Dr Oliver Mathematics
AQA Further Maths Level 2
June 2025 Paper 2
1 hour 45 minutes

The total number of marks available is 80.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

1. The 5th term in sequence A is also a term in sequence B. (3)

Sequence	nth term
A	$\dfrac{6n}{7n+25}$
B	$\dfrac{20}{n+4}$

Work out the **position** of the term in sequence B.

Solution

Well,

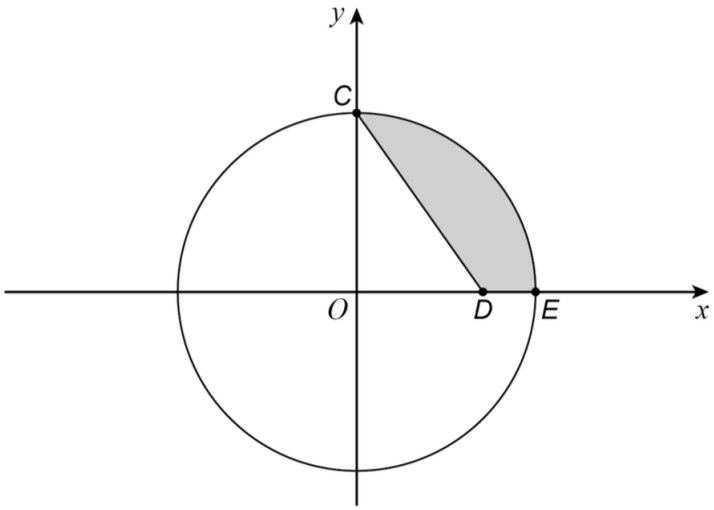
$$\begin{aligned} \text{5th term in } A &= \frac{6(5)}{7(5)+25} \\ &= \frac{6(5)}{7(5)+25} \\ &= \tfrac{1}{2}. \end{aligned}$$

Cross-multiply:

$$\begin{aligned} \frac{20}{n+4} = \tfrac{1}{2} &\Rightarrow 40 = n+4 \\ &\Rightarrow \underline{\underline{n = 36}}. \end{aligned}$$

2. C and E are points on the circle with equation (4)

$$x^2 + y^2 = 144.$$



Not drawn accurately

- CD is a straight line.
- $OD : DE = 5 : 1$.

Work out the shaded area.
Give your answer as a decimal.

Solution

Well, the radius is

$$\sqrt{144} = 12$$

and

$$\begin{aligned} \text{quadrant } OCE &= \tfrac{1}{4} \times \pi \times 12^2 \\ &= 36\pi. \end{aligned}$$

Now,

$$5 + 1 = 6$$

and

$$OD = \tfrac{5}{6} \times 12 = 10.$$

Next,

$$\begin{aligned} \text{triangle } OCD &= \tfrac{1}{2} \times 12 \times 10 \\ &= 60. \end{aligned}$$

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Hence,

$$\begin{aligned}\text{shaded area} &= \text{quadrant } OCE - \text{triangle } OCD \\ &= 36\pi - 60 \\ &= 53.097\,335\,53 \text{ (FCD)} \\ &= \underline{\underline{53.1 \text{ (3 sf)}}}.\end{aligned}$$

3. The equation of a straight line is (3)

$$2ax + y = 25, \text{ where } a \text{ is a constant.}$$

The line passes through the point $(3, 1)$.

Work out the value of the gradient of the line.

Solution

Well,

$$\begin{aligned}x = 3, y = 1 &\Rightarrow 2a(3) + 1 = 25 \\ &\Rightarrow 6a + 1 = 25 \\ &\Rightarrow 6a = 24 \\ &\Rightarrow a = 4.\end{aligned}$$

So,

$$8x + y = 25 \Rightarrow y = -8x + 25;$$

hence, the gradient of the line is $\underline{\underline{-8}}$.

4. (2)

$$-5 \leqslant n \leqslant 5.$$

Work out the range of possible values of n^2.

Solution

Well,

$$-5 \leqslant n \leqslant 5 \Rightarrow \underline{\underline{0 \leqslant n^2 \leqslant 25}}.$$

5. The equation of a curve is (3)

$$y = x^5 + px^4, \text{ where } p \text{ is a constant.}$$

When $x = -1$, the gradient of the curve is -3.

Work out the value of p.

