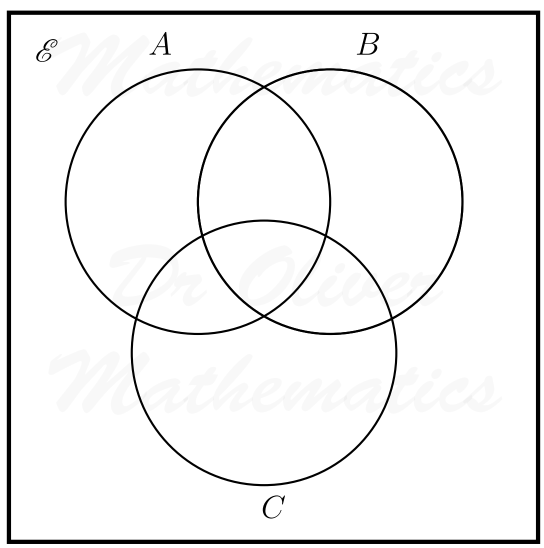
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2010 June Paper 2 Variant 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

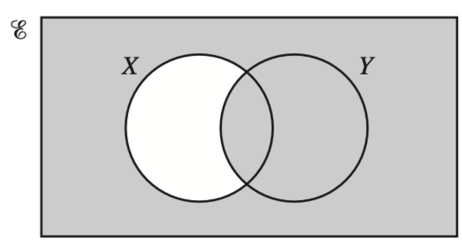
1. Find (3)

$$\int \left[2 + 5x - \frac{1}{(x-2)^2} \right] \mathrm{d}x.$$

2. (a) Copy the diagram and shade the region which represents the set $A \cup (B \cap C')$. (1)



(b) Express, in set notation, the set represented by the shaded region. (1)



(c) The universal set $\mathscr{E}$ and the sets P and Q are such that **(2)**

- $n(\mathscr{E}) = 30$,
- $n(P) = 18$, and
- $n(Q) = 16$

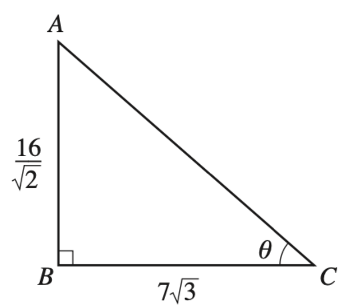
Given that $n(P \cup Q)' = 2$, find $n(P \cap Q)$.

3. The volume, V cm^3 of a spherical ball of radius r cm is given by **(4)**

$$V = \tfrac{4}{3}\pi r^3.$$

Given that the radius is increasing at a constant rate of $\frac{1}{\pi}$ cm s^{-1}, find the rate at which the volume is increasing when $V = 288\pi$.

4. The diagram shows a right-angled triangle ABC in which



- the length of AB is $\dfrac{16}{\sqrt{2}}$,
- the length of BC is $7\sqrt{3}$, and
- angle BCA is θ.

(a) Find $\tan\theta$ in the form $\dfrac{a\sqrt{6}}{b}$, where a and b are integers. **(2)**

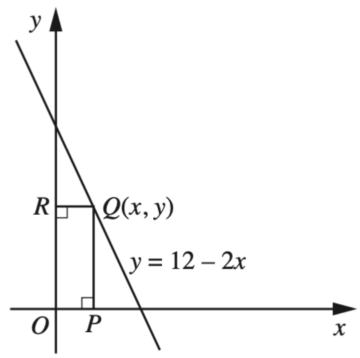
(b) Calculate the length of AC, giving your answer in the form $c\sqrt{d}$, where c and d are integers and d is as small as possible. **(3)**

5. Solve the equation **(6)**

$$2x^3 - 3x^2 - 11x + 6 = 0.$$

6. The diagram shows part of the line

$$y = 12 - 2x.$$



The point $Q(x, y)$ lies on this line and the points P and R lie on the coordinate axes such that $OPQR$ is a rectangle.

(a) Write down an expression, in terms of x, for the area A of the rectangle $OPQR$. (2)

(b) Given that x can vary, find the value of x for which A has a stationary value. (3)

(c) Find this stationary value of A and determine its nature. (2)

7. (a) Sketch the graph of (3)

$$y = |3x + 9| \text{ for } -5 < x < 2,$$

showing the coordinates of the points where the graph meets the axes.

(b) On the same diagram, sketch the graph of (1)

$$y = x + 6.$$

(c) Solve the equation (3)

$$|3x + 9| = x + 6.$$

8. (a) (i) Write down the first 4 terms, in ascending powers of x, of the expansion of (3)

$$(1 - 3x)^7.$$

(ii) Find the coefficient of x^3 in the expansion of (2)

$$(5 + 2x)(1 - 3x)^7.$$

(b) Find the term which is independent of x in the expansion of (3)

$$\left(x^2 + \frac{2}{x}\right)^9.$$

9. (a) Given that (5)

$$y = \frac{x+2}{(4x+12)^{\frac{1}{2}}},$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k(x+4)}{(4x+12)^{\frac{3}{2}}},$$

where k is a constant to be found.

(b) Hence evaluate (3)

$$\int_1^{13} \frac{x+4}{(4x+12)^{\frac{3}{2}}}\,\mathrm{d}x.$$

10. (a) Given that

$$\log_p X = 6 \text{ and } \log_p Y = 4,$$

find the value of

(i) $\log_p \left(\dfrac{X^2}{Y}\right)$, (2)

(ii) $\log_Y X$. (2)

(b) Find the value of (3)

$$2^z,$$

where

$$z = 5 + \log_2 3.$$

(c) Express (2)

$$\sqrt{512}$$

as a power of 4.

11. (a) Solve, for $0 < x < 3$ radians, the equation (3)

$$4\sin x - 3 = 0,$$

giving your answers correct to 2 decimal places.

(b) Solve, for $0^\circ < y < 360^\circ$, the equation (6)

$$4\operatorname{cosec} y = 6\sin y + \cot y.$$

EITHER

12. It is given that

$$\mathrm{f}(x) = 4x^2 + kx + k.$$

(a) Find the set of values of k for which the equation (5)

$$\mathrm{f}(x) = 3$$

has no real roots.

In the case where $k = 10$,

(b) express $\mathrm{f}(x)$ in the form (3)

$$(ax + b)^2 + c,$$

(c) find the least value of $\mathrm{f}(x)$ and the value of x for which this least value occurs. (2)

OR

13. The functions f, g, and h are defined, for $x \in \mathbb{R}$, by

$$\begin{aligned}\mathrm{f}(x) &= x^2 + 1,\\ \mathrm{g}(x) &= 2x - 5, \text{ and}\\ \mathrm{h}(x) &= 2^x.\end{aligned}$$

(a) Write down the range of f. (1)

(b) Find the value of $\mathrm{g\,f}(3)$. (2)

(c) Solve the equation (5)

$$\mathrm{f\,g}(x) = \mathrm{g}^{-1}(15).$$

(d) On the same axes, sketch the graph of $y = \mathrm{h}(x)$ and the graph of the inverse function $y = \mathrm{h}^{-1}(x)$, indicating clearly which graph represents h and which graph represents $\mathrm{h}^{-1}(x)$. (2)