Dr Oliver Mathematics
Mathematics
Expectation and Variance
Past Examination Questions

This booklet consists of 31 questions across a variety of examination topics.
The total number of marks available is 338.

1. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(2-x), & x = 0, 1, 2, \\ k(x-2), & x = 3, \\ 0, & \text{otherwise}, \end{cases}$$

where k is a positive constant.

(a) Show that $k = 0.25$. (2)

(b) Find $E(X)$ and show that $E(X^2) = 2.5$. (4)

(c) Find $\text{Var}(3X - 2)$. (3)

Two independent observations X_1 and X_2 are made of X.

(d) Show that $P(X_1 + X_2 = 5) = 0$. (1)

(e) Find the complete probability function for $X_1 + X_2$. (3)

(f) Find $P(1.3 \leqslant X_1 + X_2 \leqslant 3.2)$. (3)

2. A discrete random variable X has a probability function as shown in the table below, where a and b are constants.

x	0	1	2	3
$P(X = x)$	0.2	0.3	b	a

Given that $E(X) = 1.7$,

(a) find the value of a and the value of b. (5)

Find

(b) $P(0 < X < 1.5)$, (1)

(c) $E(2X - 3)$. (2)

(d) Show that $\text{Var}(X) = 1.41$. (3)

(e) Evaluate $\text{Var}(2X?3)$. (2)

3. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, \\ k(x+1), & x = 4, 5, \end{cases}$$

where k is a constant.

(a) Find the value of k. (2)

(b) Find the exact value of $\text{E}(X)$. (2)

(c) Show that, to 3 significant figures, $\text{Var}(X) = 1.47$. (4)

(d) Find, to 1 decimal place, $\text{Var}(4 - 3X)$. (2)

4. The random variable X has probability distribution

x	1	2	3	4	5
$P(X = x)$	0.10	p	0.20	q	0.30

(a) Given that $\text{E}(X) = 3.5$, write down two equations involving p and q. (3)

Find

(b) the value of p and the value of q, (3)

(c) $\text{Var}(X)$, (4)

(d) $\text{Var}(3 - 2X)$. (2)

5. The random variable X has the discrete uniform distribution

$$P(X = x) = \tfrac{1}{5}, \ x = 1, 2, 3, 4, 5.$$

(a) Write down the value of $\text{E}(X)$ and show that $\text{Var}(X) = 2$. (3)

Find

(b) $\text{E}(3X - 2)$, (2)

(c) $\text{Var}(4 - 3X)$. (2)

6. The random variable X has probability function

$$P(X = x) = \frac{2x - 1}{36}, \ x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of X. (3)

Find

(b) $\mathrm{P}(2 < X \leqslant 5)$, (2)

(c) the exact value of $\mathrm{E}(X)$. (2)

(d) Show that $\mathrm{Var}(X) = 1.97$ to 3 significant figures. (4)

(e) Find $\mathrm{Var}(2 - 3X)$. (2)

7. The random variable X has probability distribution

x	1	3	5	7	9
$\mathrm{P}(X = x)$	0.2	p	0.2	q	0.15

(a) Given that $\mathrm{E}(X) = 4.5$, write down two equations involving p and q. (3)

Find

(b) the value of p and the value of q, (3)

(c) $\mathrm{P}(4 < X \leqslant 7)$. (2)

Given that $\mathrm{E}(X^2) = 27.4$, find

(d) $\mathrm{Var}(X)$, (2)

(e) $\mathrm{E}(19 - 4X)$, (1)

(f) $Var(19 - 4X)$. (2)

8. The table below represents the probability distribution of the random variable T.

t	0	1	2	3	4	6	9
$\mathrm{P}(T = t)$	$\frac{7}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

Find the values of

(a) $\mathrm{E}(T)$, (2)

(b) $\mathrm{Var}(T)$. (4)

9. The random variable X has probability distribution given in the table below.

x	-1	0	1	2	3
$\mathrm{P}(X = x)$	p	q	0.2	0.15	0.15

Given that $\mathrm{E}(X) = 0.55$, find

(a) the value of p and the value of q, (5)

(b) $\text{Var}(X)$, (4)

(c) $\text{E}(2X - 4)$. (2)

10. The discrete random variable X can take only the values 2, 3, or 4. For these values the cumulative distribution function is defined by

$$\text{F}(x) = \frac{(x+k)^2}{25} \text{ for } x = 2, 3, 4,$$

where k is a positive integer.

