Dr Oliver Mathematics
GCSE Mathematics
2005 June Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Make t the subject of the formula (2)

$$v = u + 5t.$$

Solution

$$v = u + 5t \Rightarrow 5t = v - u$$
$$\Rightarrow t = \frac{v - u}{5}.$$

(b) Solve (3)

$$\frac{x - 3}{5} = x - 5.$$

Solution

$$\frac{x - 3}{5} = x - 5 \Rightarrow x - 3 = 5(x - 5)$$
$$\Rightarrow x - 3 = 5x - 25$$
$$\Rightarrow 22 = 4x$$
$$\Rightarrow x = 5\tfrac{1}{2}.$$

2. Three women earned a total of £36.
   They shared the £36 in the ratio $7 : 3 : 2$.
   Donna received the largest amount.

   (a) Work out the amount Donna received. (3)

Solution

$$7 + 3 + 2 = 12$$

and the amount Donna received is

$$\frac{7}{12} \times 36 = \underline{\underline{£21}}.$$

A year ago, Donna weighed 51.5 kg.
Donna now weighs $8\frac{1}{2}\%$ less.

(b) Work out how much Donna now weighs. (4)
Give your answer to an appropriate degree of accuracy.

Solution

$$100 - 8.5 = 91.5$$

and Donna now weighs

$$51.5 \times 0.915 = \underline{\underline{47.122\,5 \text{ kg}}}.$$

3. The equation (4)

$$x^3 - 4x = 24.$$

has a solution between 3 and 4.
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 - 4X$; then you press $\boxed{=}$.
Start? and you enter 3; then you press $\boxed{=}$.
End? and you enter 4; then you press $\boxed{=}$.
Step? and enter 0.05 – 1 decimal place divided by 2; then you press $\boxed{=}$.

x	f(x)	Comment
3.3	22.737	too low
3.35	24.195	too high

Clearly,
$$3.3 < x < 3.35$$
and the answer is
$$x = 3.3 \text{ (1 dp)}.$$

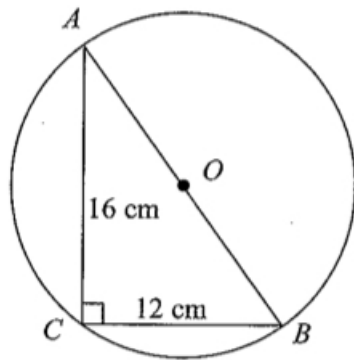4. The diagram shows triangle ABC and a circle, centre O.

Diagram **NOT**
accurately drawn

A, B, and C are points on the circumference of the circle.
AB is a diameter of the circle.
$AC = 16$ cm and $BC = 12$ cm.

(a) Angle $ACB = 90°$. (1)
    Give a reason why.

Solution

The angle in a semi-circle is a right angle.

(b) Work out the diameter AB of the circle. (3)

Solution

$$\text{Diameter} = \sqrt{16^2 + 12^2}$$
$$= 20 \text{ cm}.$$

(c) Work out the area of the circle. (3)
    Give your answer correct to 3 significant figures.

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Solution

The radius of the circle is 10 cm and

$$\text{area} = \pi \times 10^2$$
$$= 314.159\,265\,4 \text{ (FCD)}$$
$$= \underline{\underline{314 \text{ cm } (3 \text{ sf})}}.$$

5. The table shows information about the number of hours that 120 children used a computer last week.

Number of hours (h)	Frequency
$0 < h \leqslant 2$	10
$2 < h \leqslant 4$	15
$4 < h \leqslant 6$	30
$6 < h \leqslant 8$	35
$8 < h \leqslant 10$	25
$10 < h \leqslant 12$	5

(a) Work out an estimate for the mean number of hours that the children used a computer. (4)
Give your answer correct to two decimal places.

