Dr Oliver Mathematics
AQA GCSE Mathematics
2019 June Paper 3: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Work out £1.50 as a fraction of 60 p. (1)
   Circle your answer.

$$\tfrac{2}{5} \quad \tfrac{1}{4} \quad \tfrac{4}{1} \quad \tfrac{5}{2}$$

Solution

Well,

$$\frac{£1.50}{60 \text{ p}} = \frac{150}{60} = \tfrac{5}{2}$$

so

$$\tfrac{2}{5} \quad \tfrac{1}{4} \quad \tfrac{4}{1} \quad \underline{\underline{\tfrac{5}{2}}}$$

2. For a biased dice, (1)

$$\text{P}(6) = \tfrac{3}{5}.$$

Circle the probability of two sixes when the dice is rolled twice.

$$\tfrac{6}{25} \quad \tfrac{6}{10} \quad \tfrac{9}{25} \quad \tfrac{9}{5}$$

Solution

$$\text{P}(6, 6) = \tfrac{3}{5} \times \tfrac{3}{5} = \tfrac{9}{25}$$

so

$$\tfrac{6}{25} \quad \tfrac{6}{10} \quad \underline{\underline{\tfrac{9}{25}}} \quad \tfrac{9}{5}$$

3. Circle the lowest common multiple (LCM) of 5, 15, and 25 (1)

$$5 \quad 45 \quad 75 \quad 150$$

Solution

Well,

$$15 = 3 \times 5$$

and

$$25 = 5^2.$$

Now, the lowest common multiple (LCM) is

$$3 \times 5^2 = 75$$

so

$$5 \quad 45 \quad \underline{\underline{75}} \quad 150$$

4. Circle the two roots of (1)

$$(x-5)(x+3) = 0$$

$$-5 \quad -3 \quad 3 \quad 5$$

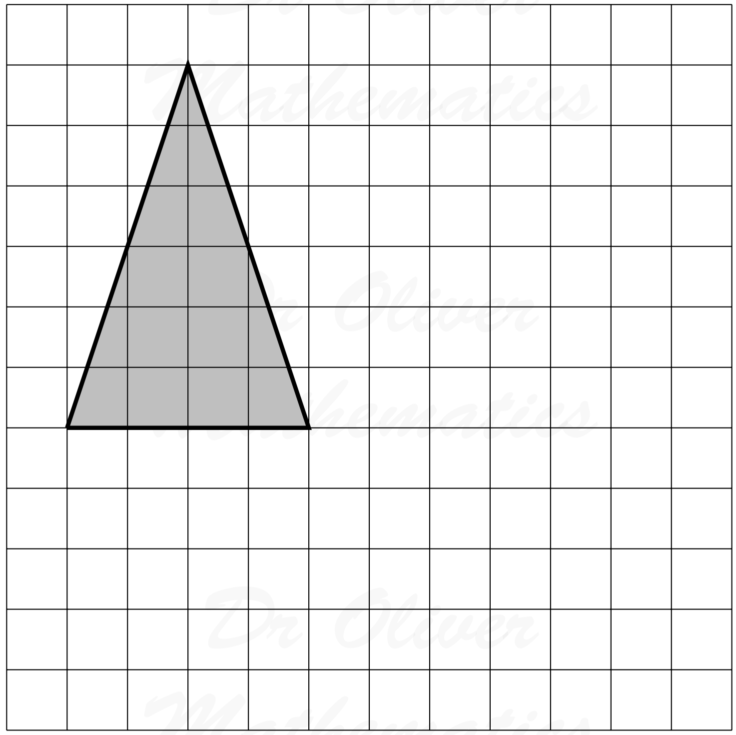
Solution

$$\begin{aligned}(x-5)(x+3) = 0 &\Rightarrow x-5 = 0 \text{ or } x+3 = 0\\ &\Rightarrow x = 5 \text{ or } x = -3\end{aligned}$$

