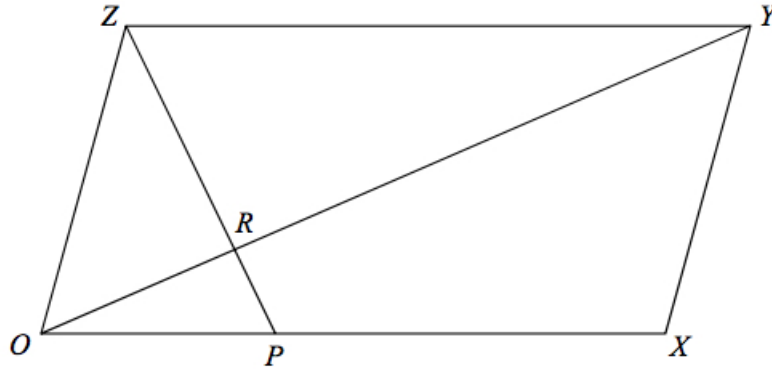


Dr Oliver Mathematics
Worked Examples
Vectors 1

From: Edexcel 2019 November Paper 3H (Calculator)

1. $OXYZ$ is a parallelogram.

(5)



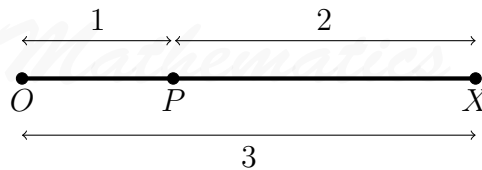
$$\begin{aligned}\overrightarrow{OX} &= \mathbf{a}. \\ \overrightarrow{OY} &= \mathbf{b}.\end{aligned}$$

P is the point on OX such that $OP : PX = 1 : 2$.
 R is the point on OY such that $OR : RY = 1 : 3$.

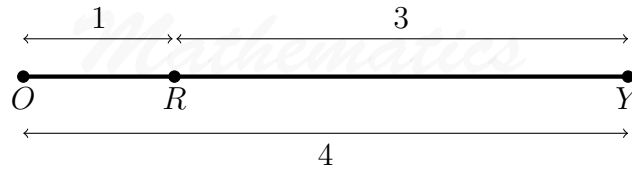
Work out, in its simplest form, the ratio $ZP : ZR$.
You must show all your working.

Solution

Well,



and



So, $\overrightarrow{OP} = \frac{1}{3}\mathbf{a}$ and $\overrightarrow{OR} = \frac{1}{4}\mathbf{b}$. Now,

$$\begin{aligned}
 \overrightarrow{ZP} &= \overrightarrow{ZO} + \overrightarrow{OP} \\
 &= \overrightarrow{YX} + \overrightarrow{OP} \\
 &= (\overrightarrow{YO} + \overrightarrow{OX}) + \overrightarrow{OP} \\
 &= (-\overrightarrow{OY} + \overrightarrow{OX}) + \overrightarrow{OP} \\
 &= (-\mathbf{b} + \mathbf{a}) + \frac{1}{3}\mathbf{a} \\
 &= \frac{4}{3}\mathbf{a} - \mathbf{b} \\
 &= 4\left(\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right)
 \end{aligned}$$

and

$$\begin{aligned}
 \overrightarrow{ZR} &= \overrightarrow{ZY} + \overrightarrow{YR} \\
 &= \overrightarrow{OX} + \frac{3}{4}\overrightarrow{YO} \\
 &= \overrightarrow{OX} + \frac{3}{4}(-\overrightarrow{OY}) \\
 &= \mathbf{a} + \frac{3}{4}(-\mathbf{b}) \\
 &= \mathbf{a} - \frac{3}{4}\mathbf{b} \\
 &= 3\left(\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right).
 \end{aligned}$$

Hence,

$$\begin{aligned}
 ZP : ZR &= |\overrightarrow{ZP}| : |\overrightarrow{ZR}| \\
 &= \left|4\left(\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right)\right| : \left|3\left(\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right)\right| \\
 &= 4\left|\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right| : 3\left|\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right| \\
 &= 4\left|\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right| : 3\left|\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right| \\
 &= \underline{\underline{4 : 3}}.
 \end{aligned}$$