Solution

Number of hours (h)	Frequency	Midpoint	Freq×Midpoint
$0 < h \leqslant 2$	10	1	10
$2 < h \leqslant 4$	15	3	45
$4 < h \leqslant 6$	30	5	150
$6 < h \leqslant 8$	35	7	245
$8 < h \leqslant 10$	25	9	225
$10 < h \leqslant 12$	5	11	55
Total	120		730

An estimate for the mean number of hours that the children used a computer

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is approximately

$$\frac{\Sigma fh}{\Sigma f} \approx \frac{730}{120}$$
$$= 6.08\dot{3}$$
$$= \underline{\underline{6.08 \text{ hours (2 dp)}}}.$$
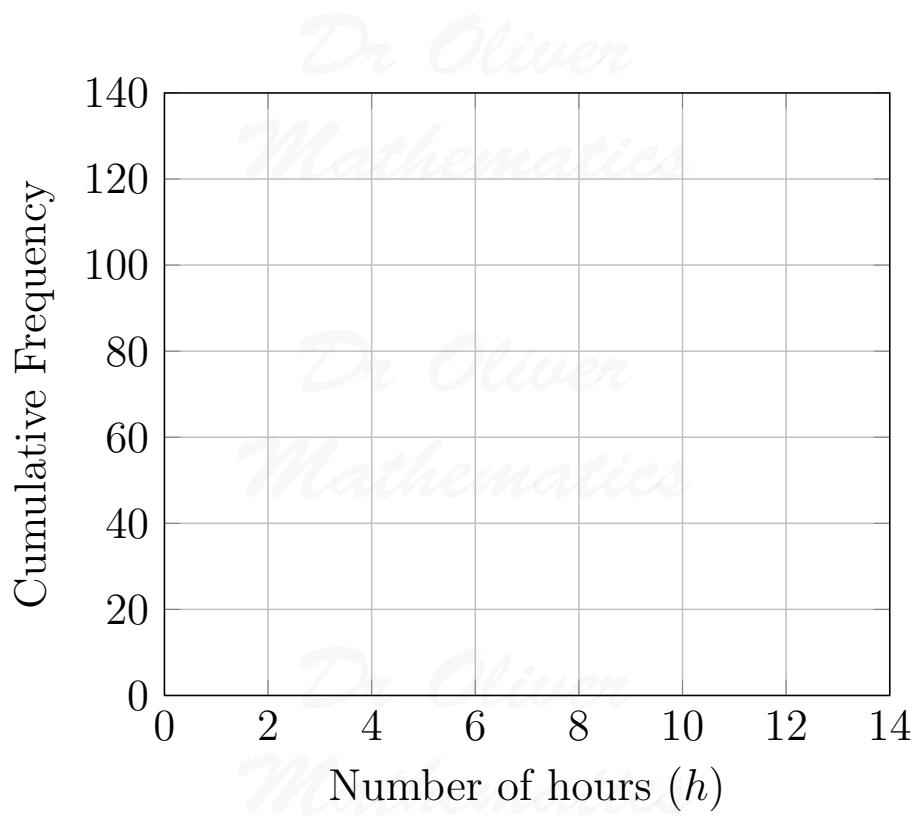
(b) Complete the cumulative frequency table. (1)

Number of hours (h)	Cumulative frequency
$0 < h \leqslant 2$	10
$2 < h \leqslant 4$	
$4 < h \leqslant 6$	
$6 < h \leqslant 8$	
$8 < h \leqslant 10$	
$10 < h \leqslant 12$	

