Dr Oliver Mathematics
GCSE Mathematics
2003 November Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. Use your calculator to work out the value of (3)

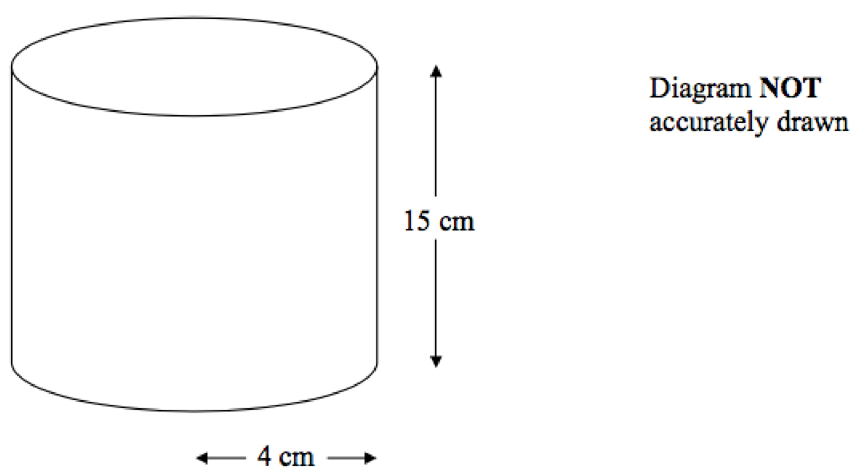
$$\frac{(7.91 - \sqrt[3]{81}) \times 4.32}{6.23 + 1.491}.$$

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\frac{(7.91 - \sqrt[3]{81}) \times 4.32}{6.23 + 1.491} &= 2.004\,875\,737 \text{ (FCD)} \\ &= \underline{\underline{2.00 \text{ (3 sf)}}}.\end{aligned}$$

2. A can of drink is in the shape of a cylinder. (3)
The can has a radius of 4 cm and a height of 15 cm.

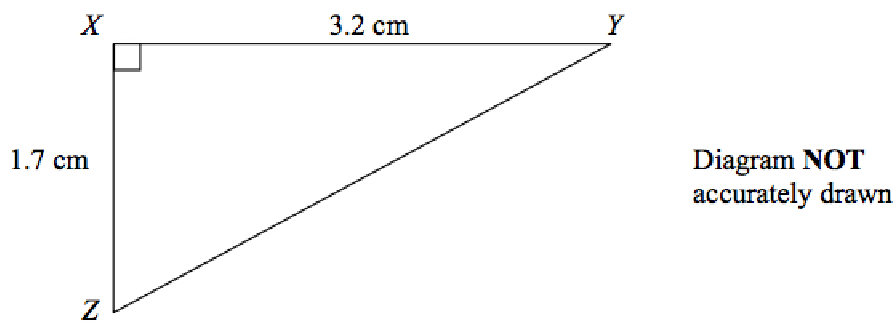


Calculate the volume of the cylinder.
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Volume} &= \pi \times 4^2 \times 15 \\ &= 753.982\,236\,9 \text{ (FCD)} \\ &= \underline{\underline{754 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

3. XYZ is a right-angled triangle. (3)



$XY = 3.2$ cm.
$XZ = 1.7$ cm.
Calculate the length of YZ.
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}YZ &= \sqrt{XY^2 + XZ^2} \\ &= \sqrt{3.2^2 + 1.7^2} \\ &= 3.623\,534\,186 \text{ (FCD)} \\ &= \underline{\underline{3.62 \text{ cm (3 sf)}}}.\end{aligned}$$

4. (a) Expand and simplify (2)

$$3(2x - 1) - 2(2x - 3).$$

Solution

$$\begin{aligned}3(2x-1)-2(2x-3) &= 6x-3-4x+6\\ &= \underline{\underline{2x+3}}.\end{aligned}$$

(b) Factorise (1)

$$y^2+y.$$

Solution

$$y^2+y=\underline{\underline{y(y+1)}}.$$

5. Charles found out the length of reign of each of 41 kings.
He used the information to complete the frequency table.

Length of reign (L years)	Number of kings
$0 < L \leqslant 10$	14
$10 < L \leqslant 20$	13
$20 < L \leqslant 30$	8
$30 < L \leqslant 40$	4
$40 < L \leqslant 50$	2

(a) Write down the class interval that contains the median. (2)

Solution

Length of reign (L years)	Number of kings	Cumulative Frequency
$0 < L \leqslant 10$	14	14
$10 < L \leqslant 20$	13	$14+13=27$
$20 < L \leqslant 30$	8	$27+8=35$
$30 < L \leqslant 40$	4	$35+4=39$
$40 < L \leqslant 50$	2	$39+2=41$

The median is in

$$\frac{41+1}{2}=21\text{st value}$$

and hence the class interval that contains the median is $\underline{\underline{10 < L \leqslant 20}}$.