(a) Find k. (2)

(b) Find the probability distribution of X. (3)

11. When Rohit plays a game, the number of points he receives is given by the discrete random variable X with the following probability distribution.

x	0	1	2	3
$\text{P}(X = x)$	0.4	0.3	0.2	0.1

(a) Find $\text{E}(X)$. (2)

(b) Find $\text{F}(1.5)$. (2)

(c) Show that $\text{Var}(X) = 1$. (4)

(d) Find $\text{Var}(5 - 3X)$. (2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10. After 3 games he has a total of 6 points. You may assume that games are independent.

(e) Find the probability that Rohit wins the prize. (6)

12. The discrete random variable X has probability function

$$\text{P}(X = x) = \begin{cases} a(3 - x), & x = 0, 1, 2, \\ b, & x = 3. \end{cases}$$

(a) Find $\text{P}(X = 2)$ and complete the table below. (1)

x	0	1	2	3
$\text{P}(X = x)$	$3a$	$2a$		b

Given that $\text{E}(X) = 1.6$,

(b) Find the value of a and the value of b. (5)

Find

(c) $\text{P}(0.5 < X < 3)$, (2)

(d) $\text{E}(3X - 2)$. (2)

(e) Show that the $\text{Var}(X) = 1.64$. (3)

(f) Calculate $\text{Var}(3X - 2)$. (2)

13. The probability function of a discrete random variable X is given by

$$p(x) = kx^2,\ x = 1, 2, 3,$$

where k is a positive constant.

(a) Show that $k = \frac{1}{14}$. (2)

Find

(b) $\text{P}(X \geqslant 2)$, (2)

(c) $\text{E}(X)$, (2)

(d) $\text{Var}(1 - X)$. (4)

14. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$\text{P}(X = x)$	$\frac{1}{5}$	a	$\frac{1}{10}$	a	$\frac{1}{5}$

where a is a constant.

(a) Find the value of a. (2)

(b) Write down $\text{E}(X)$. (1)

(c) Find $\text{Var}(X)$. (3)

The random variable $Y = 6 - 2X$.

(d) Find $\text{Var}(Y)$. (2)

(e) Calculate $\text{P}(X \geqslant Y)$. (3)

15. The discrete random variable X has the probability distribution

x	1	2	3	4
$\text{P}(X = x)$	k	$2k$	$3k$	$4k$

(a) Show that $k = 0.1$. (1)

Find

(b) $E(X)$, (2)

(c) $E(X^2)$, (2)

(d) $\text{Var}(2 - 5X)$. (3)

Two independent observations X_1 and X_2 are made of X.

(e) Show that $P(X_1 + X_2 = 4) = 0.1$. (2)

(f) Complete the probability distribution table for $X_1 + X_2$. (2)

y	2	3	4	5	6	7	8
$P(X_1 + X_2 = y)$	0.01	0.04	0.10		0.25	0.24	

(g) Find $P(1.5 < X_1 + X_2 \leqslant 3.5)$. (2)

16. The discrete random variable Y has the probability distribution

y	1	2	3	4
$P(Y = y)$	a	b	0.3	c

where a, b, and c are constants.

The cumulative distribution function $F(y)$ of Y is given in the following table

y	1	2	3	4
$F(y)$	0.1	0.5	d	1.0

where d is a constant.

(a) Find the value of a, the value of b, the value of c, and the value of d. (5)

(b) Find $P(3Y + 2 \geqslant 8)$. (2)

17. A spinner is designed so that the score S is given by the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	p	0.25	0.25	0.20	0.20

(a) Find the value of p. (2)

(b) Find $\mathrm{E}(S)$. (2)

(c) Show that $\mathrm{E}(S^2) = 9.45$. (2)

(d) Find $\mathrm{Var}(S)$. (2)

18. The discrete random variable X can take only the values 2, 3, 4, or 6. For these values the probability distribution function is given by

x	2	3	4	6
$\mathrm{P}(X = x)$	$\frac{5}{21}$	$\frac{2k}{21}$	$\frac{7}{21}$	$\frac{k}{21}$

where k is a positive integer.

