Dr Oliver Mathematics
Mathematics: Advanced Higher
2012 Paper
3 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Given (3)

$$f(x) = \frac{3x+1}{x^2+1},$$

obtain $f'(x)$.

Solution

$$\begin{aligned} f'(x) &= \frac{(x^2+1)\cdot 3 - (3x+1)\cdot 2x}{(x^2+1)^2} \\ &= \frac{(3x^2+3)-(6x^2+2x)}{(x^2+1)^2} \\ &= \underline{\underline{\frac{3-2x-3x^2}{(x^2+1)^2}}}. \end{aligned}$$

(b) Let (4)

$$g(x) = \cos^2 x \exp(\tan x).$$

Obtain an expression for $g'(x)$ and simplify your answer.

Solution

$$\begin{aligned} g'(x) &= (-2\cos x \sin x)\cdot \exp(\tan x) + \cos^2 x \cdot (\sec^2 x \exp(\tan x)) \\ &= -\sin 2x \exp(\tan x) + \exp(\tan x)) \\ &= \underline{\underline{(1-\sin 2x)\exp(\tan x)}}. \end{aligned}$$

2. The first and fourth terms of a geometric series are 2 048 and 256 respectively.

(a) Calculate the value of the common ratio. (2)

Solution

Well, $a = 2\,048$ and $ar^3 = 256$ and so

$$\begin{aligned} r^3 = \frac{ar^3}{a} &\Rightarrow r^3 = \frac{256}{2\,048} \\ &\Rightarrow r^3 = \tfrac{1}{8} \\ &\Rightarrow \underline{\underline{r = \tfrac{1}{2}}}. \end{aligned}$$

(b) Given that the sum of the first n terms is $4\,088$, find the value of n. (3)

Solution

$$\begin{aligned} \frac{a(1-r^n)}{1-r} = S_n &\Rightarrow \frac{2\,048(1-(\frac{1}{2})^n)}{1-\frac{1}{2}} = 4\,088 \\ &\Rightarrow 1-(\tfrac{1}{2})^n = \tfrac{511}{512} \\ &\Rightarrow (\tfrac{1}{2})^n = \tfrac{1}{512} \\ &\Rightarrow \ln(\tfrac{1}{2})^n = \ln(\tfrac{1}{512}) \\ &\Rightarrow n\ln(\tfrac{1}{2}) = \ln(\tfrac{1}{512}) \\ &\Rightarrow n = \frac{\ln(\frac{1}{512})}{\ln(\frac{1}{2})} \\ &\Rightarrow \underline{\underline{n = 9}}. \end{aligned}$$

3. (a) Given that $(-1+2\mathrm{i})$ is a root of the equation (4)

$$z^3 + 5z^2 + 11z + 15 = 0,$$

obtain all the roots.

Solution

Now, $\underline{\underline{(-1-2\mathrm{i})}}$ is another root.

$\times$	z	$+1$	$-2\mathrm{i}$
z	z^2	$+z$	$-2z\mathrm{i}$
$+1$	$+z$	$+1$	$-2\mathrm{i}$
$+2\mathrm{i}$	$+2z\mathrm{i}$	$+2\mathrm{i}$	$+4$

