Dr Oliver Mathematics
$a\cos\theta + b\sin\theta$: Part 1

1. $$\mathrm{f}(x) = \sqrt{3}\cos x + 3\sin x.$$

Express this in the form of

$$R\cos(x-\alpha),$$

where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Solution

$$\begin{aligned}\sqrt{3}\cos x + 3\sin x &\equiv R\cos(x-\alpha)\\ &\equiv R(\cos x\cos\alpha + \sin x\sin\alpha)\\ &\equiv R\cos x\cos\alpha + R\sin x\sin\alpha\end{aligned}$$

and so

$$R\cos\alpha = \sqrt{3} \text{ and } R\sin\alpha = 3.$$

Now,

$$\begin{aligned}R &= \sqrt{R^2}\\ &= \sqrt{R^2(\cos^2\alpha + \sin^2\alpha)}\\ &= \sqrt{(R\cos\alpha)^2 + (R\sin\alpha)^2}\\ &= \sqrt{(\sqrt{3})^2 + 3^2}\\ &= \sqrt{12}\\ &= \underline{\underline{2\sqrt{3}}}\end{aligned}$$

and

$$\begin{aligned}\tan\alpha = \frac{R\sin\alpha}{R\cos\alpha} &\Rightarrow \tan\alpha = \frac{3}{\sqrt{3}}\\ &\Rightarrow \tan\alpha = \sqrt{3}\\ &\Rightarrow \underline{\underline{\alpha = \tfrac{1}{3}\pi}}.\end{aligned}$$