Solution

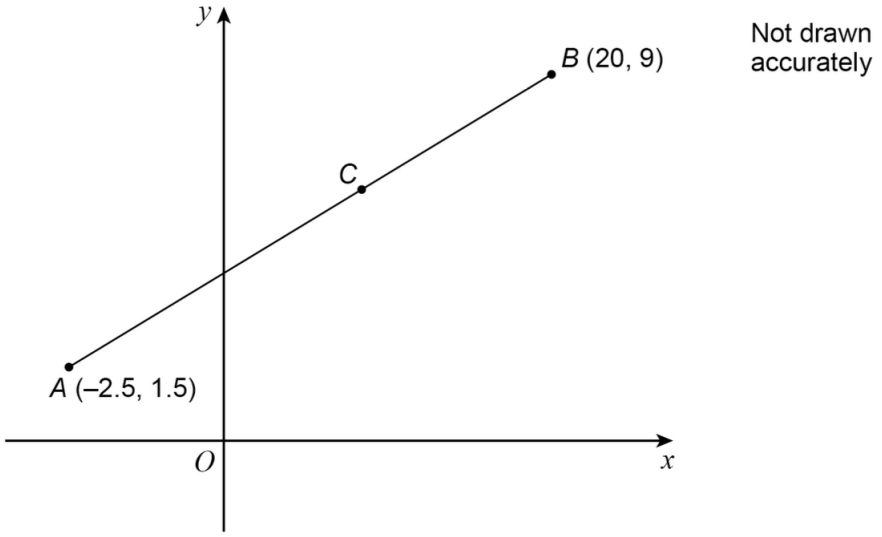
Well,

$$y = x^5 + px^4 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 5x^4 + 4px^3$$

and

$$\begin{aligned} x = -1, \frac{\mathrm{d}y}{\mathrm{d}x} = -3 &\Rightarrow 5 - 4p = -3 \\ &\Rightarrow -4p = -8 \\ &\Rightarrow \underline{\underline{p = 2}}. \end{aligned}$$

6. C is a point on the straight line AB. (3)



$AC : CB = 3 : 2$.

Work out the equation of the straight line passing through O and C.

Solution

Well,

$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AC} \\ &= \overrightarrow{OA} + \tfrac{3}{5}\overrightarrow{AB} \\ &= \begin{pmatrix} -2.5 \\ 1.5 \end{pmatrix} + \tfrac{3}{5}\begin{pmatrix} 20-(-2.5) \\ 9-1.5 \end{pmatrix} \\ &= \begin{pmatrix} -2.5 \\ 1.5 \end{pmatrix} + \tfrac{3}{5}\begin{pmatrix} 22.5 \\ 7.5 \end{pmatrix} \\ &= \begin{pmatrix} -2.5 \\ 1.5 \end{pmatrix} + \begin{pmatrix} 13.5 \\ 4.5 \end{pmatrix} \\ &= \begin{pmatrix} 11 \\ 6 \end{pmatrix};\end{aligned}$$

so, $C(6, 11)$. Finally, the equation of the straight line is

$$\underline{\underline{y = \tfrac{6}{11}x}}.$$

7. (a) (2)

$$2^{5a+1} = 2^a.$$

Work out the value of a.

Solution

Well,

$$\begin{aligned}5a + 1 = a &\Rightarrow 4a = -1 \\ &\Rightarrow \underline{\underline{a = -\tfrac{1}{4}}}.\end{aligned}$$

(b) (3)

$$3^{-7} \div 3^c = (3^{-3})^{2c}.$$

Work out the value of c.

Solution

Now,

$$\begin{aligned} 3^{-7} \div 3^{c} = (3^{-3})^{2c} &\Rightarrow 3^{-7-c} = 3^{-6c} \\ &\Rightarrow -7 - c = -6c \\ &\Rightarrow 5c = 7 \\ &\Rightarrow \underline{\underline{c = \tfrac{7}{5}}}. \end{aligned}$$

8. k is a positive integer. (3)

Prove that

$$4k(k+8) - (4k+9)(k-1)$$

is a multiple of 9.

