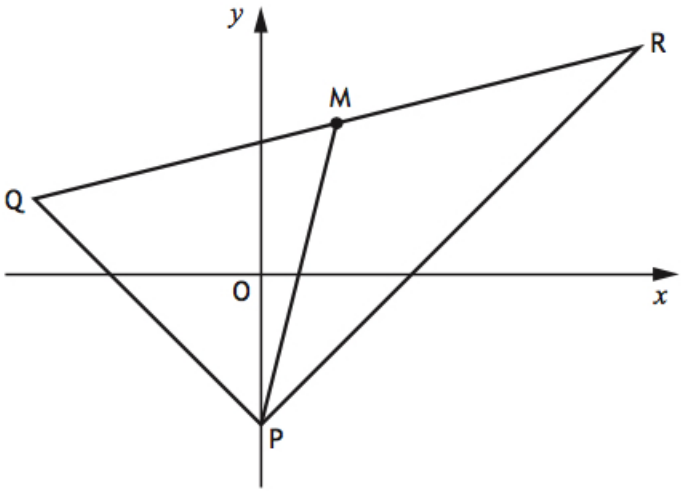
Dr Oliver Mathematics
Mathematics: Higher
2016 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. PQR is a triangle with vertices $P(0, -4)$, $Q(-6, 2)$, and $R(10, 6)$.



   (a) (i) State the coordinates of M, the midpoint of QR. (1)
   (ii) Hence find the equation of PM, the median through P. (2)
   (b) Find the equation of the line, L, passing through M and perpendicular to PR. (3)
   (c) Show that line L passes through the midpoint of PR. (3)

2. Find the range of values for p such that (3)

$$x^2 - 2x + 3 - p = 0$$

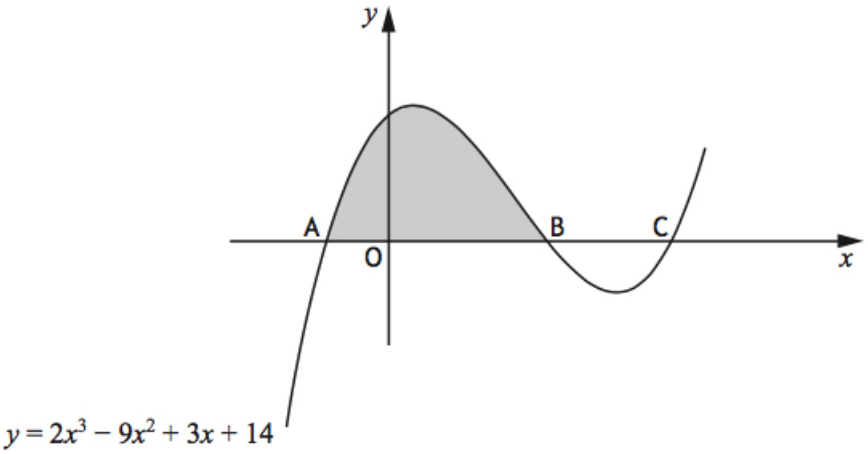
has no real roots.

3. (a) (i) Show that $(x + 1)$ is a factor of (2)

$$2x^3 - 9x^2 + 3x + 14.$$

   (ii) Hence solve the equation (3)

$$2x^3 - 9x^2 + 3x + 14.$$



(b) (i) Write down the coordinates of the points A and B. (1)

(ii) Hence calculate the shaded area in the diagram. (4)

4. Circles C_1 and C_2 have equations

$$(x + 5)^2 + (y - 6)^2 = 9$$

and

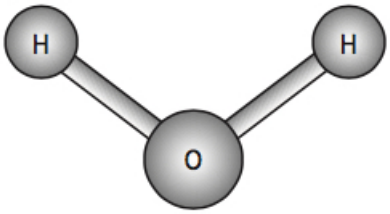
$$x^2 + y^2 - 6x - 16 = 0$$

respectively.

