Dr Oliver Mathematics
Worked Examples
Irrational Expressions 1

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Pearson Edexcel GCSE Mathematics: Grade 9+

1. (a) Write

$$\sqrt{24}$$

as the product

$$2\sqrt{p}\sqrt{q},$$

where p and q are both primes numbers, with $p < q$.

Solution
Now,

$$\begin{aligned}\sqrt{24} &= \sqrt{4 \times 2 \times 3} \\ &= \sqrt{4} \times \sqrt{2} \times \sqrt{3} \\ &= \underline{\underline{2\sqrt{2}\sqrt{3}}}.\end{aligned}$$

(b) Expand

$$(a + b + c)^2.$$

Solution
Well,

$\times$	a	$+b$	$+c$
a	a^2	$+ab$	$+ac$
$+b$	$+ab$	$+b^2$	$+bc$
$+c$	$+ac$	$+bc$	$+c^2$

so

$$(a + b + c)^2 = \underline{\underline{a^2 + b^2 + c^2 + 2ab + 2ac + 2bc}}.$$

(c) Writing

$$10 = 2 + 3 + 5,$$

or otherwise, simplify

$$\sqrt{10 + \sqrt{24} + \sqrt{40} + \sqrt{60}}.$$

Solution

Well,

$$\begin{aligned}\sqrt{40} &= \sqrt{4 \times 2 \times 5} \\ &= \sqrt{4} \times \sqrt{2} \times \sqrt{5} \\ &= 2\sqrt{2}\sqrt{5}\end{aligned}$$

and

$$\begin{aligned}\sqrt{60} &= \sqrt{4 \times 3 \times 5} \\ &= \sqrt{4} \times \sqrt{3} \times \sqrt{5} \\ &= 2\sqrt{3}\sqrt{5}.\end{aligned}$$

Now,

$$\begin{aligned} & \sqrt{10 + \sqrt{24} + \sqrt{40} + \sqrt{60}} \\ = \quad & \sqrt{2 + 3 + 5 + 2\sqrt{2}\sqrt{3} + 2\sqrt{2}\sqrt{5} + 2\sqrt{3}\sqrt{5}}\end{aligned}$$

let $a = \sqrt{2}$, $b = \sqrt{3}$, and $c = \sqrt{5}$.

Then $a^2 = 2$, $b^2 = 3$, and $c^2 = 5$:

$$\begin{aligned} &= \sqrt{a^2 + b^2 + c^2 + 2ab + 2ac + 2bc} \\ &= \sqrt{(a + b + c)^2} \\ &= a + b + c\end{aligned}$$

as $a + b + c > 0$

$$= \underline{\underline{\sqrt{2} + \sqrt{3} + \sqrt{5}}}.$$