

Dr Oliver Mathematics
Integration: Part 4

1. If

$$f'(x) = -f(x) \text{ and } f(1) = 1,$$

then find $f(x)$.

Solution

$$\begin{aligned} f'(x) = -f(x) &\Rightarrow \frac{f'(x)}{f(x)} = -1 \\ &\Rightarrow \int \frac{f'(x)}{f(x)} dx = \int (-1) dx \\ &\Rightarrow \ln f(x) = -x + c. \end{aligned}$$

Now,

$$\begin{aligned} \ln f(x) = -x + c &\Rightarrow \ln 1 = -1 + c \\ &\Rightarrow 0 = -1 + c \\ &\Rightarrow c = 1. \end{aligned}$$

Finally,

$$\begin{aligned} \ln f(x) = 1 - x &\Rightarrow e^{\ln f(x)} = e^{1-x} \\ &\Rightarrow \underline{\underline{f(x) = e^{1-x}}}. \end{aligned}$$