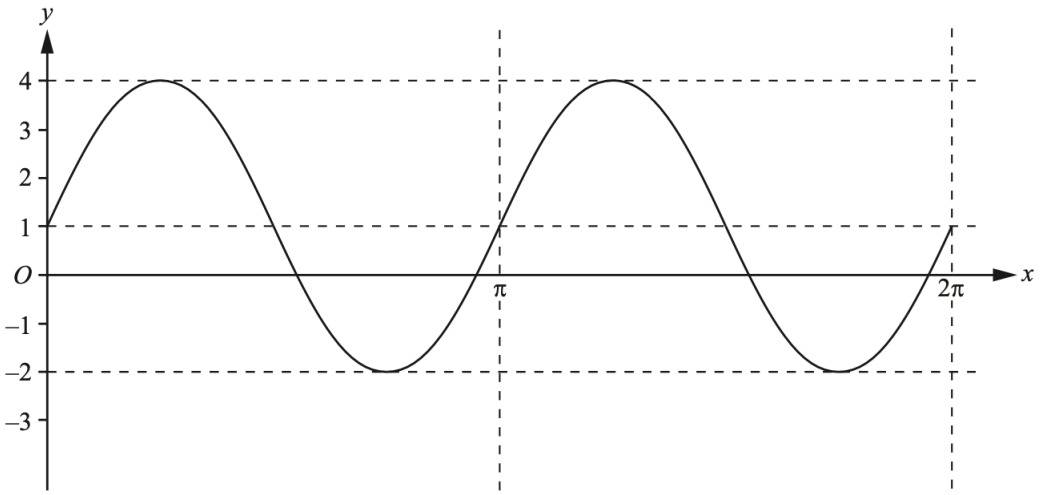
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2013 June Paper 1 Variant 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. The diagram shows the graph of (3)

$$y = a\sin(bx + c), \text{ for } 0 \leqslant x \leqslant 2\pi,$$

where a, b, and c are positive integers.



State the values of a, b, and c.

2. Find the set of values of k for which the curve (4)

$$y = (k + 1)x^2 - 3x + (k + 1)$$

lies below the x-axis.

3. Show that (4)

$$\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} \equiv 2\sec\theta.$$

4. The sets A and B are such that

$$A = \left\{x : \cos x = \tfrac{1}{2},\ 0^\circ \leqslant x \leqslant 620^\circ\right\},$$
$$B = \left\{x : \tan x = \sqrt{3},\ 0^\circ \leqslant x \leqslant 620^\circ\right\}.$$

(a) Find $\mathrm{n}(A)$. (1)

(b) Find $\mathrm{n}(B)$. (1)

(c) Find the elements of $A \cup B$. (1)

(d) Find the elements of $A \cap B$. (1)

5. (a) Find (3)

$$\int (9 + \sin 3x)\,\mathrm{d}x.$$

(b) Hence show that (3)

$$\int_{-\frac{1}{9}\pi}^{\pi} (9 + \sin 3x)\,\mathrm{d}x = a\pi + b,$$

where a and b are constants to be found.

6. The function (6)

$$\mathrm{f}(x) = ax^3 + 4x^2 + bx - 2,$$

where a and b constants, is such that $(2x - 1)$ is a factor.

Given that the remainder when $\mathrm{f}(x)$ is divided by $(x - 2)$ is twice the remainder when $\mathrm{f}(x)$ is divided by $(x + 1)$, find the value of a and of b.

7. (a) (i) Find how many different 4-digit numbers can be formed from the digits 1, 3, 5, 6, 8, and 9 if each digit may only be used only once. (1)

(ii) Find how many of the 4-digit numbers are even. (1)

(b) A team of 6 people is to be selected from 8 men and 4 women.