(b) Calculate an estimate for the mean length of reign. (4)

Solution

Length of reign (L years)	Number of kings	Midpoint	Kings×Midpoint
$0 < L \leqslant 10$	14	5	$14 \times 5 = 70$
$10 < L \leqslant 20$	13	15	$13 \times 15 = 195$
$20 < L \leqslant 30$	8	25	$8 \times 25 = 200$
$30 < L \leqslant 40$	4	35	$4 \times 35 = 140$
$40 < L \leqslant 50$	2	45	$2 \times 45 = 90$
Total	41		695

Hence, an estimate for the mean length of reign is

$$\begin{aligned}\bar{x} &= \frac{\Sigma fx}{\Sigma f} \\ &\approx \frac{695}{41} \\ &= 16.\dot{9}512\dot{1} \\ &= \underline{\underline{17.0 \text{ years (3 sf)}}}.\end{aligned}$$

6. A floppy disk can store 1 440 000 bytes of data.

(a) Write the number 1 440 000 in standard form. (1)

Solution

$$1\,440\,000 = \underline{\underline{1.44 \times 10^6}}.$$

A hard disk can store 2.4×10^9 bytes of data.

(b) Calculate the number of floppy disks needed to store the 2.4×10^9 bytes of data. (3)

Solution

$$\frac{2.4 \times 10^9}{1.44 \times 10^6} = 1\,666\tfrac{2}{3}$$
$$= \underline{\underline{1\,667 \text{ floppy disks}}}.$$

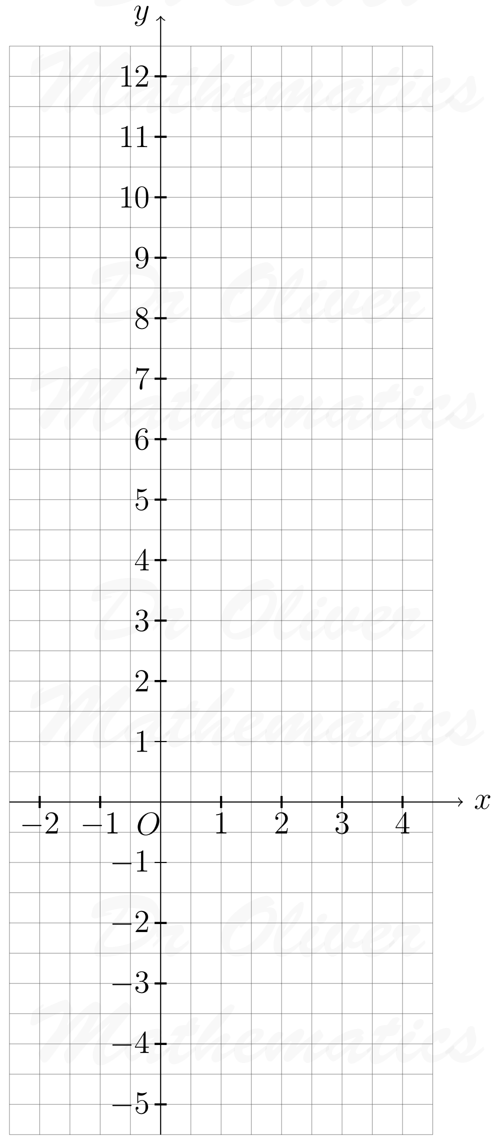
7. (a) Complete the table for $y = x^2 - 3x + 1$. (2)