Solution

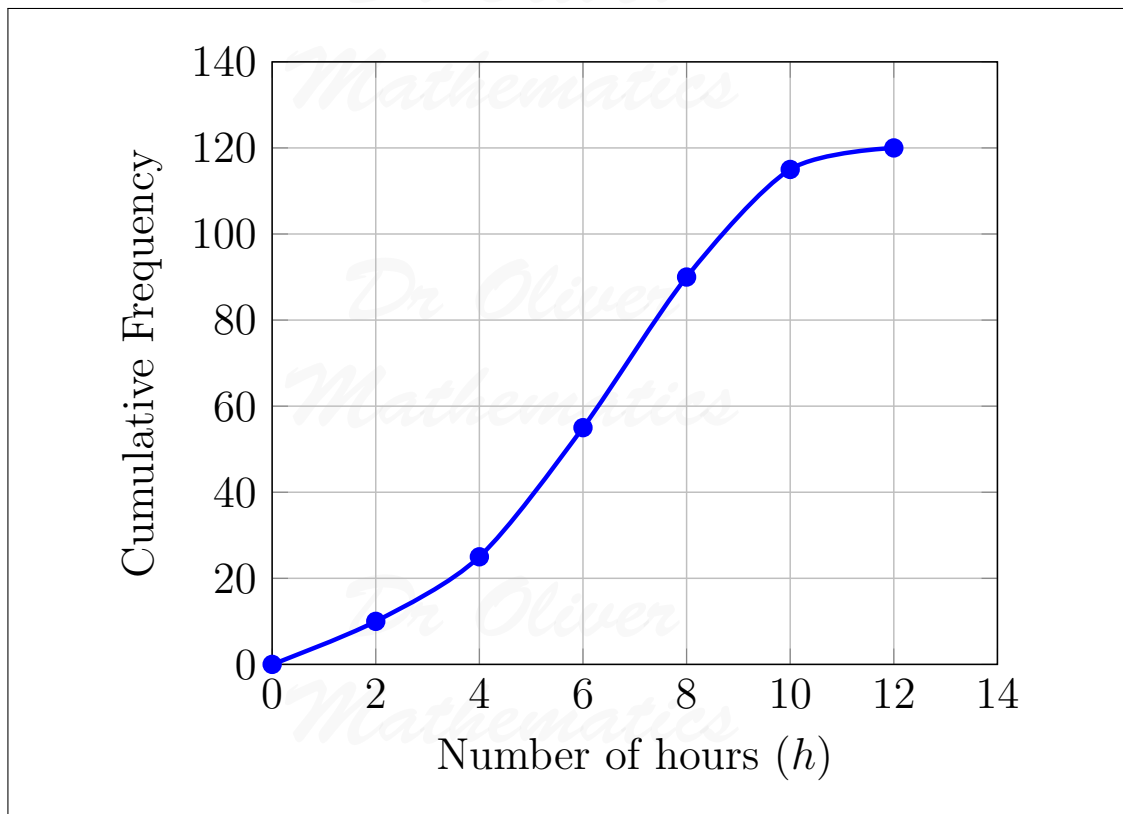| Number of hours (h)	Cumulative frequency
$0 < h \leqslant 2$	10
$2 < h \leqslant 4$	$10 + 15 = \underline{25}$
$4 < h \leqslant 6$	$25 + 30 = \underline{55}$
$6 < h \leqslant 8$	$55 + 35 = \underline{\underline{90}}$
$8 < h \leqslant 10$	$90 + 25 = \underline{115}$
$10 < h \leqslant 12$	$115 + 5 = \underline{120}$

(c) Draw a cumulative frequency graph for your table. (2)

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Solution

(d) Use your graph to find an estimate for the number of children who used a computer   (2)
for less than 7 hours last week
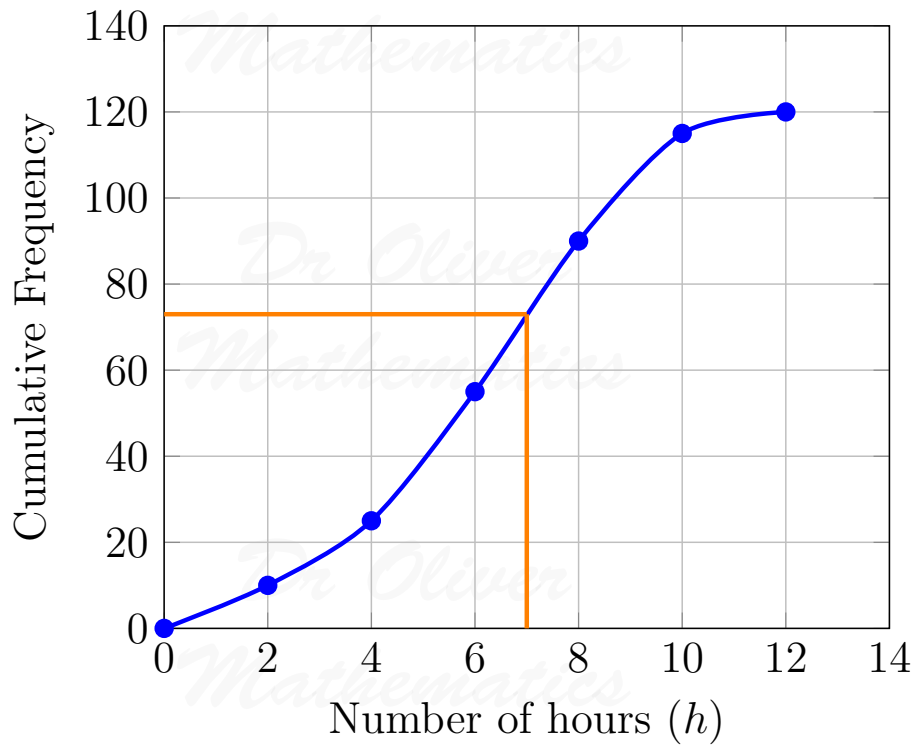
Solution