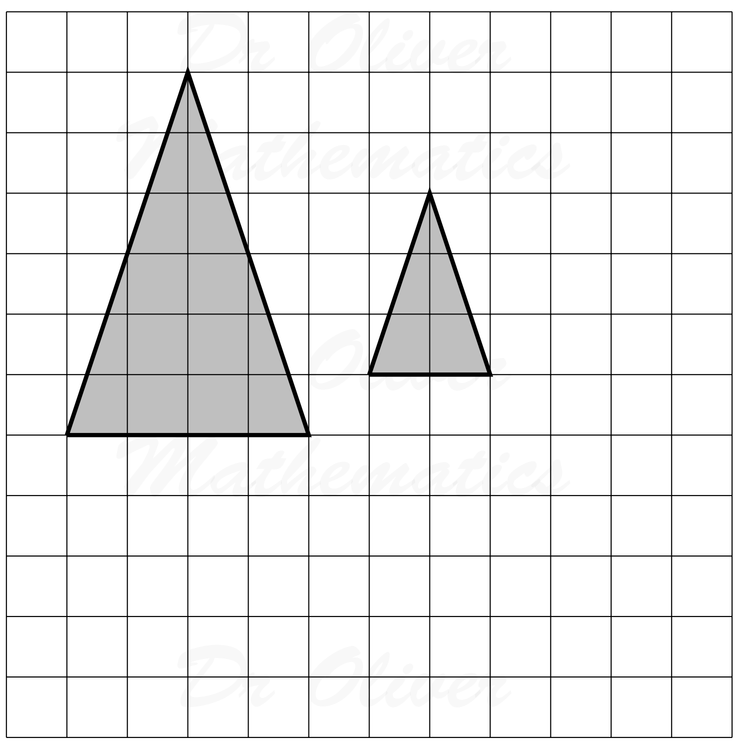
so

$$-5 \quad \underline{\underline{-3}} \quad 3 \quad \underline{\underline{5}}$$

5. On the grid, draw an enlargement of the triangle with scale factor $\frac{1}{2}$. (2)



Solution



6. To the nearest pound, Jon has £9. (3)

To the nearest 50 p, Ellie has £6.50.

Work out the maximum possible total amount of money.

Solution

Well,

$$\pounds 8.50 \leqslant \text{Jon} \leqslant \pounds 9.49$$

and

$$\pounds 6.25 \leqslant \text{Ellie} \leqslant \pounds 7.24.$$

Finally, the maximum possible total amount of money is

$$9.49 + 7.24 = \underline{\underline{\pounds 16.73}}.$$

7. Two solids, J and K, have the same density. (3)

	J	K
Mass	48 g	78 g
Volume	8 cm^3	
Density		

Complete the table.
Include units in your answers.

Solution

Well, for J,

$$\begin{aligned} \text{density} &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{48}{8} \\ &= 6. \end{aligned}$$

For K,

$$\begin{aligned} \text{volume} &= \frac{\text{mass}}{\text{density}} \\ &= \frac{78}{6} \\ &= 13. \end{aligned}$$

So,

	J	K
Mass	48 g	78 g
Volume	8 cm^3	$\underline{\underline{13\text{ cm}^3}}$
Density	$\underline{\underline{6\text{ g/cm}^3}}$	$\underline{\underline{6\text{ g/cm}^3}}$

8. Rearrange (1)

$$y = 3x - 2$$

to make x the subject.
Circle your answer.

$$x = \tfrac{1}{3}y - 2 \quad x = \frac{y+2}{3} \quad x = \frac{y-2}{3} \quad x = \tfrac{1}{3}y + 2$$

Solution

Well,

$$\begin{aligned} y = 3x - 2 &\Rightarrow y + 2 = 3x \\ &\Rightarrow \frac{y+2}{3} = x \end{aligned}$$

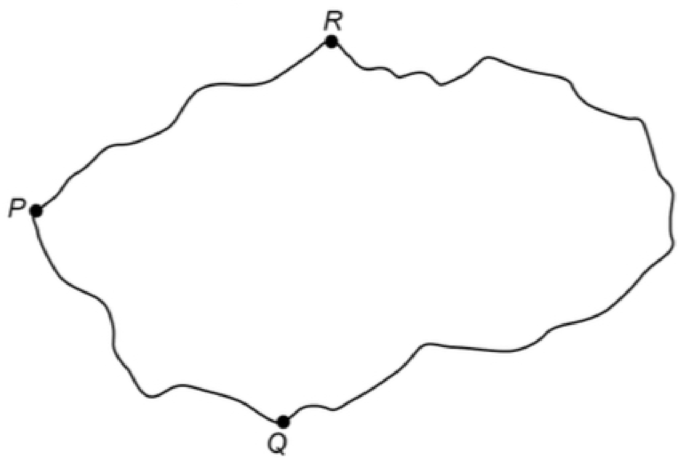
so

$$x = \tfrac{1}{3}y - 2 \quad \underline{\underline{x = \frac{y+2}{3}}} \quad x = \frac{y-2}{3} \quad x = \tfrac{1}{3}y + 2$$

9. Towns P, Q, and R are connected by roads PQ, PR, and QR. (4)

- PR is 10 km longer than PQ.

- QR is twice as long as PR.
- The total length of the three roads is 170 km.



Not drawn accurately

Work out the length of PQ.

Solution

Now,

$$PR = PQ + 10 \quad (1)$$
$$QR = 2PR \quad (2)$$
$$PQ + PR + QR = 170 \quad (3).$$

Next,

$$\begin{aligned} PQ + PR + QR = 170 &\Rightarrow PQ + (PQ + 10) + 2PR = 170 \\ &\Rightarrow 2PQ + 2PR = 160 \\ &\Rightarrow 2PQ + 2(PQ + 10) = 160 \\ &\Rightarrow 4PQ + 20 = 160 \\ &\Rightarrow 4PQ = 140 \\ &\Rightarrow \underline{\underline{PQ = 35 \text{ km}}}. \end{aligned}$$

10. Mia wants to borrow £6 000 and repay it, with interest, after two years. (3)

She sees two offers for loans.

Offer 1	Offer 2
Compound interest	Compound interest
3% per year	First year 1%
	Second year 5%

Mia says, "I will pay back the same amount because the average of 1% and 5% is 3%."

Is she correct?
You **must** show your working.

Solution

Offer 1:

$$6\,000 \times 1.03^2 = 6\,365.40.$$

Offer 2:

$$6\,000 \times 1.01 \times 1.05 = 6\,363.$$

No, she is wrong: Offer 2 is the best one.