x	-2	-1	0	1	2	3	4
y	11		1	-1		1	5

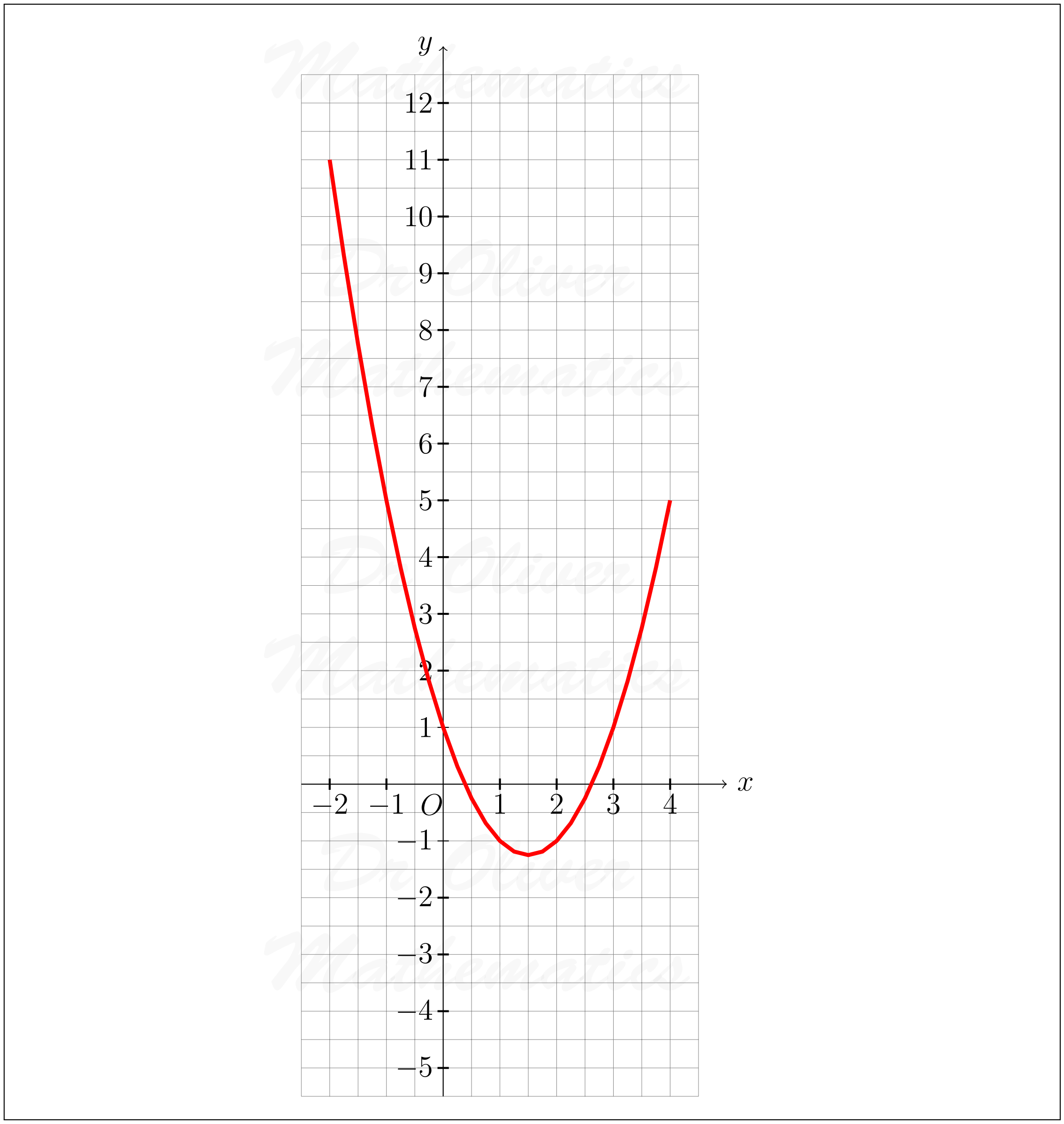
Solution

x	-2	-1	0	1	2	3	4
y	11	$\underline{\underline{5}}$	1	-1	$\underline{\underline{-1}}$	1	5

(b) On the grid, draw the graph of $y = x^2 - 3x + 1$. (2)

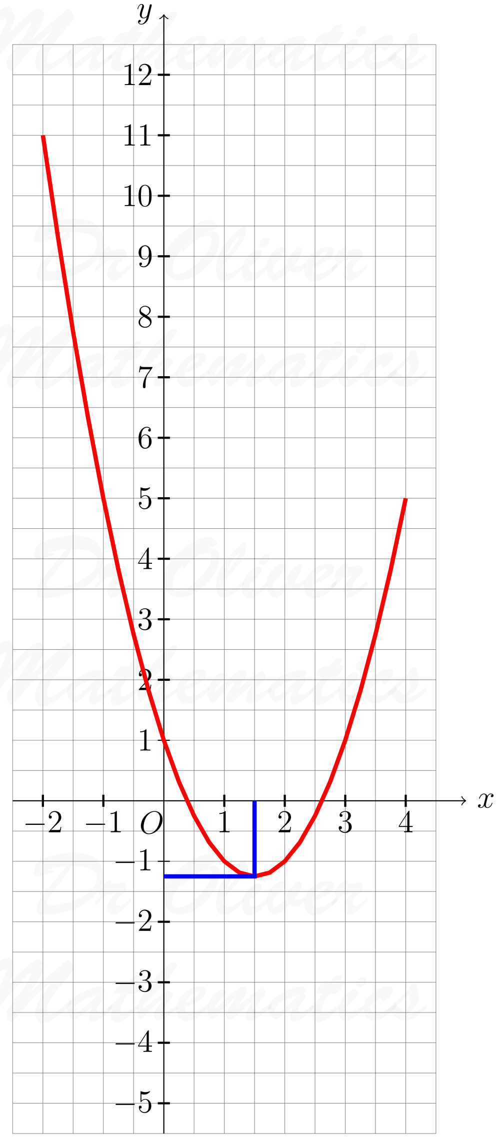


Solution



(c) Use your graph to find an estimate for the minimum value of y. (1)

Solution

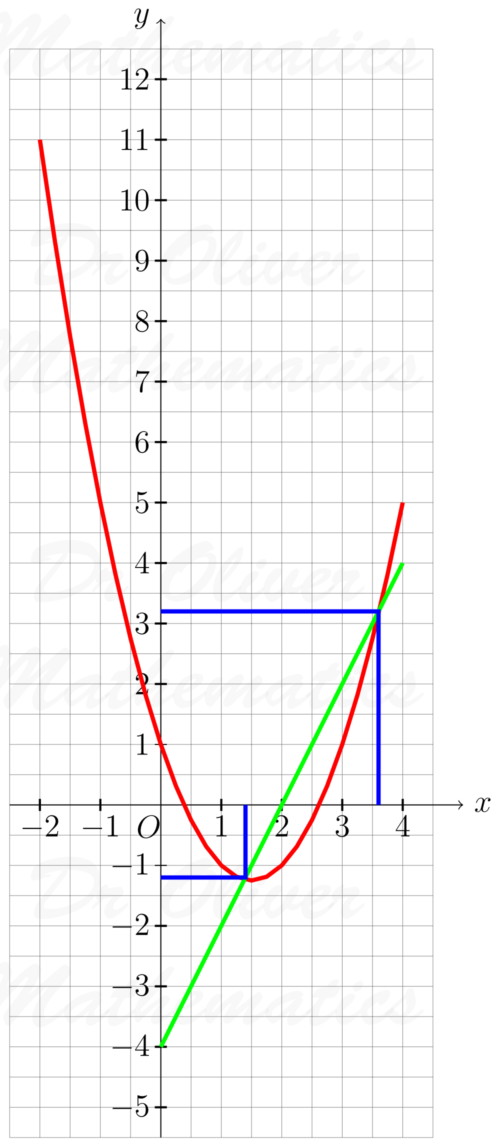


Correct read-off: approximately -1.25.