Correct read-off: approximately $\underline{\underline{73}}$ children.

6. (a) Simplify $a^3 \times a^4$. (1)

> **Solution**
> $$a^3 \times a^4 = \underline{\underline{a^7}}.$$

(b) Simplify $3x^2y \times 5xy^3$. (2)

> **Solution**
> $$3x^2y \times 5xy^3 = \underline{\underline{15x^3y^4}}.$$

(c) Simplify $\dfrac{(x-1)^2}{x-1}$. (1)

> **Solution**
> $$\frac{(x-1)^2}{x-1} = \underline{\underline{x-1}}.$$

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(d) Factorise $a^2 - 9b^2$. (2)

Solution

$$a^2 - 9b^2 = (a)^2 - (3b)^2$$
$$= \underline{\underline{(a + 3b)(a - 3b)}}.$$

7. In a sale, normal prices are reduced by 20%. (3)
   Andrew bought a saddle for his horse in the sale.
   The sale price of the saddle was £220.
   Calculate the normal price of the saddle.

Solution

$100 - 20 = 80$ and the normal price of the saddle is

$$\frac{220}{0.8} = \underline{\underline{£275}}.$$

8. Solve (3)

$$x + 2y = 4$$
$$3x - 4y = 7.$$

Solution

$$x + 2y = 4 \quad (1)$$
$$3x - 4y = 7 \quad (2)$$

Add $2 \times (1) + (2)$:

$$5x = 15 \Rightarrow \underline{\underline{x = 3}}$$
$$\Rightarrow 3 + 2y = 4$$
$$\Rightarrow 2y = 1$$
$$\Rightarrow \underline{\underline{y = \tfrac{1}{2}}}.$$

9. Work out (2)
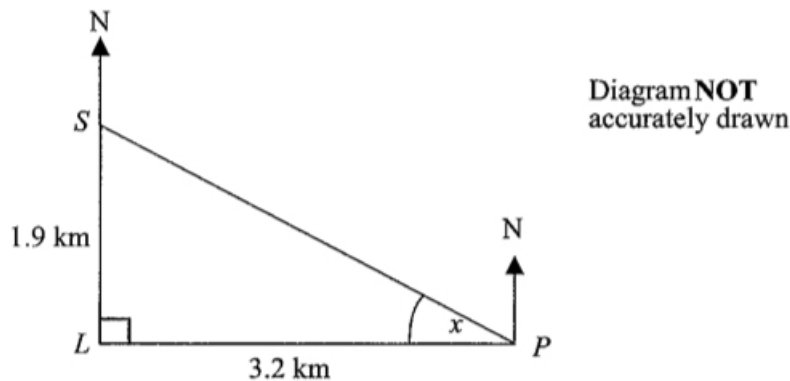$$(3.2 \times 10^5) \times (4.5 \times 10^4).$$

Give your answer in standard form correct to 2 significant figures.