11. Here are two sets of numbers, A and B. (4)

Set A

200	160
104	100

Set B

270	400	483
300	x	

$$\text{Mean of Set } A : \text{mean of Set } B = 3 : 8.$$

Work out the value of x.

Solution

Well,

$$\begin{aligned}\text{mean of Set } A &= \frac{200 + 160 + 104 + 100}{4} \\ &= 141\end{aligned}$$

and

$$\begin{aligned}\text{mean of Set } B &= \frac{270 + 400 + 483 + 300 + x}{5} \\ &= \frac{1\,453 + x}{5}.\end{aligned}$$

Now,

$$\begin{aligned} & \text{mean of Set } A : \text{mean of Set } B = 3 : 8 \\ \Rightarrow\quad & 141 : \frac{1\,453 + x}{5} = 3 : 8 \\ \Rightarrow\quad & \frac{1\,453 + x}{5 \times 141} = \frac{8}{3} \\ \Rightarrow\quad & 1\,453 + x = 1\,880 \\ \Rightarrow\quad & \underline{\underline{x = 427}}. \end{aligned}$$

12. A straight line has gradient 4 and passes through the point $(5, 23)$. (3)

Work out the equation of the line.
Give your answer in the form $y = mx + c$.

Solution

Well, the equation of the line is

$$y = 4x + c,$$

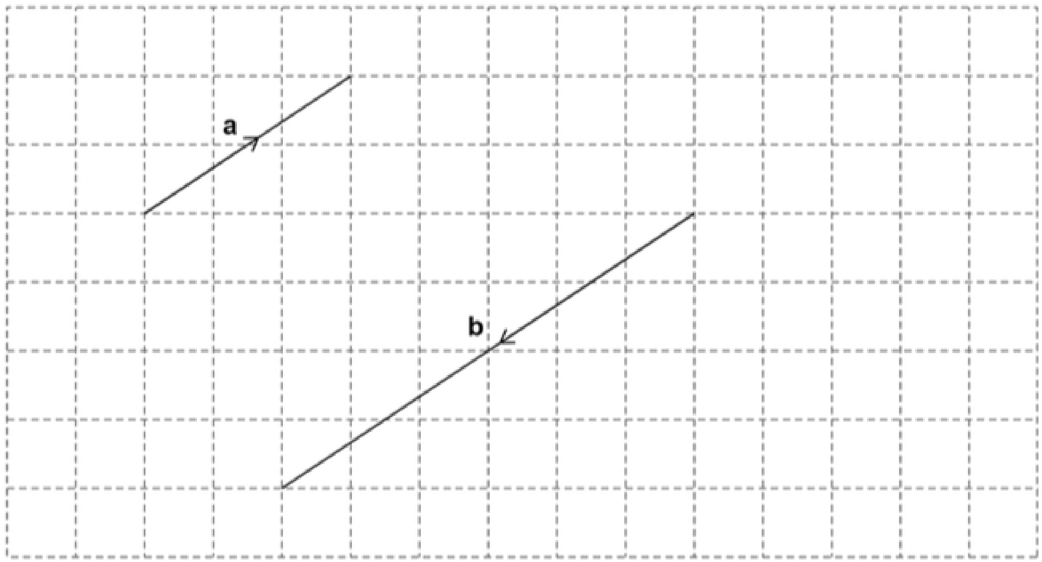
where c is some constant. Now,

$$\begin{aligned} x = 5, y = 23 \Rightarrow 23 &= 4(5) + c \\ \Rightarrow 23 &= 20 + c \\ \Rightarrow c &= 3; \end{aligned}$$

hence,

$$\underline{\underline{y = 4x + 3}}.$$

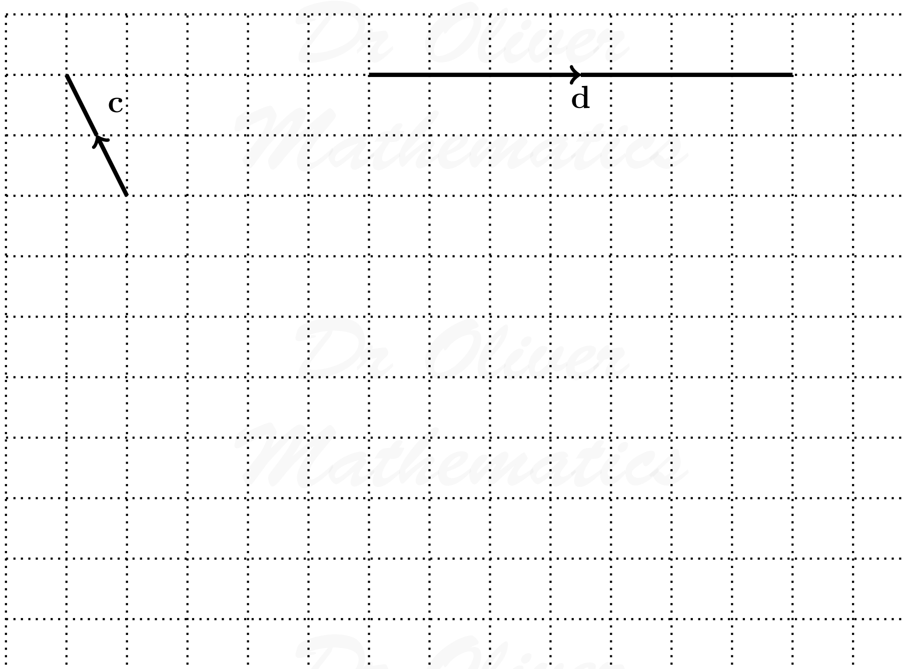
13. (a) Vectors **a** and **b** are drawn on a grid. (1)



Write **b** in terms of **a**.