Hence,

$$(z + 1 - 2\mathrm{i})(z + 1 + 2\mathrm{i}) = z^2 + 2z + 5.$$

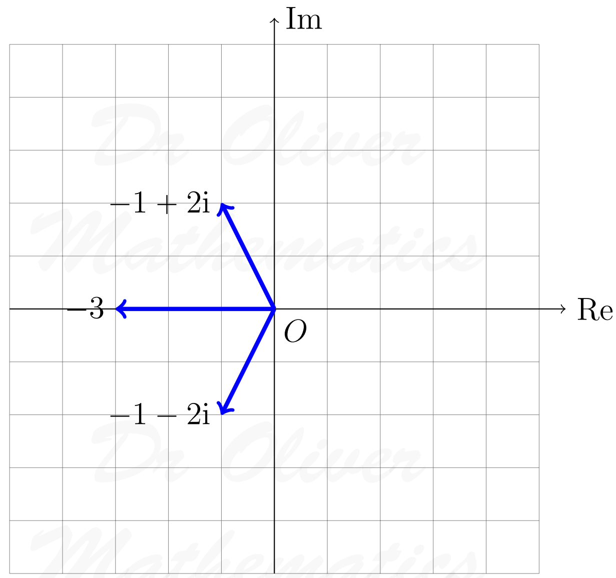
Now,

$$z^3 + 5z^2 + 11z + 15 = (z^2 + 2z + 5)(z + 3)$$

which means the third root is $\underline{\underline{-3}}$.

(b) Plot all the roots on an Argand diagram. (2)

Solution



4. (a) Write down and simplify the general term in the expansion of (3)

$$\left(2x - \frac{1}{x^2}\right)^9.$$

Solution

The general term is

$$\underline{\underline{\binom{9}{r}(2x)^r\left(-\frac{1}{x^2}\right)^{9-r}}}.$$

(b) Hence, or otherwise, obtain the term independent of x. (2)

Solution

$$\binom{9}{r}(2x)^r\left(-\frac{1}{x^2}\right)^{9-r} = \binom{9}{r}(2x)^r\left(-x^{-2}\right)^{9-r}$$
$$= (-1)^{9-r}\binom{9}{r}2^r x^r x^{-2(9-r)}$$
$$= (-1)^{9-r}\binom{9}{r}2^r x^r x^{2r-18}$$
$$= (-1)^{9-r}\binom{9}{r}2^r x^{3r-18}$$

which means

$$3r - 18 = 0 \Rightarrow r = 6.$$

Hence,

$$(-1)^3\binom{9}{6}2^6 = \underline{\underline{-5\,376}}.$$

5. Obtain an equation for the plane passing through the points $P(-2,1,-1)$, $Q(1,2,3)$, and $R(3,0,1)$. (5)

Solution

$$\overrightarrow{PQ} = 3\mathbf{i}+\mathbf{j}+4\mathbf{k} \text{ and } \overrightarrow{PR} = 5\mathbf{i}-\mathbf{j}+2\mathbf{k}.$$

Now,

$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 4 \\ 5 & -1 & 2 \end{vmatrix}$$
$$= 6\mathbf{i} + 14\mathbf{j} - 8\mathbf{k}.$$

So, an equation is

$$6x + 14y - 8z = (6\mathbf{i} + 14\mathbf{j} - 8\mathbf{k}).(3\mathbf{i} + \mathbf{k})$$
$$= 18 + 0 - 8$$

and so

$$6x + 14y - 8z = 10 \Rightarrow \underline{\underline{3x + 7y - 4z = 5}}.$$

6. (a) Write down the Maclaurin expansion of e^x as far as the term in x^3. (1)

Solution

$$e^x = \underline{\underline{1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + \ldots}}.$$

(b) Hence, or otherwise, obtain the Maclaurin expansion of $(1 + e^x)^2$ as far as the term in x^3. (4)

Solution

Now,

$$1 + e^x = 2 + x + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + \ldots.$$

$\times$	2	$+x$	$+\frac{1}{2}x^2$	$+\frac{1}{6}x^3$
2	4	$+2x$	$+x^2$	$+\frac{1}{3}x^3$
$+x$	$+2x$	$+x^2$	$+\frac{1}{2}x^3$	$\ldots$
$+\frac{1}{2}x^2$	$+x^2$	$+\frac{1}{2}x^3$	$\ldots$	$\ldots$
$+\frac{1}{6}x^3$	$+\frac{1}{3}x^3$	$\ldots$	$\ldots$	$\ldots$