Solution

$$(3.2 \times 10^5) \times (4.5 \times 10^4) = 1.44 \times 10^{10}$$
$$= \underline{\underline{1.4 \times 10^{10}}} \text{ (2 sf)}.$$

10. A lighthouse, L, is 3.2 km due West of a port, P.
A ship, S, is 1.9 km due North of the lighthouse, L.

Diagram **NOT** accurately drawn

(a) Calculate the size of the angle marked x. (3)
Give your answer correct to 3 significant figures.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan LPS = \frac{1.9}{3.2}$$
$$\Rightarrow \angle LPS = 30.699\,722\,55 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle LPS = 30.7^\circ}} \text{ (3 sf)}.$$

(b) Find the bearing of the port, P, from the ship, S. (1)
Give your answer correct to 3 significant figures.

Solution

$$90 + 30.699\ldots = \underline{\underline{121°}} \ \text{(3 sf)}.$$
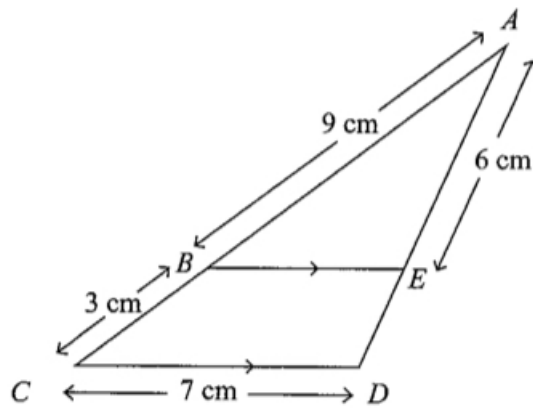
11. Here is a diagram.

Diagram **NOT** accurately drawn

BE is parallel to CD.
$AB = 9$ cm.
$BC = 3$ cm.
$CD = 7$ cm.
$AE = 6$ cm.

(a) Calculate the length of ED. (2)

Solution

$$\frac{ED + 6}{6} = \frac{12}{9} \Rightarrow ED + 6 = 8$$
$$\Rightarrow \underline{\underline{ED = 2 \ \text{cm}}}.$$

(b) Calculate the length of BE. (2)

Solution

$$\frac{BE}{7} = \frac{9}{12} \Rightarrow BE = 7 \times \tfrac{3}{4}$$
$$\Rightarrow \underline{\underline{BE = 5\tfrac{1}{4} \ \text{cm}}}.$$

12.

$$P = \pi r + 2r + 2a.$$

$P = 84.$
$r = 6.7.$

(a) Work out the value of a. (3)
Give your answer correct to 3 significant figures.

Solution

$$84 = 6.7\pi + 13.4 + 2a \Rightarrow 2a = 70.6 - 6.7\pi$$
$$\Rightarrow a = 24.775\,664\,61 \text{ (FCD)}$$
$$\Rightarrow a = 24.8 \text{ (3 sf)}.$$

(b) Make r the subject of the formula (3)

$$P = \pi r + 2r + 2a.$$

Solution

$$P = \pi r + 2r + 2a \Rightarrow r(\pi + 2) = P - 2a$$
$$\Rightarrow r = \frac{P - 2a}{\pi + 2}.$$

13. ABC is a triangle. (3)
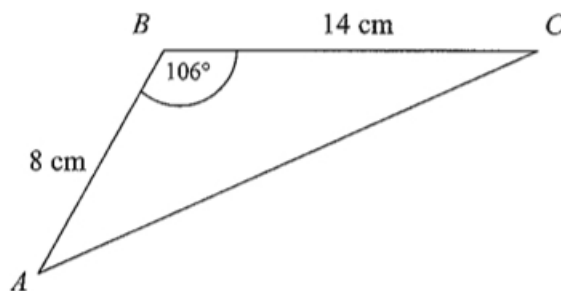
Diagram **NOT**
accurately drawn

$AB = 8$ cm.
$BC = 14$ cm.

Angle $ABC = 106°$.
Calculate the area of the triangle.
Give your answer correct to 3 significant figures.

Solution

$$\text{Area} = \tfrac{1}{2} \times 8 \times 14 \times \sin 106°$$
$$= 53.830\,654\,97 \text{ (FCD)}$$
$$= \underline{\underline{53.8 \text{ cm}^2 \text{ (3 sf)}}}.$$

14. Bill invests £500 on 1st January 2004 at a compound interest rate of $R\%$ per annum.
    The value, $£V$, of this investment after n years is given by the formula

$$V = 500 \times (1.045)^n.$$

   (a) Write down the value of R.   (1)

Solution
$\underline{\underline{4.5\%}}$.