Solution

Well,

$\times$	$4k$	$+9$
k	$4k^2$	$+9k$
-1	$-4k$	-9

and

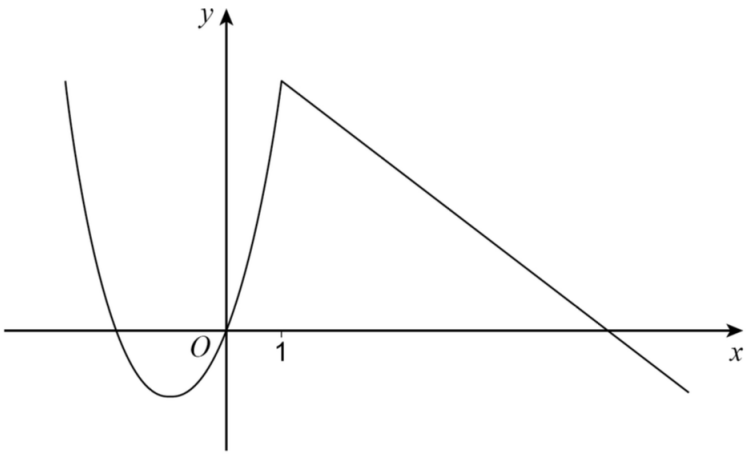
$$\begin{aligned} 4k(k+8) - (4k+9)(k-1) &= 4k^2 + 32k - (4k^2 + 5k - 9) \\ &= 4k^2 + 32k - 4k^2 - 5k + 9 \\ &= 27k + 9 \\ &= 9(3k+1); \end{aligned}$$

hence the expression is a <u>multiple of 9</u>.

9. (2)

$$\mathrm{f}(x) = \begin{cases} x^2 + px, & x \leqslant 1 \\ 2p - \frac{1}{2}x, & x > 1. \end{cases}$$

A sketch of $y = \mathrm{f}(x)$ is shown.



Work out the value of p.

Solution

Well,

$$x = 1 \Rightarrow \mathrm{f}(1) = 1^2 + p(1)$$
$$\Rightarrow \mathrm{f}(1) = p + 1$$

and

$$x = 1 \Rightarrow \mathrm{f}(1) = 2p - \tfrac{1}{2}(1)$$
$$\Rightarrow \mathrm{f}(1) = 2p - \tfrac{1}{2}.$$

Now, both sides are equal:

$$p + 1 = 2p - \tfrac{1}{2} \Rightarrow \underline{\underline{p = \tfrac{3}{2}}}.$$

10. The first four terms of a quadratic sequence are (2)

$$-1 \qquad 11 \qquad 31 \qquad 59.$$

Work out an expression for the nth term.
Give your answer in the form $an^2 + b$, where a and b are constants.

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Solution

Let the

$$n\text{th term} = an^2 + c.$$

Then

$$\begin{array}{ccccccc} -1 & & 11 & & 31 & & 59 \\ & 12 & & 20 & & 28 & \\ & & 8 & & 8 & & \end{array}$$

and

$$\begin{array}{ccccccc} a+c & & 4a+c & & 9a+c & & 16a+c \\ & 3a & & 5a & & 7a & \\ & & 2a & & 2a & & \end{array}$$

We compare terms:

$$2a = 8 \Rightarrow a = 4,$$

and

$$\begin{aligned} a + c = -1 &\Rightarrow 4 + c = -1 \\ &\Rightarrow c = -5; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{4n^2 - 5}}.$$

11. (a) Simplify fully (2)

$$\frac{3}{4w} + \frac{1}{6w}.$$

Solution

Well,

$$\begin{aligned} \frac{3}{4w} + \frac{1}{6w} &= \frac{9}{12w} + \frac{2}{12w} \\ &= \underline{\underline{\frac{11}{12w}}}. \end{aligned}$$

(b) Factorise fully (2)

$$(x-1)^9 + (x-1)^8(x+4).$$

Solution

Now,

$$\begin{aligned}(x-1)^9 + (x-1)^8(x+4) &= (x-1)^8[(x-1)+(x+4)] \\ &= \underline{\underline{(2x+3)(x-1)^8}}.\end{aligned}$$

(c) Factorise fully (2)

$$8y^2 - 50.$$

Solution

Difference of two squares:

$$\begin{aligned}8y^2 - 50 &= 2(4y^2 - 25) \\ &= 2[(2y)^2 - 5^2] \\ &= \underline{\underline{2(2y-5)(2y+5)}}.\end{aligned}$$

12. (a) Circle the matrix that represents an enlargement, scale factor 3, centre O. (1)

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \quad \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \quad \begin{pmatrix} 3 & 0 \\ 3 & 0 \end{pmatrix}$$

Solution

$$\underline{\underline{\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}}} \quad \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \quad \begin{pmatrix} 3 & 0 \\ 3 & 0 \end{pmatrix}$$

(b) $OABC$ is the unit square, where $O(0,0)$, $A(1,0)$, $B(1,1)$, and $C(0,1)$. (3)

- **K** is the transformation represented by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- **L** is the transformation represented by

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

$OABC$ is mapped to $OA'B'C'$ under a combined transformation of **K** followed by **L**.