Hence,

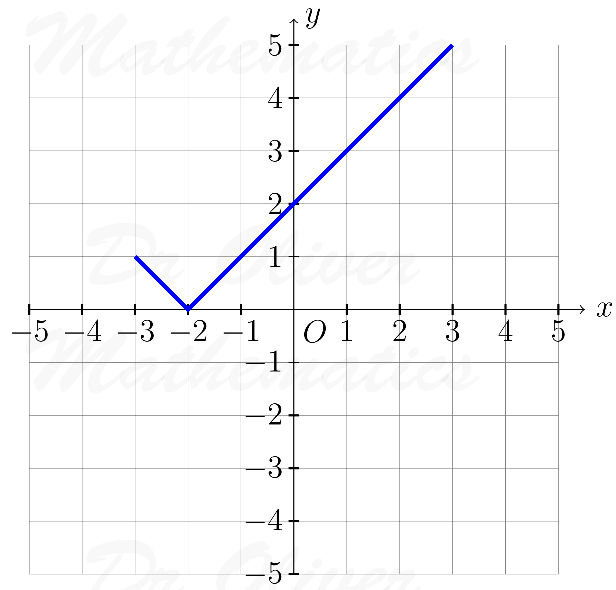
$$(1 + e^x)^2 = \underline{\underline{4 + 4x + 3x^2 + +\tfrac{5}{3}x^3 + \ldots}}.$$

7. A function is defined by

$$f(x) = |x + 2| \text{ for all } x.$$

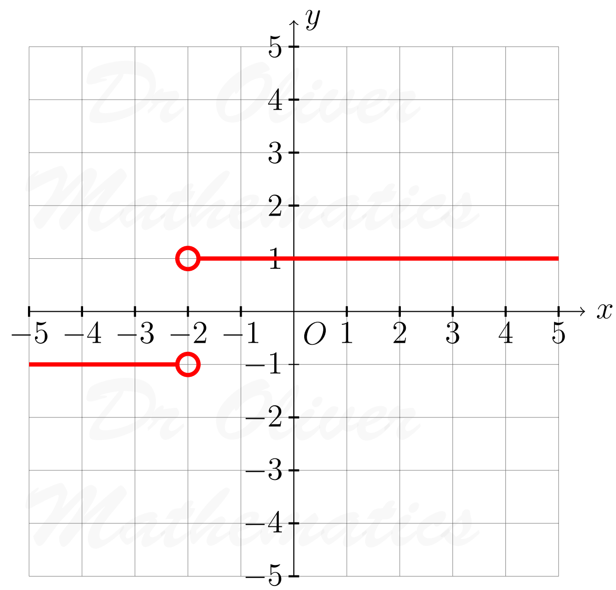
(a) Sketch the graph of the function for $-3 \leqslant x \leqslant 3$. (2)

Solution



(b) On a separate diagram, sketch the graph of $f'(x)$. (2)

Solution



8. Use the substitution $x = 4\sin\theta$ to evaluate (6)

$$\int_0^2 \sqrt{16 - x^2}\,\mathrm{d}x.$$

Solution

$$\begin{aligned} x = 4\sin\theta &\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\cos\theta \\ &\Rightarrow \mathrm{d}x = 4\cos\theta\,\mathrm{d}\theta. \end{aligned}$$

Now,

$$\begin{aligned} x = 0 &\Rightarrow \sin\theta = 0 \Rightarrow \theta = 0, \\ x = 2 &\Rightarrow \sin\theta = \tfrac{1}{2} \Rightarrow \theta = \tfrac{1}{6}\pi. \end{aligned}$$