   (b) Use your calculator to find the value of Bill's investment after 20 years.   (2)

Solution
The value of Bill's investment after 20 years is

$$V = 500 \times (1.045)^{20}$$
$$= 1\,205.857\,012 \text{ (FCD)}$$
$$= \underline{\underline{£1\,205.85 \text{ (2 dp, rounding down)}}}$$

15. The diagram below shows a 6-sided shape.
    All the corners are right angles.
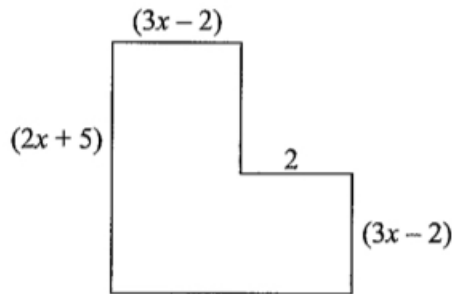   All measurements are given in centimetres.

The area of the shape is 25 cm².

(a) Show that (3)
$$6x^2 + 17x - 39 = 0.$$

Solution

$$3x(2x + 5) - 2(7 - x) = 25 \Rightarrow 6x^2 + 15x - 14 + 2x = 25$$
$$\Rightarrow \underline{\underline{6x^2 + 17x - 39 = 0}},$$

as required.

(b) (i) Solve the equation (4)
$$6x^2 + 17x - 39 = 0.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & +17 \\ \text{multiply to:} & (+6) \times (-39) = -234 \end{array}\right\} - 9, +26$$

$$6x^2 + 17x - 39 = 0 \Rightarrow 6x^2 - 9x + 26x - 39 = 0$$
$$\Rightarrow 3x(2x - 3) + 13(2x - 3) = 0$$
$$\Rightarrow (3x + 13)(2x - 3) = 0$$
$$\Rightarrow 3x + 13 = 0 \text{ or } 2x - 3 = 0$$
$$\Rightarrow \underline{\underline{x = -4\tfrac{1}{3} \text{ or } x = 1\tfrac{1}{2}}}.$$

(ii) Hence work out the length of the longest side of the shape.
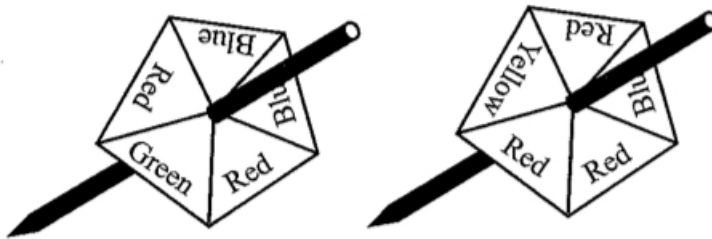
Solution

Is it
$$2 \times 1\frac{1}{2} + 5 = 8$$
and
$$5 \times 1\frac{1}{2} = 7\frac{1}{2}?$$

It must be <u>8 cm</u>.

16. Jeremy designs a game for a school fair.

He has two 5-sided spinners.
The spinners are equally likely to land on each of their sides.
One spinner has 2 red sides, 1 green side, and 2 blue sides.
The other spinner has 3 red sides, 1 yellow side and 1 blue side.

(a) Calculate the probability that the two spinners will land on the same colour. (3)

Solution

$$\mathrm{P}(RR) + \mathrm{P}(BB) = \left(\tfrac{2}{5} \times \tfrac{3}{5}\right) + \left(\tfrac{2}{5} \times \tfrac{1}{5}\right)$$
$$= \tfrac{6}{25} + \tfrac{2}{25}$$
$$= \underline{\underline{\tfrac{8}{25}}}.$$

The game consists of spinning each spinner once.
It costs 20p to play the game.
To win a prize both spinners must land on the same colour.
The prize for a win is 50p.
100 people play the game.