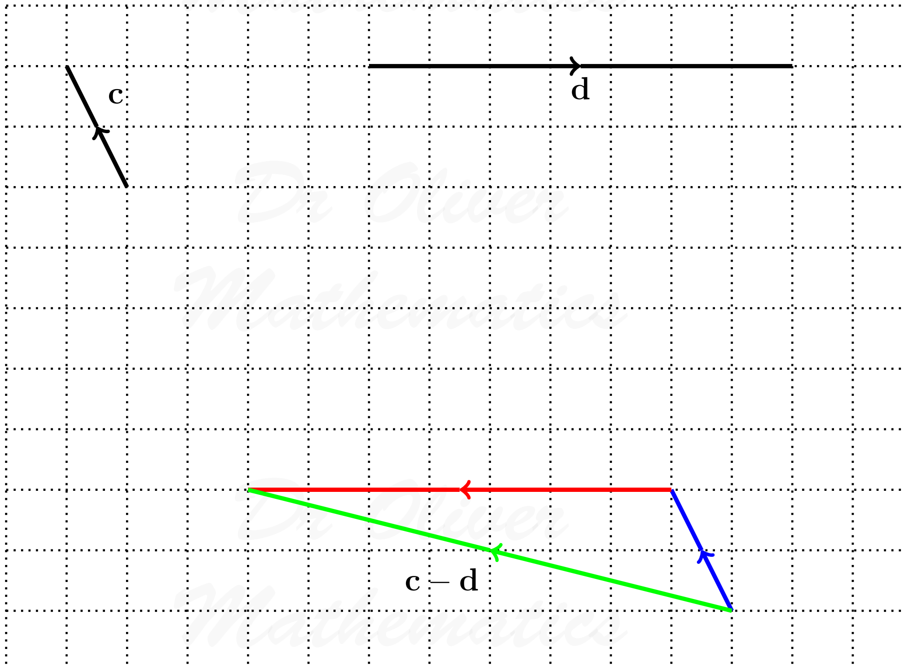
Solution
$\mathbf{b} = -2\mathbf{a}$.

(b) Vectors **c** and **d** are drawn on a grid. (2)



On the grid above, draw a vector representing $\mathbf{c} - \mathbf{d}$.

Solution



14. For Class X, (1)

$$\text{number of boys} : \text{number of girls} = 7 : 8.$$

For Class Y,

$$\text{number of boys} : \text{number of girls} = 3 : 4.$$

Which statement must be true?
Tick **one** box.

- [ ] Class X has more boys than class Y
- [ ] Class X has twice as many girls as class Y
- [ ] Class X has a greater proportion of boys than class Y
- [ ] Class X has the same proportion of boys as class Y

Solution

Class X has a greater proportion of boys than class Y.

15. Simplify fully (3)

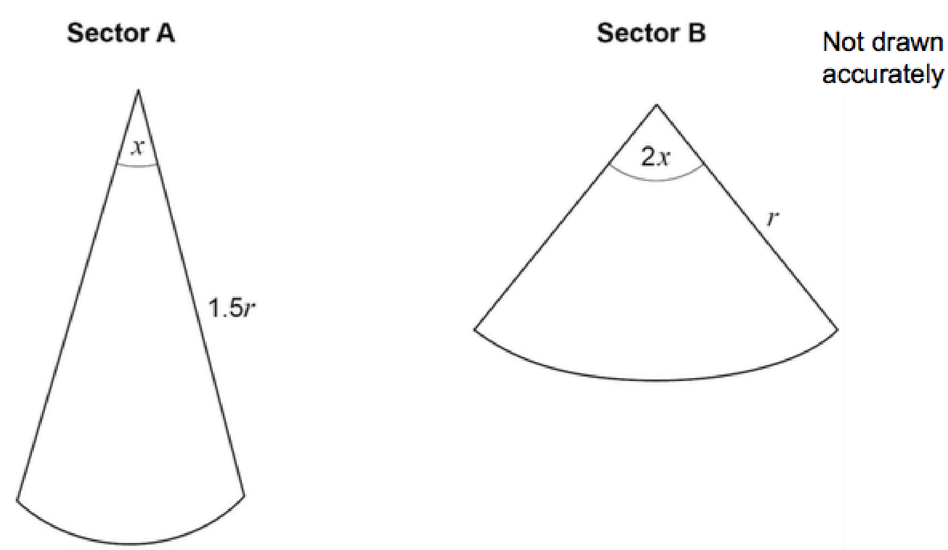
$$\frac{a^3b^2}{cd} \times \frac{c}{ab^5}.$$

Solution

Well,

$$\begin{aligned}\frac{a^3b^2}{cd} \times \frac{c}{ab^5} &= \frac{a^2}{d} \times \frac{1}{b^3} \\ &= \frac{a^2}{b^3 d}.\end{aligned}$$

16. Here are two sectors from different circles. (2)



Which sector has the bigger area?
Tick a box.