(d) Use a graphical method to find estimates of the solutions to the equation (3)

$$x^2 - 3x + 1 = 2x - 4.$$

Solution



Correct read-off: approximately 1.4 and 3.6.

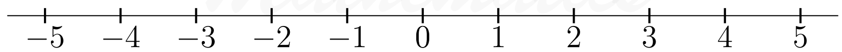
8. (a) (i) Solve the inequality (3)

$$5x - 7 < 2x - 1.$$

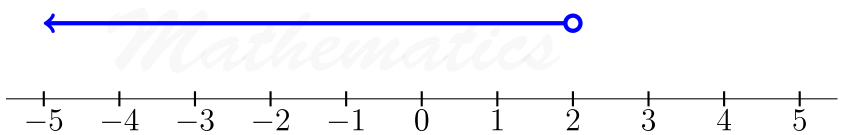
Solution

$$5x - 7 < 2x - 1 \Rightarrow 3x < 6$$
$$\Rightarrow \underline{\underline{x < 2}}.$$

(ii) On the number line, represent the solution set to part (i).



Solution



n is an integer such that $-4 \leqslant 2n < 3$.

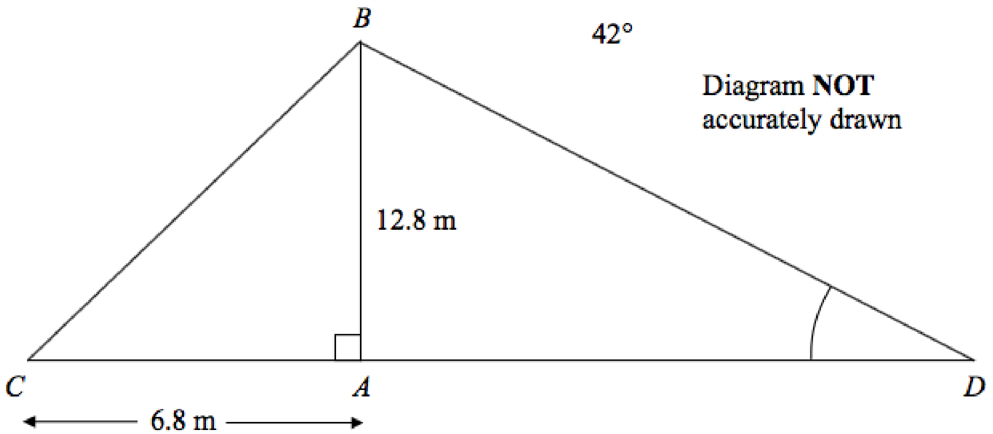
(b) Write down the possible values of n. (3)

Solution

$$-4 \leqslant 2n < 3 \Rightarrow -2 \leqslant n < 1\tfrac{1}{2}$$

so n is $\underline{\underline{-2, -1, 0, 1}}$.

9. The diagram represents a vertical flagpole, AB.
The flagpole is supported by two ropes, BC and BD, fixed to the horizontal ground at C and D.



$AB = 12.8$ m.
$AC = 6.8$ m.
Angle $BDA = 42°$.

(a) Calculate the size of angle BCA. (3)
Give your answer correct to 3 significant figures.

Solution

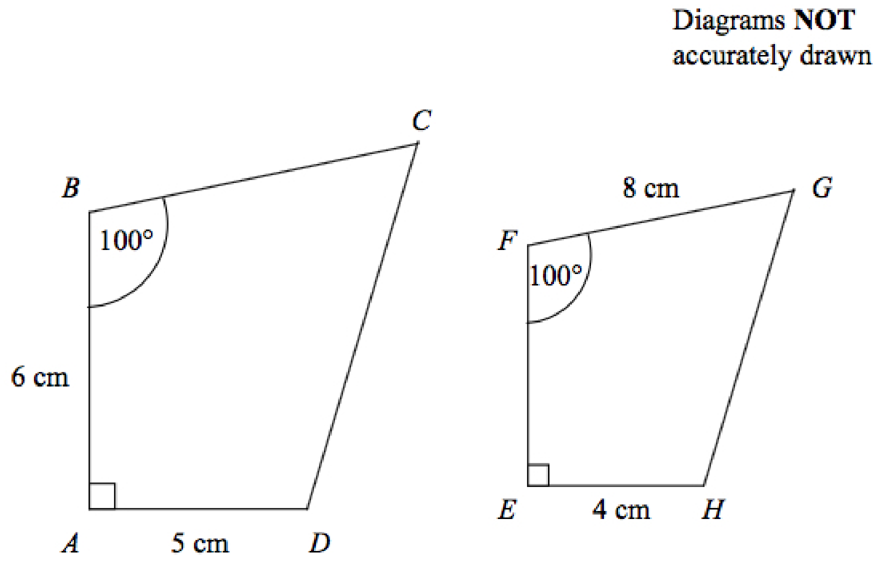
$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan BCA = \frac{12.8}{6.8}$$
$$\Rightarrow BCA = 62.020\,525\,61 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{BCA = 62.0° \text{ (3 sf)}}}.$$

(b) Calculate the length of the rope BD. (3)
Give your answer correct to 3 significant figures.