(b) Work out an estimate of the profit that Jeremy should expect to make. (2)

$$100(0.2 - 0.5 \times \tfrac{8}{25}) = 100 \times \tfrac{1}{25}$$
$$= \underline{\underline{£4}}.$$

17. Peter transports metal bars in his van. (4)
    The van has a safety notice "Maximum Load 1200 kg."
    Each metal bar has a label "Weight 60 kg."
    For safety reasons Peter assumes that 1200 is rounded correct to 2 significant figures and 60 is rounded correct to 1 significant figure.
    Calculate the greatest number of bars that Peter can safely put into the van if his assumptions are correct.

Solution

$$1\,150 \leqslant \text{maximum load} < 1\,250$$

and

$$55 \leqslant \text{weight} < 65.$$

Therefore, the greatest number of bars that Peter can safely put into the van if his assumptions are correct is

$$\frac{1\,150}{65} = 17\tfrac{9}{13};$$

hence, he can put <u>17 bars</u> into the van.

18. Simplify fully
    (a) $(3xy^2)^4$, (2)

    Solution

    $$(3xy^2)^4 = \underline{\underline{81x^4y^8}}.$$

    (b) $\dfrac{x^2 - 3x}{x^2 - 8x + 15}.$ (3)

    Solution

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$$\frac{x^2 - 3x}{x^2 - 8x + 15} = \frac{x(x-3)}{(x-3)(x-5)}$$
$$= \underline{\underline{\frac{x}{x-5}}}.$$

19. The diagram shows a pyramid.
The apex of the pyramid is V.
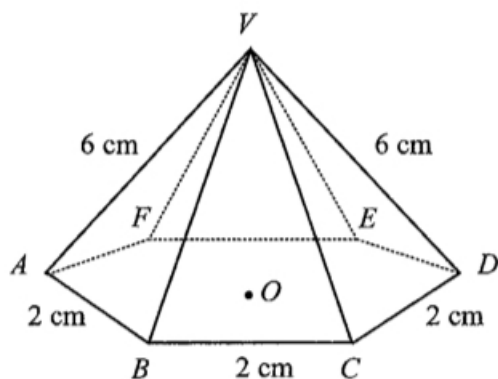Each of the sloping edges is of length 6 cm.

Diagram **NOT**
accurately drawn

The base of the pyramid is a regular hexagon with sides of length 2 cm.
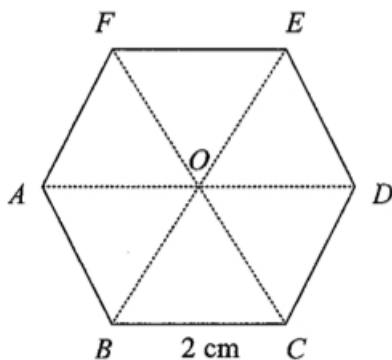O is the centre of the base.

Diagram **NOT**
accurately drawn

(a) Calculate the height of V above the base of the pyramid. (2)
Give your answer correct to 3 significant figures.

Solution

$\triangle OAV$ is a right-angled triangle and

$$OV = \sqrt{6^2 - 2^2}$$
$$= 5.656\,854\,249 \ \text{(FCD)}$$
$$= \underline{\underline{5.66 \text{ cm (3 sf)}}}.$$

(b) Calculate the size of angle DVA. (3)
Give your answer correct to 3 significant figures.

Solution

$$\angle DVA = 2 \times \angle DVO$$
$$= 2 \times \sin^{-1}\left(\frac{2}{6}\right)$$
$$= 38.942\,441\,27 \ \text{(FCD)}$$
$$= \underline{\underline{38.9° \text{ (3 sf)}}}.$$

(c) Calculate the size of angle AVC. (4)
Give your answer correct to 3 significant figures.