☐ Sector A ☐ Sector B

Show working to support your answer.

Solution

Sector A:

$$\begin{aligned} \text{area}_A &= \frac{x}{360} \times \pi \times (1.5r)^2 \\ &= \tfrac{1}{160}\pi r^2 x. \end{aligned}$$

Sector A:

$$\begin{aligned} \text{area}_B &= \frac{2x}{360} \times \pi \times r^2 \\ &= \tfrac{1}{180}\pi r^2 x. \end{aligned}$$

Hence, Sector A has more area.

17. A factory makes kettles. (3)

- Four samples of kettles are tested for faults.
- Each sample has size 200.

Here are the relative frequencies of faulty kettles in the samples.

Sample	P	Q	R	S
Relative frequency	0.03	0.035	0.015	0.01

Work out the range of the number of faulty kettles in the four samples.

Solution

We take the biggest from the lowest:

$$\begin{aligned} 200(0.035 - 0.01) &= 200 \times 0.025 \\ &= \underline{\underline{5}}. \end{aligned}$$

18. (a) Write (1)

$$x(3x - 9) = 4$$

in the form

$$ax^2 + bx + c = 0,$$

where a, b, and c are integers.

Solution

$$\begin{aligned} x(3x - 9) = 4 &\Rightarrow 3x^2 - 9x = 4 \\ &\Rightarrow \underline{\underline{3x^2 - 9x - 4 = 0}}; \end{aligned}$$

so, $\underline{\underline{a = 3}}$, $\underline{\underline{b = -9}}$, and $\underline{\underline{c = -4}}$.

(b) Solve (2)

$$x(3x - 9) = 4.$$

Give your answers to 2 decimal places.

Solution

$$x(3x - 9) = 4 \Rightarrow 3x^2 - 9x - 4 = 0$$

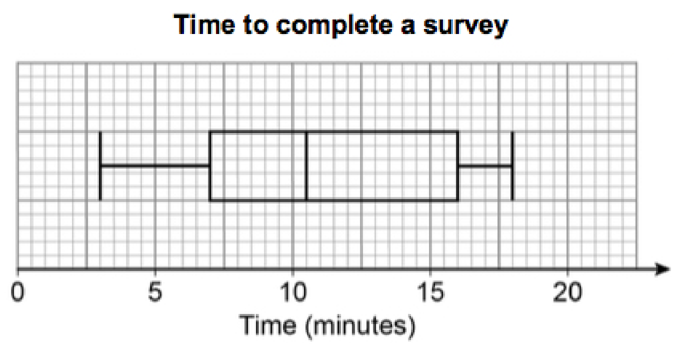
Quadratic formula: $a = 3$, $b = -9$, and $c = -4$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{9 \pm \sqrt{9^2 - 4 \times 3 \times (-4)}}{2 \times 3} \\ &= \frac{9 \pm \sqrt{129}}{6} \\ &= -0.392\,969\,448\,6,\ 3.392\,969\,449 \text{ (FCD)} \\ &= \underline{\underline{-0.39,\ 3.39 \text{ (2 dp)}}}. \end{aligned}$$

19. Here is some information about the times people took to complete a survey. (2)

Fastest time	3 minutes
Slowest time	18 minutes
Median	11 minutes
Lower quartile	7 minutes
Interquartile range	8 minutes

Ben draws this box plot to show the information.



Make **two** criticisms of his box plot.

Solution

E.g.,

$$\text{UQ} = \text{LQ} + \text{IQR} = 7 + 8 = 15;$$

the median is 11, not the 10.5 as indicated.

20. d is directly proportional to the square of v.

$d = 6$ when $v = 20$.

(a) Work out an equation connecting d and v. (3)

Solution

Well,

$$d \propto v^2 \Rightarrow d = kv^2,$$

for some constant d. Now,

$$d = 6, v = 20 \Rightarrow 6 = k(20^2)$$
$$\Rightarrow k = \tfrac{3}{200}$$

and

$$\underline{\underline{d = \tfrac{3}{200}v^2}}.$$

(b) Work out the value of d when $v = 30$. (2)

Solution

$$\begin{aligned} v = 30 \Rightarrow d &= \tfrac{3}{200}(30^2) \\ \Rightarrow \underline{\underline{d}} &\underline{\underline{= 13\tfrac{1}{2}}}. \end{aligned}$$

21. Hanif makes green paint by mixing blue paint and yellow paint in the ratio (5)

$$\text{blue} : \text{yellow} = 7 : 3.$$

- He buys blue paint in 50-litre containers, each costing £225.
- He buys yellow paint in 20-litre containers, each costing £80.

He wants to

- sell the green paint in 5-litre tins and
- make 40% profit on each tin.

How much should he sell each tin for?