(a) Show that $k = 3$. (2)

Find

(b) $\mathrm{F}(3)$, (1)

(c) $\mathrm{E}(X)$, (2)

(d) $\mathrm{E}(X^2)$, (2)

(e) $\mathrm{Var}(7X - 5)$. (4)

19. A discrete random variable X has probability function

$$\mathrm{P}(X = x) = \begin{cases} k(1-x)^2, & x = -1, 0, 1, \text{ and } 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{1}{6}$. (3)

(b) Find $\mathrm{E}(X)$. (2)

(c) Show that $\mathrm{E}(X^2) = \frac{4}{3}$. (2)

(d) Find $\mathrm{Var}(1 - 3X)$. (3)

20. The discrete random variable X can take only the values 1, 2, and 3. For these values the cumulative distribution function is defined by

$$\mathrm{F}(x) = \frac{x^3 + k}{40}, \ x = 1, 2, 3.$$

(a) Show that $k = 13$. (2)

(b) Find the probability distribution of X. (4)

Given that $\mathrm{Var}(X) = \frac{259}{320}$,

(c) find the exact value of $\text{Var}(4X - 5)$. (2)

21. A fair blue die has faces numbered 1, 1, 3, 3, 5, and 5. The random variable B represents the score when the blue die is rolled.

(a) Write down the probability distribution for B. (2)

(b) State the name of this probability distribution. (1)

(c) Write down the value of $\text{E}(B)$. (1)

A second die is red and the random variable R represents the score when the red die is rolled. The probability distribution of R is

r	2	4	6
$\text{P}(R = r)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(d) Find $\text{E}(R)$. (2)

(e) Find $\text{Var}(R)$. (3)

22. A biased die with six faces is rolled. The discrete random variable X represents the score on the uppermost face. The probability distribution of X is shown in the table below.

x	1	2	3	4	5	6
$\text{P}(X = x)$	a	a	a	b	b	0.3

(a) Given that $\text{E}(X) = 4.2$, find the value of a and the value of b. (5)

(b) Show that $\text{E}(X^2) = 20.4$. (1)

(c) Find $\text{Var}(5 - 3X)$. (3)

A biased die with five faces is rolled. The discrete random variable Y represents the score which is uppermost. The cumulative distribution function of Y is shown in the table below.

y	1	2	3	4	5
$\text{F}(y)$	$\frac{1}{10}$	$\frac{2}{10}$	$3k$	$4k$	$5k$

(d) Find the value of k. (1)

(e) Find the probability distribution of Y. (3)

Each die is rolled once. The scores on the two dice are independent.

(f) Find the probability that the sum of the two scores equals 2. (2)

23. The discrete random variable X takes the values 1, 2, and 3 and has cumulative distribution function $\mathrm{F}(x)$ given by

x	1	2	3
$\mathrm{F}(x)$	0.4	0.65	1

(a) Find the probability distribution of X. (3)

(b) Write down the value of $\mathrm{F}(1.8)$. (1)

24. The score S when a spinner is spun has the following probability distribution.

s	0	1	2	4	5
$\mathrm{P}(S = s)$	0.2	0.2	0.1	0.3	0.2

(a) Find $\mathrm{E}(S)$. (2)

(b) Show that $\mathrm{E}(S^2) = 10.4$. (2)

(c) Hence find $\mathrm{Var}(S)$. (2)

(d) Find (4)

(i) $\mathrm{E}(5S - 3)$,

(ii) $\mathrm{Var}(5S - 3)$.

(e) Find $\mathrm{P}(5S - 3 > S + 3)$. (3)

The spinner is spun twice. The score from the first spin is S_1 and the score from the second spin is S_2. The random variables S_1 and S_2 are independent and the random variable $X = S_1 \times S_2$.

(f) Show that $\mathrm{P}(\{S_1 = 1\} \cap X < 5) = 0.16$. (2)

(g) Find $\mathrm{P}(X < 5)$. (3)

25. The discrete random variable X has the probability function

$$\mathrm{P}(X = x) = \begin{cases} kx, & x = 2, 4, 6, \\ k(x-2), & x = 8, \\ 0, & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{18}$. (2)

(b) Find the exact value of F(5). (1)

(c) Find the exact value of E(X). (2)

(d) Find the exact value of E(X^2). (2)

(e) Calculate Var($3 - 4X$), giving your answer to 3 significant figures. (3)

