 2^3 &= \frac{2^2 \times 3^2}{4}, \\ 1^3 + 2^3 + 3^3 &= \frac{3^2 \times 4^2}{4}. \end{aligned}$$

(a) Write down a similar expression for (1)

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3.$$

Solution

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = \underline{\underline{\frac{5^2 \times 6^2}{4}}}.$$

(b) Write down a similar expression for (2)

$$1^3 + 2^3 + 3^3 + \ldots + n^3.$$

Solution

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \underline{\underline{\frac{n^2 \times (n+1)^2}{4}}}.$$

(c) Hence **evaluate** (2)

$$1^3 + 2^3 + 3^3 + \ldots + 9^3.$$

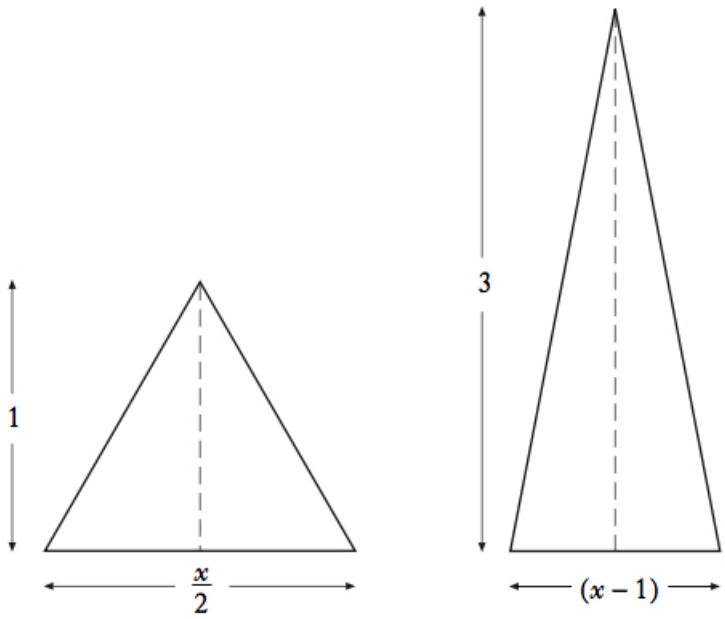
Solution

$$\begin{aligned} 1^3 + 2^3 + 3^3 + \ldots + 9^3 &= \frac{9^2 \times 10^2}{4} \\ &= \frac{81 \times 100}{4} \\ &= 81 \times 25 \end{aligned}$$

$\times$	80	1
20	1 600	20
5	400	5

$$\begin{aligned} &= 1\,600 + 20 + 400 + 5 \\ &= 2\,000 + 25 \\ &= \underline{\underline{2\,025}}. \end{aligned}$$

11. Two triangles have dimensions as shown. (4)



The triangles are equal in area.
Calculate the value of x.

Solution

$$\tfrac{1}{2} \times \frac{x}{2} \times 1 = \tfrac{1}{2} \times (x-1) \times 3 \Rightarrow \frac{x}{2} = 3(x-1)$$
$$\Rightarrow \frac{x}{2} = 3x - 3$$
$$\Rightarrow \frac{5x}{2} = 3$$
$$\Rightarrow 5x = 6$$
$$\Rightarrow \underline{\underline{x = 1\tfrac{1}{5}}}.$$