Dr Oliver Mathematics
Mathematics: Advanced Higher
2022 Paper 1: Non-Calculator
1 hour

The total number of marks available is 36.
You must write down all the stages in your working.

1. (a) Given (3)

$$y = \frac{1-3x}{x^2+4},$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.
Simplify your answer.

(b) Given (2)

$$\mathrm{f}(x) = \operatorname{cosec} 5x,$$

find $\mathrm{f}'(x)$.

2. Use Gaussian elimination to solve the following system of equations: (4)

$$\begin{aligned} x - 2y + z &= 4 \\ 2x + y - 3z &= 3 \\ x - 7y - 4z &= 9. \end{aligned}$$

3. Given that (3)

$$z_1 = 5 + 3\mathrm{i} \text{ and } z_2 = 6 + 2\mathrm{i},$$

express $z_1\bar{z}_2$ in the form $a + \mathrm{i}b$, where a and b are real numbers.

4. A curve is defined by the equation

$$y^3 + 4y = 2xy + 1.$$

(a) Use implicit differentiation to find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (3)

(b) Find the gradient of the tangent to the curve when $y = -1$. (1)

(c) Show that the curve has no stationary point. (2)

5. (a) Find, and simplify, the Maclaurin expansion for e^{-4x}, up to and including the term in x^3. (2)

(b) Hence find the first four terms of the Maclaurin expansion of (2)

$$\frac{3+2x}{\mathrm{e}^{4x}}.$$

6. (a) Consider the statement: (1)

For all odd numbers n, n^2+4 is prime.

Find a counterexample to show that the statement is false.

(b) Prove directly that the difference between the cubes of any two consecutive integers is not divisible by 3. (3)

7. (a) Use the substitution $u = y^2+1$, or otherwise, to find the exact value of (4)

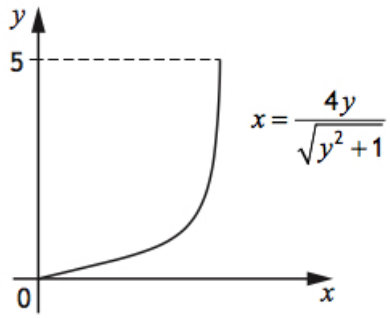
$$\int_0^5 \frac{4y}{\sqrt{y^2+1}}\,\mathrm{d}y.$$

Student engineers are using a 3D printer to make a model.

Relative to a suitable set of axes, the cross-section of the model is **symmetrical about the y-axis** and is represented **in the first quadrant** by the curve with equation

$$x = \frac{4y}{\sqrt{y^2+1}},\ 0 \leqslant y \leqslant 5,$$

as shown in the diagram.



(b) State the area of the cross-section. (1)

(c) Express (1)

$$\frac{y^2}{y^2+1}$$

in the form

$$a + \frac{b}{y^2 + 1},$$

where a and b are real numbers.

The curve

$$x = \frac{4y}{\sqrt{y^2 + 1}},\ 0 \leqslant y \leqslant 5,$$

will be rotated through 2π radians about the y-axis to make the model.

(d) Find the volume of the model. (4)