Work out the coordinates of A', B', and C'.

Solution

Well,

$$\begin{aligned} \mathbf{LK} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \end{aligned}$$

so, it is a reflection in $y = -x$.

Finally,

$$\underline{\underline{A'(0,-1),\ B'(-1,-1),\ \text{and}\ C'(-1,0)}}.$$

13. Rearrange (4)

$$x = \frac{3k^2}{5k^2 - 2}$$

to make k the subject.

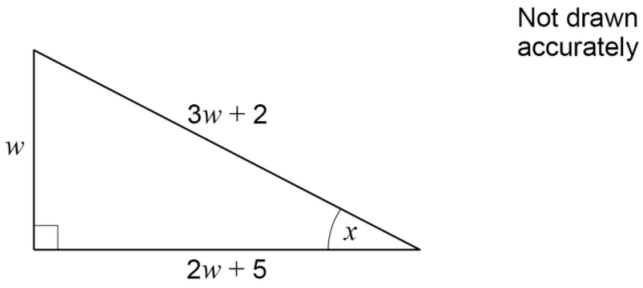
Solution

Well,

$$\begin{aligned} x = \frac{3k^2}{5k^2-2} &\Rightarrow x(5k^2-2) = 3k^2 \\ &\Rightarrow 5k^2x - 2x = 3k^2 \\ &\Rightarrow 5k^2x - 3k^2 = 2x \\ &\Rightarrow k^2(5x-3) = 2x \\ &\Rightarrow k^2 = \frac{2x}{5x-3} \\ &\Rightarrow \underline{\underline{k = \pm\sqrt{\frac{2x}{5x-3}}}}; \end{aligned}$$

as we are not given any information about k.

14. All lengths shown are in centimetres.



(a) Use Pythagoras' theorem to show that (2)

$$4w^2 - 8w - 21 = 0.$$

Solution

Well,

$\times$	$3w$	$+2$
$3w$	$9w^2$	$+6w$
$+2$	$+6w$	$+4$

and

$\times$	$2w$	$+5$
$2w$	$4w^2$	$+10w$
$+5$	$+10w$	$+25$

So,

$$(3w+2)^2 = w^2 + (2w+5)^2 \Rightarrow 9w^2 + 12w + 4 = w^2 + (4w^2 + 20w + 25)$$
$$\Rightarrow \underline{\underline{4w^2 - 8w - 21 = 0}},$$

as required.

(b) Work out the size of angle x. (3)

Solution

Now,

$$\left.\begin{array}{ll} \text{add to:} & -8 \\ \text{multiply to:} & (+4)\times(-21) = -84 \end{array}\right\} +6,\ -14$$

e.g.,

$$\begin{aligned} 4w^2 - 8w - 21 = 0 &\Rightarrow 4w^2 + 6w - 14w - 21 = 0 \\ &\Rightarrow 2w(w+3) - 7(w+3) = 0 \\ &\Rightarrow (2w-7)(w+3) = 0 \\ &\Rightarrow 2w - 7 = 0 \text{ or } w + 3 = 0 \\ &\Rightarrow w = 3.5 \text{ or } w = -3. \end{aligned}$$

Next, $w > 0$ (why?) so $w = 3.5$ and $2w + 5 = 12$.

Finally,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan x = \tfrac{3.5}{12} \\ &\Rightarrow x = 16.260\,204\,471 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 16.3^\circ \text{ (1 dp)}}}. \end{aligned}$$

15. For a curve $y = \mathrm{f}(x)$,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \frac{9}{x^2}.$$

(a) Show that the curve has a turning point where $x = 3$. (1)

Solution

Well,

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow 1 - \frac{9}{x^2} = 0 \\ &\Rightarrow 1 = \frac{9}{x^2} \\ &\Rightarrow x^2 = 9 \\ &\Rightarrow x = \pm 3. \end{aligned}$$

so, the curve has a <u>turning point</u> where $x = 3$.

(b) Use $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to work out the nature of the turning point. (2)
You **must** show your working.