Solution

1 litre of blue paint costs

$$\frac{225}{50} = 4.5$$

and 1 litre of yellow paint costs

$$\frac{80}{20} = 4.$$

Now, the green paint costs

$$\begin{aligned} (7 \times 4.5) + (3 \times 4) &= 31.5 + 12 \\ &= 43.5. \end{aligned}$$

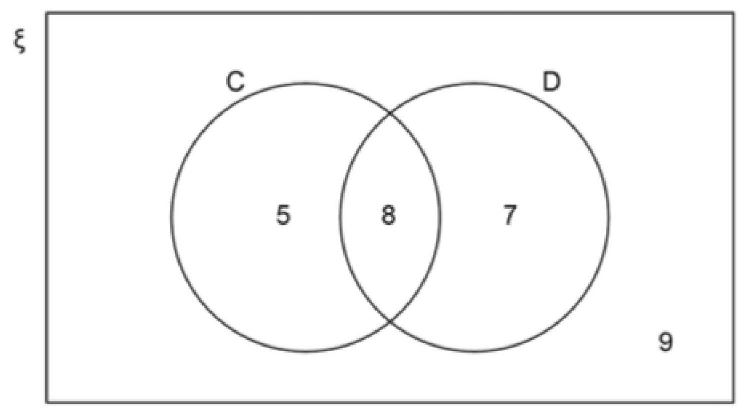
But that is 10 units worth! So, 5 litres costs

$$\frac{43.5}{2} = 21.75.$$

Finally,

$$\text{he sell each tin} = 21.75 \times 1.4$$
$$= \underline{\underline{£30.45}}.$$

22.
- $\mathscr{E} = 29$ students in a class.
- C = students who own a cat.
- D = students who own a dog.



(a) A student is chosen at random. (1)
Circle the probability that the student owns a cat or a dog but not both.

$$\frac{12}{29} \quad \frac{13}{29} \quad \frac{15}{29} \quad \frac{20}{29}$$

Solution
Well,

$$5 + 7 = 12$$

so

$$\underline{\underline{\frac{12}{29}}} \quad \frac{13}{29} \quad \frac{15}{29} \quad \frac{20}{29}$$

(b) A student who owns a dog is chosen at random. (1)
Circle the probability that the student also owns a cat.

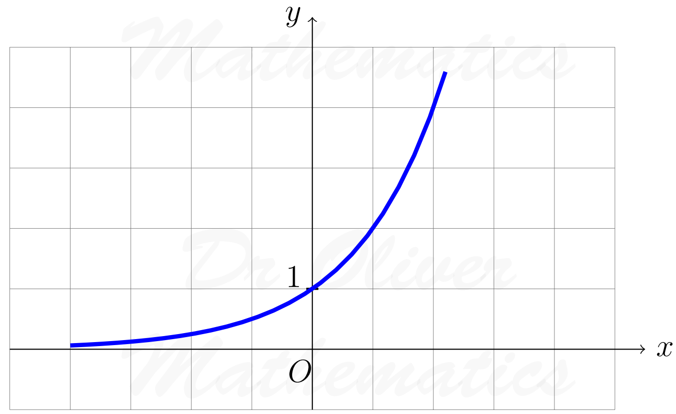
$$\frac{7}{15} \quad \frac{8}{15} \quad \frac{7}{29} \quad \frac{8}{29}$$

Solution

$$\frac{7}{15} \quad \underline{\underline{\frac{8}{15}}} \quad \frac{7}{29} \quad \frac{8}{29}$$

23. Here is a sketch of the curve (2)

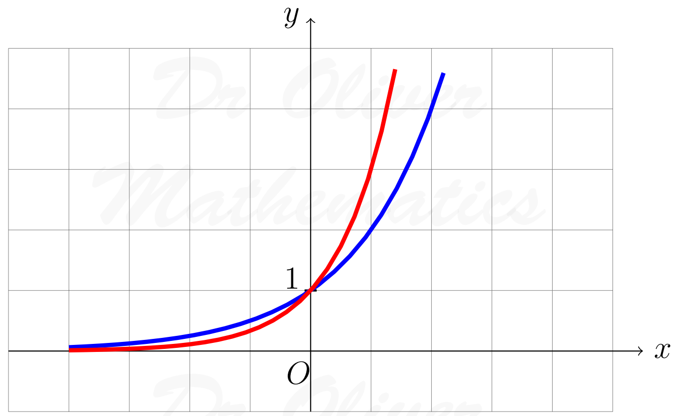
$$y = 2^x.$$



On the axes above, sketch the curve

$$y = 3^x.$$

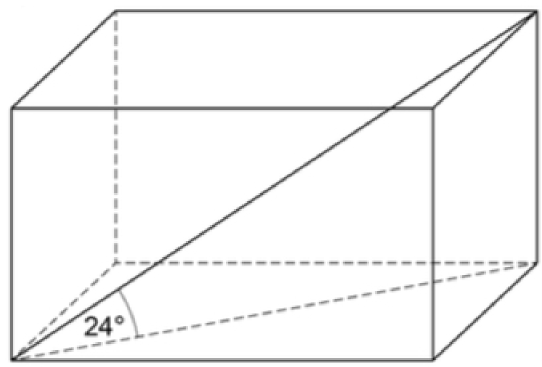
Solution



24. The length of a diagonal of a cuboid is 20 cm. (3)

The diagonal makes an angle of 24° with the base.

The area of the base is 150 cm^2.



Work out the volume of the cuboid.