Solution

$$\text{hyp} = \frac{\text{opp}}{\sin} \Rightarrow BD = \frac{12.8}{\sin 42°}$$
$$\Rightarrow BD = 19.129\,299\,84 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{BD = 19.1 \text{ m (3 sf)}}}.$$

10. Shapes $ABCD$ and $EFGH$ are mathematically similar. (5)



(i) Calculate the length of BC.

Solution

$$\frac{BC}{FG} = \frac{AD}{EH} \Rightarrow BC = \frac{5 \times 8}{4}$$
$$\Rightarrow \underline{\underline{BC = 10 \text{ cm}}}.$$

(ii) Calculate the length of EF.

Solution

$$\frac{AB}{AD} = \frac{EF}{EH} \Rightarrow EF = \frac{4 \times 6}{5}$$
$$\Rightarrow \underline{\underline{BC = 4.8 \text{ cm}}}.$$

11. Solve the simultaneous equations (4)

$$2x + 3y = -3$$
$$3x - 2y = 28.$$

Solution

$$2x + 3y = -3 \quad (1)$$
$$3x - 2y = 28 \quad (2)$$

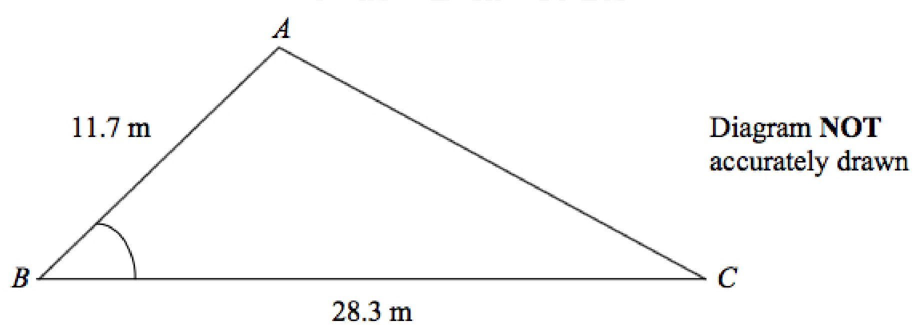
E.g.,

$$2 \times (1): \quad 4x + 6y = -6 \quad (3)$$
$$3 \times (2): \quad 9x - 6y = 84 \quad (4)$$

Add (3) + (4):

$$13x = 76 \Rightarrow x = 6$$
$$\Rightarrow \underline{\underline{x = 6}}$$
$$\Rightarrow 12 + 3y = -3$$
$$\Rightarrow 3y = -15$$
$$\Rightarrow \underline{\underline{y = -5}}.$$

12. Here is a triangle ABC.



$AB = 11.7$ m.
$BC = 28.3$ m.
Angle $ABC = 67°$.

(a) Calculate the area of the triangle ABC. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 11.7 \times 28.3 \times \sin 67^\circ \\ &= 152.394\,181 \text{ (FCD)} \\ &= \underline{\underline{152 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

(b) Calculate the length of AC. (3)
Give your answer correct to 3 significant figures.

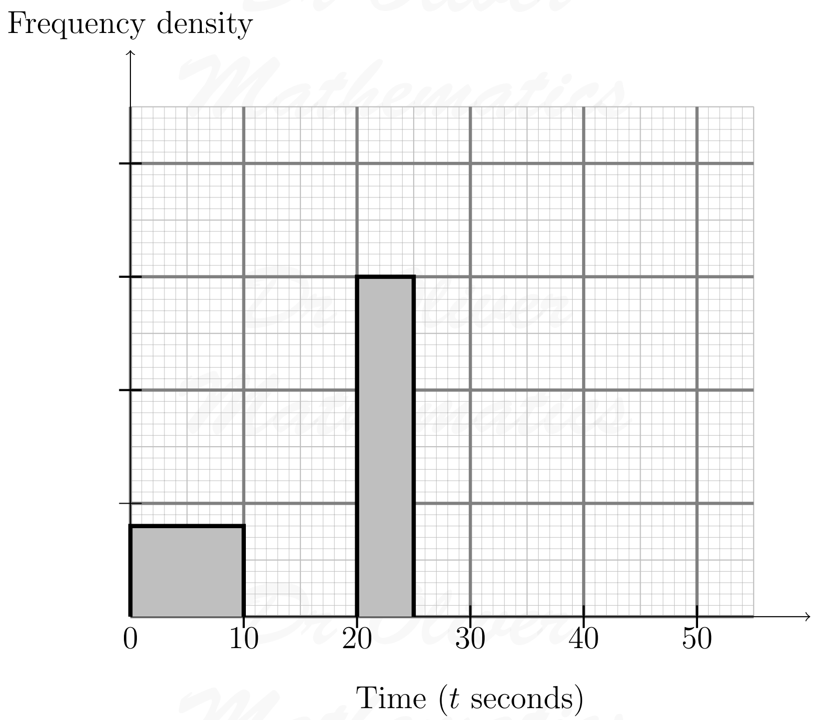
Solution

$$\begin{aligned}c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c^2 &= 11.7^2 + 28.3^2 - 2 \times 11.7 \times 28.3 \times \cos 67^\circ \\ \Rightarrow c &= 26.058\,204\,7 \text{ (FCD)} \\ &= \underline{\underline{26.1 \text{ cm (3 sf)}}}.\end{aligned}$$

13. One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table.
She used this information to complete the frequency table.