Solution

Now,

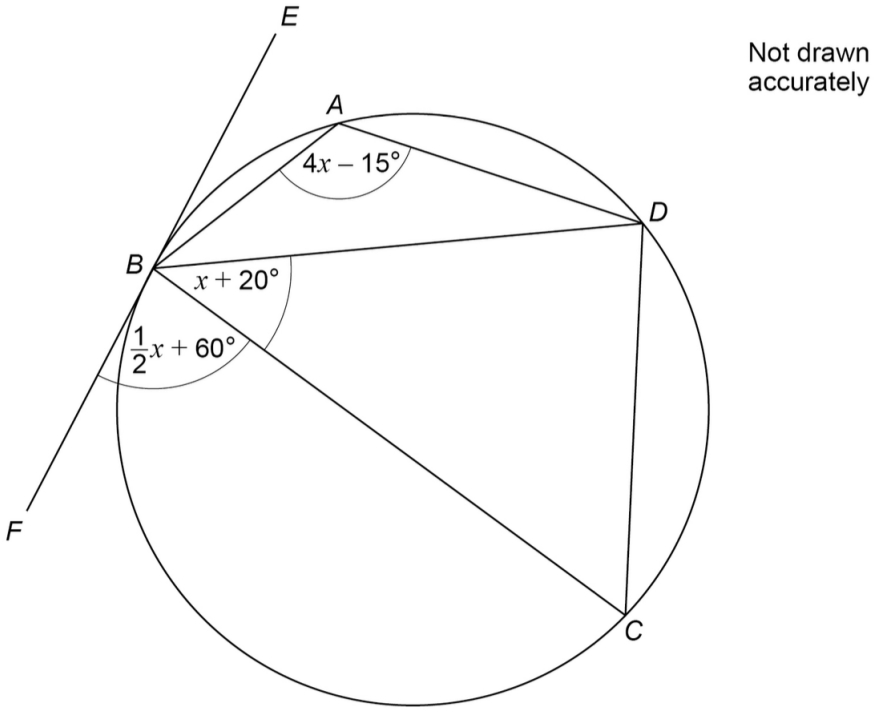
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \frac{9}{x^2} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 9x^{-2}$$
$$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 18x^{-3}.$$

Next,

$$x = 3 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \tfrac{2}{3} > 0;$$

hence, it is a minimum point.

16. A, B, C, and D are points on a circle. (4)



EBF is a tangent.

Work out the value of x.

Solution

Alternate segment theorem:

$$4x - 15 = (x + 20) + (\tfrac{1}{2}x + 60) \Rightarrow \tfrac{5}{2}x = 95$$
$$\Rightarrow \underline{\underline{x = 38^\circ}}.$$

17. Solve (5)

$$2a + 4b + c = 12$$
$$3a - b + 2c = -2$$
$$a - 5b - 3c = 6.$$

Do **not** use trial and improvement.
You **must** show your working.

Solution

We convert into matrices:

$$\begin{pmatrix} 2 & 4 & 1 \\ 3 & -1 & 2 \\ 1 & -5 & -3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 12 \\ -2 \\ 6 \end{pmatrix}$$

Determinant:

$$2(3 + 5) - 4(-9 - 2) + 1(-5 + 1) = 56.$$

Matrix of minors:

$$\begin{pmatrix} 13 & -11 & -14 \\ -7 & -7 & -14 \\ 9 & 1 & -14 \end{pmatrix}$$

Matrix of cofactors:

$$\begin{pmatrix} 13 & 11 & -14 \\ 7 & -7 & 14 \\ 9 & -1 & -14 \end{pmatrix}$$

Transpose:

$$\begin{pmatrix} 13 & 7 & 9 \\ 11 & -7 & -1 \\ -14 & 14 & -14 \end{pmatrix}$$

So,

$$\text{inverse} = \tfrac{1}{56} \begin{pmatrix} 13 & 7 & 9 \\ 11 & -7 & -1 \\ -14 & 14 & -14 \end{pmatrix}.$$

Now,

$$\begin{pmatrix} 2 & 4 & 1 \\ 3 & -1 & 2 \\ 1 & -5 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 12 \\ -2 \\ 6 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \tfrac{1}{56} \begin{pmatrix} 13 & 7 & 9 \\ 11 & -7 & -1 \\ -14 & 14 & -14 \end{pmatrix} \begin{pmatrix} 12 \\ -2 \\ 6 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \tfrac{1}{56} \begin{pmatrix} 196 \\ 140 \\ -280 \end{pmatrix}$$