26. The discrete random variable X has probability distribution

x	-4	-2	1	3	5
$P(X = x)$	0.4	p	0.05	0.15	p

(a) Show that $p = 0.2$. (2)

Find

(b) E(X), (2)

(c) F(0), (1)

(d) $P(3X + 2 > 5)$. (2)

Given that $\text{Var}(X) = 13.35$,

(e) find the possible values of a such that $\text{Var}(aX + 3) = 53.4$. (2)

27. The discrete random variable X has probability distribution

$$P(X = x) = \tfrac{1}{10},\ x = 1, 2, 3, \ldots, 10.$$

(a) Write down the name given to this distribution. (2)

(b) Write down the value of (2)

(i) $P(X = 10)$,

(ii) $P(X \leqslant 10)$.

28. In a quiz, a team gains 10 points for every question it answers correctly and loses 5 points for every question it does not answer correctly. The probability of answering a question correctly is 0.6 for each question. One round of the quiz consists of 3 questions.

The discrete random variable X represents the total number of points scored in one round. The table shows the incomplete probability distribution of X

x	30	15	0	-15
$P(X = x)$	0.216			0.064

(a) Show that the probability of scoring 15 points in a round is 0.432. (2)

(b) Find the probability of scoring 0 points in a round. (1)

(c) Find the probability of scoring a total of 30 points in 2 rounds. (3)

(d) Find $\mathrm{E}(X)$. (2)

(e) Find $\mathrm{Var}(X)$. (3)

In a bonus round of 3 questions, a team gains 20 points for every question it answers correctly and loses 5 points for every question it does not answer correctly.

(f) Find the expected number of points scored in the bonus round. (3)

29. The discrete random variable X has the following probability distribution, where p and q are constants.

x	-2	-1	$\frac{1}{2}$	$\frac{3}{2}$	2
$\mathrm{P}(X = x)$	p	q	0.2	0.3	p

(a) Write down an equation in p and q. (1)

Given that $\mathrm{E}(X) = 0.4$,

(b) find the value of q, (3)

(c) hence find the value of p. (2)

Given also that $\mathrm{E}(X^2) = 2.275$,

(d) find $\mathrm{Var}(X)$. (2)

Sarah and Rebecca play a game. A computer selects a single value of X using the probability distribution above. Sarah's score is given by the random variable $S = X$ and Rebecca's score is given by the random variable $R = \dfrac{1}{X}$.

(e) Find $\mathrm{E}(R)$. (3)

(f) Find the probability that (4)

(i) Sarah is the winner,

(ii) Rebecca is the winner.

30. The discrete random variable X has probability distribution

x	-1	0	1	2
$\mathrm{P}(X = x)$	a	b	b	c

The cumulative distribution function of X is given by

x	-1	0	1	2
$\mathrm{F}(x)$	$\frac{1}{3}$	d	$\frac{5}{6}$	e

(a) Find the values of a, b, c, d, and e. (5)

(b) Write down the value of $\mathrm{P}(X^2 = 1)$. (1)

31. The score, X, for a biased spinner is given by the probability distribution

x	0	3	6
$\mathrm{P}(X = x)$	$\frac{1}{12}$	$\frac{2}{3}$	$\frac{1}{4}$

Find

(a) $\mathrm{E}(X)$, (2)

(b) $\mathrm{Var}(X)$. (3)

A biased coin has one face labelled 2 and the other face labelled 5. The score, Y, when the coin is spun has

$$\mathrm{P}(Y = 5) = p \text{ and } E(Y) = 3.$$

(c) Form a linear equation in p and show that $p = \frac{1}{3}$. (3)

(d) Write down the probability distribution of Y. (1)

Sam plays a game with the spinner and the coin. Each is spun once and Sam calculates his score, S, as follows

$$\text{if } X = 0, \text{ then } S = Y^2$$
$$\text{if } X \neq 0, \text{ then } S = XY.$$

(e) Show that $\mathrm{P}(S = 30) = \frac{1}{12}$. (2)

(f) Find the probability distribution of S. (3)

(g) Find $\mathrm{E}(S)$. (2)

Charlotte also plays the game with the spinner and the coin. Each is spun once and Charlotte ignores the score on the coin and just uses X^2 as her score. Sam and Charlotte each play the game a large number of times.

(h) State, giving a reason, which of Sam and Charlotte should achieve the higher total score. (2)