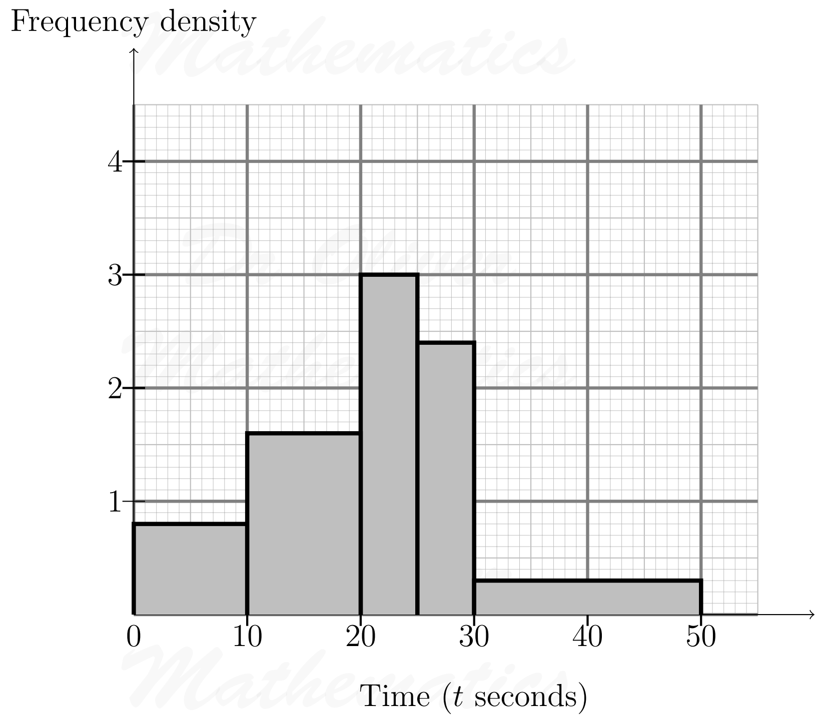
Time (t seconds)	Frequency
$0 < t \leqslant 10$	8
$10 < t \leqslant 20$	16
$20 < t \leqslant 25$	15
$25 < t \leqslant 30$	12
$30 < t \leqslant 50$	6

(a) Use the table to complete the histogram. (3)

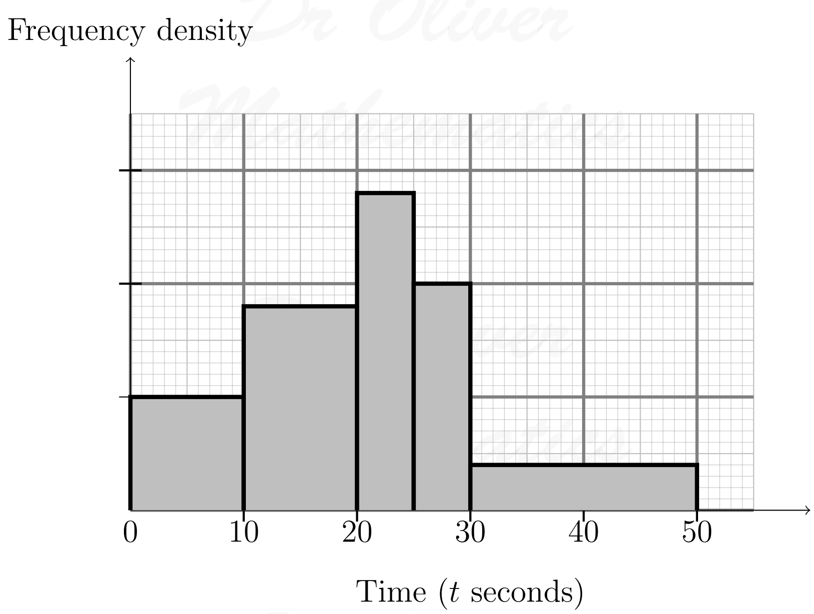


Solution

Time (t seconds)	Frequency	Width	Frequency Density
$0 < t \leqslant 10$	8	10	$\frac{8}{10} = 0.8$
$10 < t \leqslant 20$	16	10	$\frac{16}{10} = 1.6$
$20 < t \leqslant 25$	15	5	$\frac{15}{5} = 3$
$25 < t \leqslant 30$	12	5	$\frac{12}{5} = 2.4$
$30 < t \leqslant 50$	6	20	$\frac{6}{20} = 0.3$



On Tuesday she conducted a similar survey and drew the following histogram from her results.



(b) Use the histogram for Tuesday to complete the table. (2)

Time (t seconds)	Frequency
$0 < t \leqslant 10$	10
$10 < t \leqslant 20$	
$20 < t \leqslant 25$	
$25 < t \leqslant 30$	
$30 < t \leqslant 50$	

Solution

Time (t seconds)	Frequency	Width	Frequency Density
$0 < t \leqslant 10$	10	10	$\frac{10}{10} = 1$
$10 < t \leqslant 20$	$\underline{\underline{18}}$	10	$\frac{18}{10} = 1.8$
$20 < t \leqslant 25$	$\underline{\underline{14}}$	5	$\frac{2.8}{5} = 14$
$25 < t \leqslant 30$	$\underline{\underline{10}}$	5	$\frac{2}{5} = 10$
$30 < t \leqslant 50$	$\underline{\underline{8}}$	20	$\frac{8}{20} = 0.4$

14. Prove that, (3)

$$(n+1)^2 - (n-1)^2$$

is a multiple of 4, for all positive integer values of n.