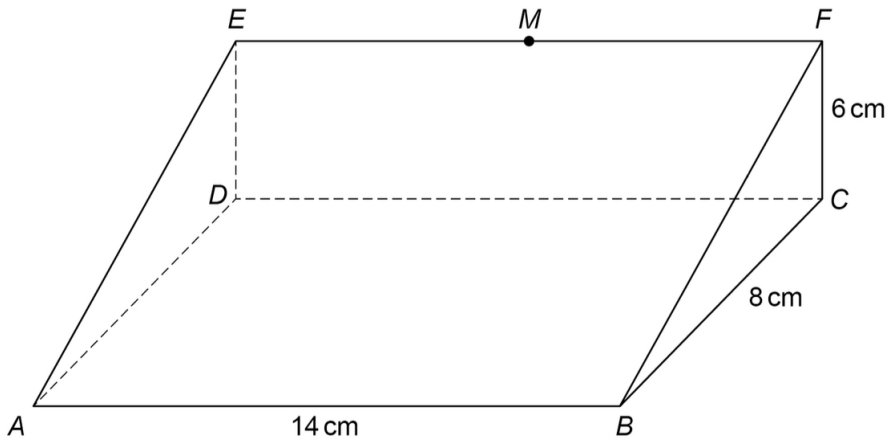
$$\Rightarrow \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \tfrac{1}{56} \begin{pmatrix} 196 \\ 140 \\ -280 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3.5 \\ 2.5 \\ -5 \end{pmatrix},$$

hence,

$$\underline{\underline{a = 3.5,\ b = 2.5,\ \text{and}\ c = -5}}.$$

18. $ABCDEF$ is a triangular prism. (5)



- $ABCD$ is a rectangle and angle BCF is $90°$.
- $AB = 14$ cm.
- $BC = 8$ cm

- $CF = 6$ cm.
- M is the midpoint of EF.

Work out the size of angle BMD.

Solution

Pythagoras' theorem twice and 3D Pythagoras' theorem :

$$\begin{aligned} BD^2 = BC^2 + CD^2 &\Rightarrow BD^2 = 8^2 + 14^2 \\ &\Rightarrow BD^2 = 260 \\ &\Rightarrow BD = 2\sqrt{65}, \end{aligned}$$

$$\begin{aligned} DM^2 = DE^2 + EM^2 &\Rightarrow DM^2 = DE^2 + (\tfrac{1}{2}EF)^2 \\ &\Rightarrow DM^2 = 6^2 + 7^2 \\ &\Rightarrow DM^2 = 85 \\ &\Rightarrow DM = \sqrt{85}, \end{aligned}$$

and

$$\begin{aligned} BM^2 = BC^2 + CF^2 + FM^2 &\Rightarrow BM^2 = 8^2 + 6^2 + 7^2 \\ &\Rightarrow BM^2 = 149 \\ &\Rightarrow DM = \sqrt{149}. \end{aligned}$$

Cosine rule:

$$\begin{aligned} & BD^2 = BM^2 + DM^2 - 2 \times BM \times DM \times \cos BMD \\ \Rightarrow \quad & 260 = 85 + 149 - 2 \times \sqrt{85} \times \sqrt{149} \times \cos BMD \\ & \Rightarrow 2\sqrt{12\,665} \cos BMD = -26 \\ & \Rightarrow \cos BMD = -\frac{26}{2\sqrt{12\,665}} \\ & \Rightarrow \cos BMD = 96.633\,366\,675 \text{ (FCD)} \\ & \Rightarrow \underline{\underline{\cos BMD = 96.6^\circ \text{ (3 sf)}}}. \end{aligned}$$

19. (4)