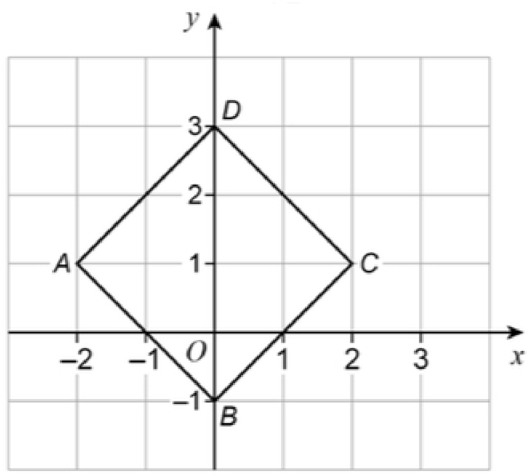
Solution

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 24^\circ = \frac{\text{opp}}{20}$$
$$\Rightarrow \text{opp} = 20 \sin 24^\circ.$$

Finally,

$$\begin{aligned} \text{volume} &= \text{area of the base} \times \text{height} \\ &= 150 \times 20 \sin 24^\circ \\ &= 1\,220.209\,929 \text{ (FCD)} \\ &= \underline{\underline{1\,220 \text{ cm}^3 b \text{ (3 sf)}}}. \end{aligned}$$

25. $ABCD$ is a square.
A is $(-2, 1)$. B is $(0, -1)$. C is $(2, 1)$. D is $(0, 3)$.



(a) A single transformation of $ABCD$ is such that (2)

- B is mapped to D,
- D is mapped to B, and
- A and C are invariant points.

Describe fully the transformation.

Solution
E.g., reflection in the line $y = 1$.

(b) A different **single** transformation of $ABCD$ is such that (3)

- B is mapped to D,
- D is mapped to B, and
- the only invariant point is $(0, 1)$.

Describe fully the transformation.

Solution
E.g., rotation, centre $(0, 1)$ through $180°$.

26. (3)

$$\mathrm{g}(x) = 16 - x \text{ and } \mathrm{h}(x) = x^3.$$

Solve

$$\mathrm{g}\,\mathrm{h}(x) = 24.$$

Solution

Well,

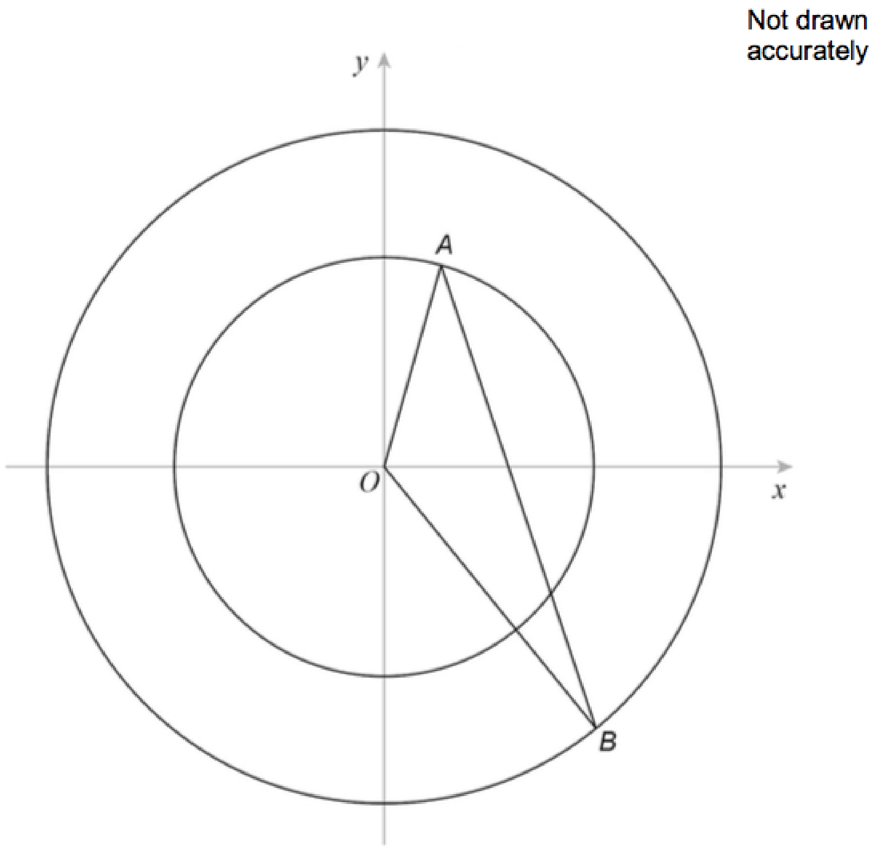
$$\begin{aligned} \mathrm{g}\,\mathrm{h}(x) &= \mathrm{g}(\mathrm{h}(x)) \\ &= \mathrm{g}(x^3) \\ &= 16 - x^3 \end{aligned}$$

and

$$\begin{aligned} \mathrm{g}\,\mathrm{h}(x) = 24 &\Rightarrow 16 - x^3 = 24 \\ &\Rightarrow x^3 = -8 \\ &\Rightarrow \underline{\underline{x = -2}}. \end{aligned}$$

27. In this question, all lengths are in centimetres. (5)

- A is a point on a circle, centre O.
- B is a point on a different circle, centre O.
- $AB = 20$.