Solution

$$\begin{aligned}(n+1)^2 - (n-1)^2 &= (n^2 + 2n + 1) - (n^2 - 2n + 1) \\ &= \underline{\underline{4n}},\end{aligned}$$

and it is a multiple of 4 because we have taken all $n \in \mathbb{N}$.

15. Two numbers have a difference of 15 and a product of 199.75.
The larger of the two numbers is x.

(a) Show that (3)

$$x^2 - 15x - 199.75 = 0.$$

Solution

Let the larger of the two numbers is x and the smaller of the two numbers is y. Then

$$x - y = 15$$

and

$$xy = 199.75.$$

Now,

$$\begin{aligned} y = \frac{199.75}{x} &\Rightarrow x - \frac{199.75}{x} = 15 \\ &\Rightarrow x^2 - 199.75 = 15x \\ &\Rightarrow \underline{\underline{x^2 - 15x - 199.75 = 0}}, \end{aligned}$$

as required.

(b) Solve the equation (3)

$$x^2 - 15x - 199.75 = 0.$$

Solution

EITHER

$$x^2 - 15x - 199.75 = 0 \Rightarrow 4x^2 - 60x - 799 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & -60 \\ \text{multiply to:} & -799 \end{array}\right\} + 34,\ -94$$

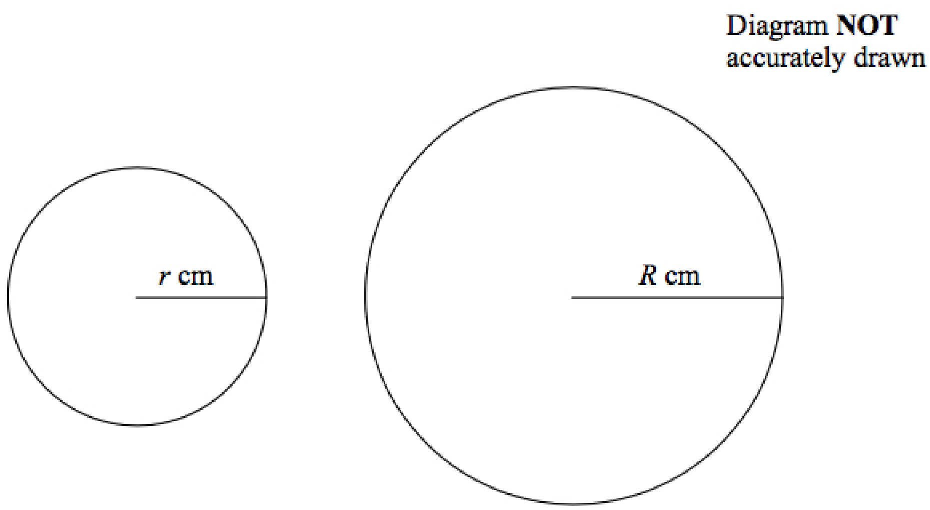
$$\begin{aligned} &\Rightarrow 4x^2 + 34x - 94x - 799 = 0 \\ &\Rightarrow 2x(2x + 17) - 47(2x + 17) = 0 \\ &\Rightarrow (2x - 47)(2x + 17) = 0 \\ &\Rightarrow 2x - 47 = 0 \text{ or } 2x + 17 = 0 \\ &\Rightarrow 2x = 47 \text{ or } 2x = -17 \\ &\Rightarrow \underline{\underline{x = 23\tfrac{1}{2} \text{ or } x = -8\tfrac{1}{2}}} \end{aligned}$$

OR

$a = 1$, $b = -15$, and $c = -199.75$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{15 \pm \sqrt{(-15)^2 - 4 \times 1 \times (-199.75)}}{2} \\ &= \frac{15 \pm \sqrt{1024}}{2} \\ &= \frac{15 \pm 32}{2} \\ &= \underline{\underline{-8\tfrac{1}{2} \text{ or } 23\tfrac{1}{2}}}. \end{aligned}$$

16. The diagram represents two metal spheres of different sizes.
The radius of the smaller sphere is r cm.
The radius of the larger sphere is R cm.



$r = 1.7$ correct to 1 decimal place.
$R = 31.0$ correct to 3 significant figures.

(a) Write down the upper and lower bounds of r and R. (2)

Solution

The lower bound or r is $\underline{\underline{1.65}}$.

The upper bound or r is $\underline{\underline{1.75}}$.

The lower bound or R is $\underline{\underline{30.95}}$.

The upper bound or R is $\underline{\underline{31.05}}$.

(b) Find the smallest possible value of $R - r$. (1)

Solution

$$R - r = 30.95 - 1.75 = \underline{\underline{29.2}}.$$

The larger sphere of radius R cm was melted down and used to make smaller spheres of radius r cm.

(c) Calculate the smallest possible number of spheres that could be made. (4)

Solution

The upper bound on the number of sphere of radius R cm is

$$\tfrac{4}{3}\pi \times 30.95^3$$

whereas the lower bound on the number of sphere of radius r cm is

$$\tfrac{4}{3}\pi \times 1.75^3.$$

Divide:

$$\frac{\frac{4}{3}\pi \times 30.95^3}{\frac{4}{3}\pi \times 1.75^3} = 5\,531.817\ldots;$$

hence, $\underline{\underline{5\,531 \text{ spheres}}}$ could be made.