$$\mathrm{f}(x) = 4x^3 - 7 \text{ and } \mathrm{g}(x) = 6x + 8.$$

Work out the value of x for which

$$\mathrm{f}(2x) = \mathrm{g}\,\mathrm{f}(x).$$

Solution

Well,

$$\begin{aligned} \mathrm{f}(2x) &= 4(2x)^3 - 7 \\ &= 32x^3 - 7 \end{aligned}$$

and

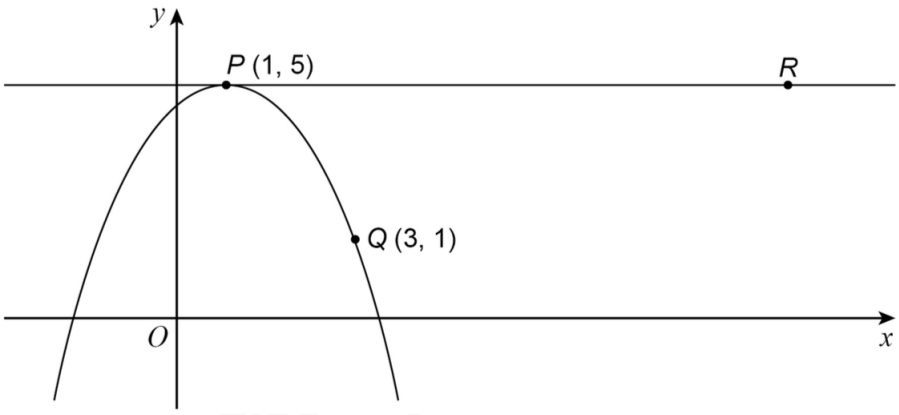
$$\begin{aligned} \mathrm{g\,f}(x) &= \mathrm{g}(\mathrm{f}(x)) \\ &= \mathrm{g}(4x^3 - 7) \\ &= 6(4x^3 - 7) + 8 \\ &= 24x^3 - 42 + 8 \\ &= 24x^3 - 34. \end{aligned}$$

Now,

$$\begin{aligned} \mathrm{f}(2x) = \mathrm{g\,f}(x) &\Rightarrow 32x^3 - 7 = 24x^3 - 34 \\ &\Rightarrow 8x^3 = -27 \\ &\Rightarrow x^3 = -\tfrac{27}{8} \\ &\Rightarrow \underline{\underline{x = -\tfrac{3}{2}}}. \end{aligned}$$

20. Here is a sketch of the curve with equation (5)

$$y = 4 + 2x - x^2.$$



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- The tangent to the curve at the turning point P is shown.
- The normal to the curve at Q meets this tangent at R.

Work out the x-coordinate of R.

Solution

Well,

$$y = 4 + 2x - x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 - 2x.$$

Now,

$$x = 3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -4$$

and

$$m_{\text{normal}} = \tfrac{1}{4}.$$

So,

$$y - 1 = \tfrac{1}{4}(x - 3).$$

But the y-coordinate of R is 5 (why?) and

$$\begin{aligned} y = 5 &\Rightarrow 5 - 1 = \tfrac{1}{4}(x - 3) \\ &\Rightarrow 4 = \tfrac{1}{4}(x - 3) \\ &\Rightarrow 16 = x - 3 \\ &\Rightarrow \underline{\underline{x = 19}}. \end{aligned}$$

21. (a) θ is an angle between 270° and 360°. (2)

$$\cos\theta = \tfrac{1}{5k} \text{ and } \tan\theta = -3k, \text{ where } k \text{ is a positive constant.}$$

Work out the value of θ.

Solution

Well, $\cos\theta$ is positive and

$$\begin{aligned} \tan\theta = \frac{\sin\theta}{\cos\theta} &\Rightarrow -3k = \frac{\sin\theta}{\frac{1}{5k}} \\ &\Rightarrow \sin\theta = -\tfrac{3}{5}. \end{aligned}$$

Finally,

$$\begin{aligned} \sin\theta = -\tfrac{3}{5} &\Rightarrow \theta = 323.130\,102\,4 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\theta = 323^\circ \text{ (3 sf)}}}. \end{aligned}$$

(b) Work out the values of x between 0° and 360° for which (3)

$$8\cos^2 x - \cos x = 0.$$

Solution

Now,

$$\begin{aligned} 8\cos^2 x - \cos x = 0 &\Rightarrow \cos x(8\cos x - 1) = 0 \\ &\Rightarrow \cos x = 0 \text{ or } 8\cos x - 1 = 0 \\ &\Rightarrow \cos x = 0 \text{ or } \cos x = \tfrac{1}{8}. \end{aligned}$$

$\underline{\cos x = 0}$:

$$\cos x = 0 \Rightarrow \underline{\underline{x = 0^\circ, 360^\circ}}.$$

$\underline{\cos x = \frac{1}{8}}$:

$$\begin{aligned} \cos x = \tfrac{1}{8} &\Rightarrow x = 82.819\,244\,22,\ 277.180\,755\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 82.8^\circ,\ 277^\circ \text{ (3 sf)}}}. \end{aligned}$$

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