Solution
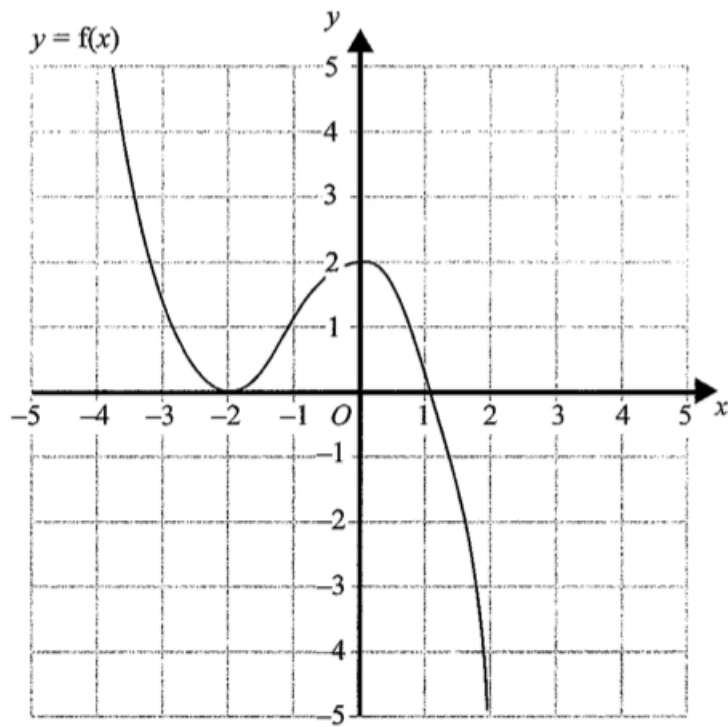$$AC = 2\sqrt{2^2 - 1^2} = 2\sqrt{3}$$
and
$$\cos AVC° = \frac{6^2 + 6^2 - (2\sqrt{3})^2}{2 \times 6 \times 6} \Rightarrow \cos AVC° = \tfrac{5}{6}$$
$$\Rightarrow \angle AVC° = 33.557\,309\,76 \ \text{(FCD)}$$
$$\Rightarrow \underline{\underline{\angle AVC° = 33.6° \text{ (3 sf)}}}.$$

20. The graph of $y = \text{f}(x)$ is shown on the grids.
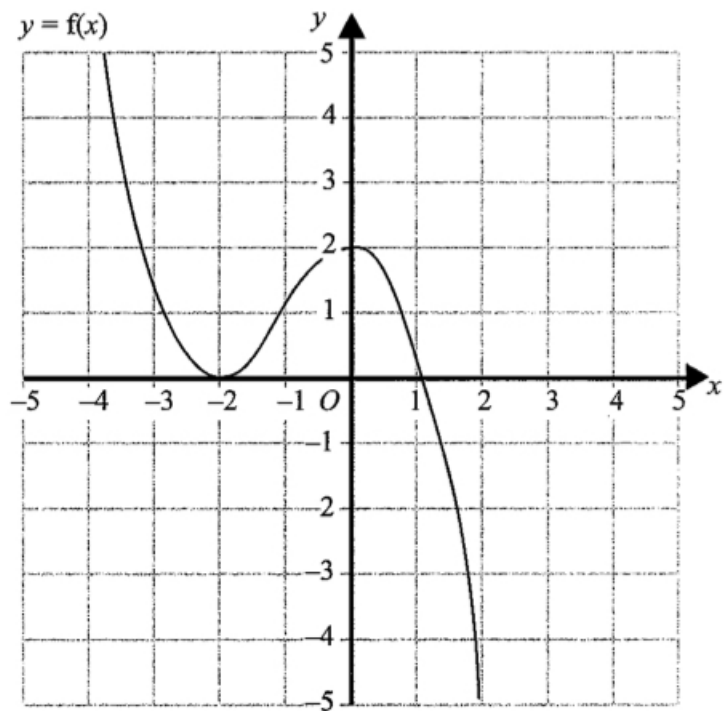   (a) Sketch the graph of $y = \text{f}(x - 1)$. (2)

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y = f(x)

Solution

Translate the graph by 1 in the x-direction: the new graph should go through, for example, $(-1, 0)$, $(1, 2)$, and slightly beyond $(2, 0)$.

(b) Sketch the graph of $y = 2\,f(x)$. (2)

$y = f(x)$

Solution

<u>Stretch by 2 in the y-direction</u>: the new graph should go through, for example, $(-2, 0)$, $(0, 4)$, and slightly beyond $(1, 0)$.