The equation of the larger circle is

$$x^2 + y^2 = 144$$

and

$$\text{radius of smaller circle} : \text{radius of larger circle} = 4 : 5.$$

Work out the size of angle AOB.

Solution

The equation of the larger circle is

$$x^2 + y^2 = 144 = 12^2$$

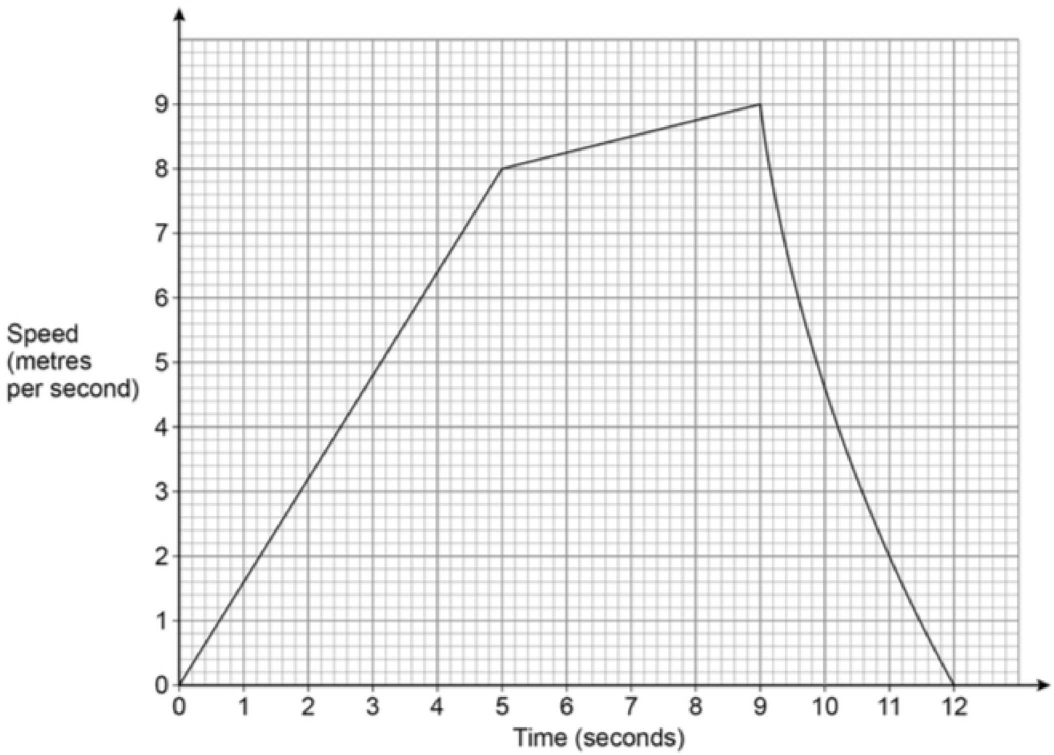
so the radius is 12. Now,

$$
\begin{aligned}
& \text{radius of smaller circle : radius of larger circle} = 4 : 5 \\
\Rightarrow\quad & \frac{\text{radius of smaller circle}}{12} = \frac{4}{5} \\
\Rightarrow\quad & \text{radius of smaller circle} = \tfrac{4}{5} \times 12 \\
\Rightarrow\quad & \text{radius of smaller circle} = 9.6.
\end{aligned}
$$

Cosine rule:

$$
\begin{aligned}
& AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \times \cos OAB \\
\Rightarrow\quad & 20^2 = 9.6^2 + 12^2 - 2 \times 9.6 \times 12 \times \cos OAB \\
\Rightarrow\quad & 400 = 92.16 + 144 - 230.4 \cos OAB \\
\Rightarrow\quad & 230.4 \cos OAB = -163.84 \\
\Rightarrow\quad & \cos OAB = -\tfrac{32}{45} \\
\Rightarrow\quad & \angle OAB = 135.325\,390\,4 \text{ (FCD)} \\
\Rightarrow\quad & \underline{\underline{\angle OAB = 135^\circ \text{ (3 sf)}}}.
\end{aligned}
$$

28. Leo runs for 12 seconds.
The graph shows his speed.



(a) Show that the distance he runs is less than 67.5 metres. (4)

Solution

Well, the distance $9 \leqslant t \leqslant 12$ is curved **inwards**, so

$$\begin{aligned}\text{distance} &= (\text{distance he runs } 0 \leqslant t \leqslant 5) + (\text{distance he runs } 5 \leqslant t \leqslant 9) \\ &\quad + (\text{distance he runs } 9 \leqslant t \leqslant 12) \\ &< (\tfrac{1}{2} \times 5 \times 8) + \left[\tfrac{1}{2}(8+9)(4)\right] + (\tfrac{1}{2} \times 3 \times 9) \\ &= 20 + 34 + 13.5 \\ &= 67.5;\end{aligned}$$

hence, the distance he runs is less than 67.5 metres

(b) Work out his average acceleration for the first 9 seconds.
State the units of your answer. (2)

Solution

$$\text{Average acceleration} = \frac{9}{9}$$
$$= \underline{\underline{1 \text{ m/s}^2}}.$$