Find the number of different teams that can be selected if

(i) there are no restrictions, (1)

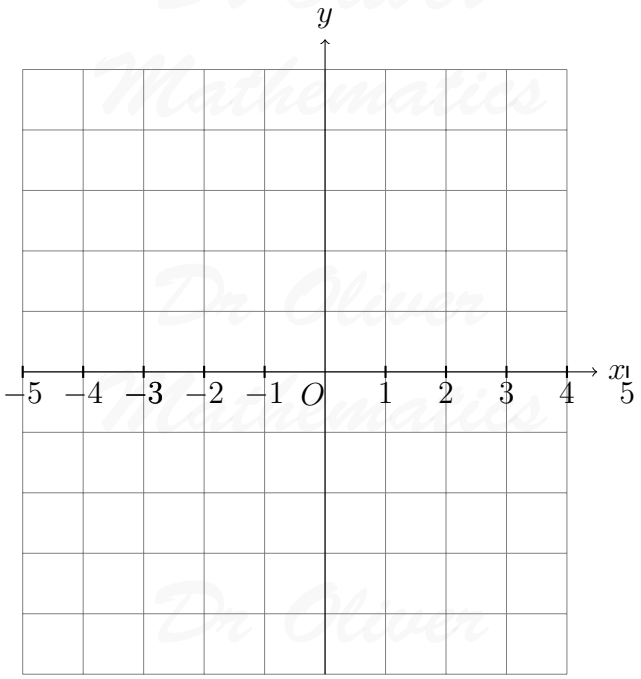
(ii) the team contains all 4 women, (1)

(iii) the team contains at least 4 men. (3)

8. (a) On the grid, sketch the graph of (4)

$$y = |(x - 2)(x + 3)| \text{ for } -5 \leqslant x \leqslant 4,$$

and state the coordinates of the points where the curve meets the coordinate axis.



(b) Find the coordinates of the stationary points on the curve (2)

$$y = |(x-2)(x+3)|.$$

(c) Given that k is a positive constant, state the set of values of k for which (1)

$$|(x-2)(x+3)| = k$$

has only two solutions.

9. (a) Differentiate (3)

$$4x^3 \ln(2x+1)$$

with respect to x.

(b) (i) Given that (4)

$$y = \frac{2x}{\sqrt{x+2}},$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+4}{(\sqrt{x+2})^3}.$$

(ii) Hence find (2)

$$\int \frac{5x+20}{(\sqrt{x+2})^3}\,\mathrm{d}x.$$

(iii) Hence evaluate (2)

$$\int_2^7 \frac{5x+20}{(\sqrt{x+2})^3}\,\mathrm{d}x.$$

10. **Solutions to this question by accurate drawing will not be accepted.**
The points $A(-3,2)$ and $B(1,4)$ are vertices of an isosceles triangle ABC, where angle $ABC = 90°$.

(a) Find the length of the line AB. (1)

(b) Find the equation of the line BC. (3)

(c) Find the coordinates of each of the two possible positions of C. (6)

11. (a) It is given that the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}.$$

(i) Find $\mathbf{A} + 2\mathbf{I}$. (1)

(ii) Find $\mathbf{A}^2$. (2)

(iii) Use your answers to parts (i) and (ii) to find the matrix $\mathbf{B}$ such that (2)

$$\mathbf{A}^2\mathbf{B} = \mathbf{I}.$$

(b) Given the matrix (4)

$$\mathbf{C} = \begin{pmatrix} x & -1 \\ x^2 - x + 1 & x - 1 \end{pmatrix},$$

show that $\det \mathbf{C} \neq 0$.

12. (a) A function f is such that

$$\mathrm{f}(x) = 3x^2 - 1 \text{ for } -10 \leqslant x \leqslant 8.$$

(i) Find the range of f. (3)

(ii) Write down a suitable domain for f for which f^{-1} exists. (1)

(b) Function g and h are defined by

$$\mathrm{g}(x) = 4\mathrm{e}^x - 2 \text{ for } x \in \mathbb{R},$$
$$\mathrm{h}(x) = \ln 5x \text{ for } x > 0.$$

(i) Find $\mathrm{g}^{-1}(x)$. (2)

(ii) Solve (3)

$$\mathrm{g}\,\mathrm{h}(x) = 18.$$