17. (a) Factorise (2)

$$9x^2 - 6x + 1.$$

Solution

$$9x^2 - 6x + 1 = \underline{\underline{(3x-1)^2}}.$$

(b) Simplify (3)

$$\frac{6x^2 + 7x - 3}{9x^2 - 6x + 1}.$$

Solution

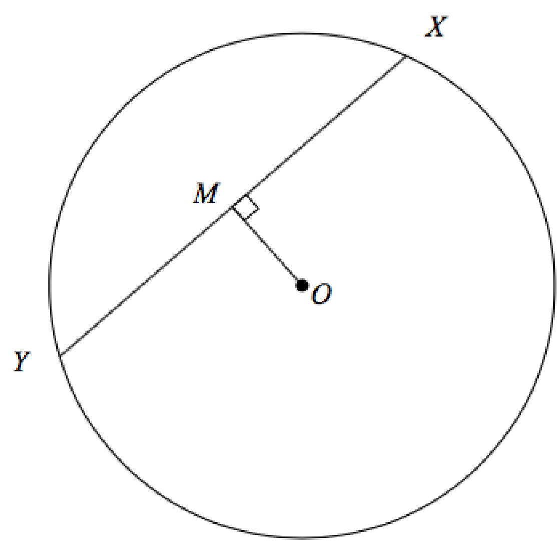
$$\left.\begin{array}{ll}\text{add to:} & +7 \\ \text{multiply to:} & 6 \times (-3) = -18\end{array}\right\} -2,\ +9$$

$$\begin{aligned} 6x^2 + 7x - 3 &= 6x^2 - 2x + 9x - 3 \\ &= 2x(3x - 1) + 3(3x - 1) \\ &= (2x + 3)(3x - 1) \end{aligned}$$

and

$$\begin{aligned} \frac{6x^2 + 7x - 3}{9x^2 - 6x + 1} &= \frac{(2x + 3)(3x - 1)}{(3x - 1)^2} \\ &= \frac{2x + 3}{3x - 1}. \end{aligned}$$

18. X and Y are points on the circle, centre O. (3)



M is the point where the perpendicular from O meets the chord XY.
Prove that M is the midpoint of the chord XY.

Solution

$OX = OY$ (radii).
OM is common.

$\angle OMX = \angle OMY (= 90^\circ)$.
Hence, $\triangle OMX = \triangle OMY$ are congruent (RHS) and $\underline{\underline{M \text{ is the midpoint of the chord } XY}}$.

19. Joan has two boxes of chocolates. (4)
The boxes are labelled **A** and **B**.
Box **A** contains 15 chocolates. There are 6 plain, 4 milk, and 5 white chocolates.
Box **B** contains 12 chocolates. There are 4 plain, 3 milk, and 5 white chocolates.
Joan takes one chocolate at random from each box.
Work out the probability that the two chocolates Joan takes are not of the same type.

Solution

$$\begin{aligned}
\text{P(both the same)} &= \text{P}(PP) + \text{P}(MM) + \text{P}(CC) \\
&= \left(\tfrac{6}{15} \times \tfrac{4}{12}\right) + \left(\tfrac{4}{15} \times \tfrac{3}{12}\right) + \left(\tfrac{5}{15} \times \tfrac{5}{12}\right) \\
&= \tfrac{24}{180} + \tfrac{12}{180} + \tfrac{25}{180} \\
&= \tfrac{61}{180}
\end{aligned}$$

and the two chocolates Joan takes are not of the same type is

$$1 - \tfrac{61}{180} = \underline{\underline{\tfrac{119}{180}}}.$$

20. Solve the simultaneous equations (7)

$$\begin{aligned}
x^2 + y^2 &= 29 \\
y - x &= 3.
\end{aligned}$$

Solution

$$
\begin{aligned}
y = x - 3 \Rightarrow x^2 + (x+3)^2 &= 29 \\
\Rightarrow x^2 + (x^2 + 6x + 9) &= 29 \\
\Rightarrow 2x^2 + 6x - 20 &= 0 \\
\Rightarrow x^2 + 3x - 10 &= 0 \\
\Rightarrow (x+5)(x-2) &= 0 \\
\Rightarrow \underline{\underline{x = -5}} &\text{ or } \underline{\underline{x = 2}} \\
\Rightarrow \underline{\underline{y = -2}} &\text{ or } \underline{\underline{y = 5}}.
\end{aligned}
$$

21. The depth, D metres, of the water at the end of a jetty in the afternoon can be modelled by this formula (4)

$$D = 5.5 + A \sin 30(t - k)^\circ$$

where t hours is the number of hours after midday and A and k are constants.
Yesterday the low tide was at 3 p.m.
The depth of water at low tide was 3.5 m.
Find the value of A and k.

Solution

$$3.5 = 5.5 + A \sin 30(3 - k)^\circ \Rightarrow A \sin 30(3 - k)^\circ = -2.$$

So, e.g., $\underline{\underline{A = 2}}$ and

$$
\begin{aligned}
\sin(90 - 30k)^\circ = -1 &\Rightarrow 90 - 30k = -90 \\
&\Rightarrow 30k = 180 \\
&\Rightarrow \underline{\underline{k = 6}}.
\end{aligned}
$$

There are various other solutions . . .