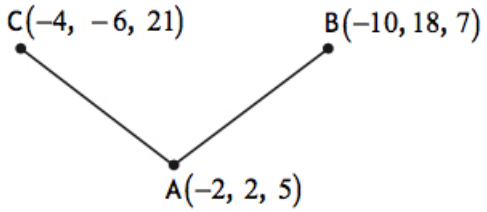
(a) Write down the centres and radii of C_1 and C_2. (4)

(b) Show that C_1 and C_2 do not intersect. (3)

5. The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point $A(-2, 2, 5)$. The two hydrogen atoms are positioned at points $B(-10, 18, 7)$ and $C(-4, -6, 21)$ as shown in the diagram below.



(a) (a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. (2)

(b) Hence, or otherwise, find the size of angle BAC. (4)

6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200\mathrm{e}^{0.107t},$$

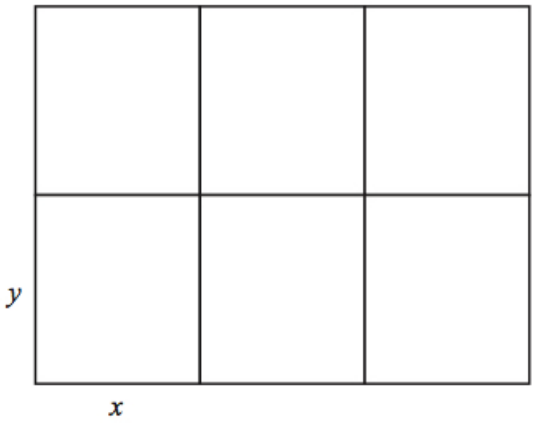
where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study. (1)

(b) Calculate the time taken for the number of bacteria to double. (4)

7. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



The area of land being set aside is 108 m^2.

(a) Show that the total length of fencing, L metres, is given by (3)

$$L(x) = 9x + \frac{144}{x}.$$

(b) Find the value of x that minimises the length of fencing required. (6)

8. (a) Express (4)

$$5\cos x - 2\sin x$$

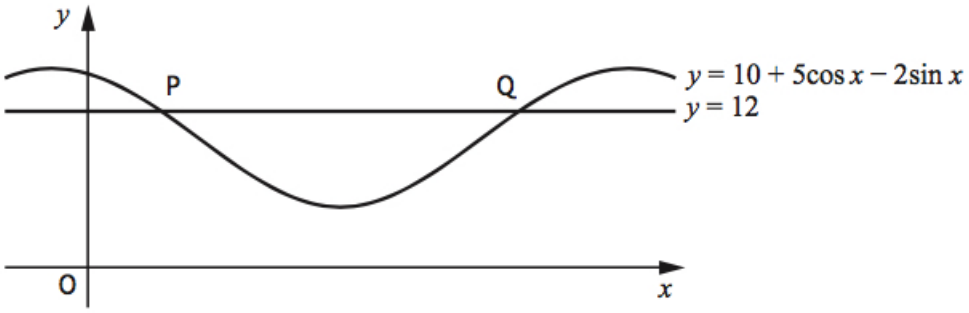
in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$.

The diagram shows a sketch of part of the graph of

$$y = 10 + 5\cos x - 2\sin x$$

and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



(b) Find the x-coordinates of P and Q. (4)

9. For a function f, defined on a suitable domain, it is known that: (4)

- $\mathrm{f}'(x) = \dfrac{2x + 1}{\sqrt{x}}$ and
- $\mathrm{f}(9) = 40$.

Express $\mathrm{f}(x)$ in terms of x.

10. (a) Given that (2)

$$y = (x^2 + 7)^{\frac{1}{2}},$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Hence find (1)

$$\int \frac{4x}{\sqrt{x^2 + 7}}\,\mathrm{d}x.$$

11. (a) Show that (4)

$$\sin 2x \tan x \equiv 1 - \cos 2x,$$

where $\frac{1}{2}\pi < x < \frac{3}{2}\pi$.

(b) Given that (2)

$$f(x) = \sin 2x \tan x,$$

find $f'(x)$.