Finally,

$$\begin{aligned} \int_0^2 \sqrt{16 - x^2}\,\mathrm{d}x &= \int_0^{\frac{1}{6}\pi} \sqrt{16 - (4\sin\theta)^2}\,(4\cos\theta)\,\mathrm{d}\theta \\ &= \int_0^{\frac{1}{6}\pi} \sqrt{16 - 16\sin^2\theta}\,(4\cos\theta)\,\mathrm{d}\theta \\ &= \int_0^{\frac{1}{6}\pi} \sqrt{16\cos^2\theta}\,(4\cos\theta)\,\mathrm{d}\theta \\ &= \int_0^{\frac{1}{6}\pi} (4\cos\theta)^2\,\mathrm{d}\theta \\ &= 16\int_0^{\frac{1}{6}\pi} \cos^2\theta\,\mathrm{d}\theta \\ &= 16\int_0^{\frac{1}{6}\pi} \tfrac{1}{2}(1 + \cos 2\theta)\,\mathrm{d}\theta \\ &= 8\int_0^{\frac{1}{6}\pi} (1 + \cos 2\theta)\,\mathrm{d}\theta \\ &= 8\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_{x=0}^{\frac{1}{6}\pi} \\ &= 8\left\{\left(\tfrac{1}{6}\pi + \tfrac{\sqrt{3}}{4}\right) - (0 + 0)\right\} \\ &= \underline{\underline{\tfrac{4}{3}\pi + 2\sqrt{3}}}. \end{aligned}$$

9. A non-singular $n \times n$ matrix $\mathbf{A}$ satisfies the equation (4)

$$\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I},$$

where $\mathbf{I}$ is the $n \times n$ identity matrix.

Show that

$$\mathbf{A}^3 = k\mathbf{I}$$

and state the value of k.

Solution

$$\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I} \Rightarrow \mathbf{A}^{-1} = \mathbf{I} - \mathbf{A}.$$

Now,

$$\begin{aligned} \mathbf{I} &= \mathbf{A}\mathbf{A}^{-1} \\ &= \mathbf{A}(\mathbf{I} - \mathbf{A}) \\ &= \mathbf{A} - \mathbf{A}^2 \end{aligned}$$

and

$$\begin{aligned} \mathbf{A}^2 &= \mathbf{A} - \mathbf{I} \\ &= -\mathbf{A}^{-1}. \end{aligned}$$

Finally,

$$\mathbf{A}^2 = -\mathbf{A}^{-1} \Rightarrow \underline{\underline{\mathbf{A}^3 = -\mathbf{I}}};$$

hence, $\underline{\underline{k = -1}}$.

10. Use the division algorithm to express 1234_{10} in base 7. (3)

Solution

$$\begin{aligned} 1234 &= 176 \times 7 + 2 \\ 176 &= 25 \times 7 + 1 \\ 25 &= 3 \times 7 + 4 \\ 3 &= 0 \times 7 + 3; \end{aligned}$$

hence,

$$1234_{10} = \underline{\underline{3412_5}}.$$

11. (a) Write down the derivative of $\sin^{-1} x$. (1)

Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin^{-1} x) = \underline{\underline{\frac{1}{\sqrt{1-x^2}}}}.$$

(b) Use integration by parts to obtain (4)

$$\int \sin^{-1} x \cdot \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x.$$

Solution

$$u = \sin^{-1} x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{\sqrt{1-x^2}},$$
$$\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{x}{\sqrt{1-x^2}} \Rightarrow v = -\sqrt{1-x^2}.$$

Now,

$$\begin{aligned}\int \sin^{-1} x \cdot \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x &= -\sin^{-1} x \cdot \sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}\,\mathrm{d}x \\ &= -\sin^{-1} x \cdot \sqrt{1-x^2} + \int 1\,\mathrm{d}x \\ &= \underline{\underline{-\sin^{-1} x \cdot \sqrt{1-x^2} + x + c}}.\end{aligned}$$

12. The radius of a cylindrical column of liquid is decreasing at the rate of 0.02 $\mathrm{m\,s^{-1}}$, while the height is increasing at the rate of 0.01 $\mathrm{m\,s^{-1}}$. (5)

Find the rate of change of the volume when the radius is 0.6 metres and the height is 2 metres.

Solution

Now,

$$V = \pi r^2 h \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = \pi\left[2r\frac{\mathrm{d}r}{\mathrm{d}t}h + r^2\frac{\mathrm{d}h}{\mathrm{d}t}\right].$$

Finally,

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -0.02, \frac{\mathrm{d}h}{\mathrm{d}t} = 0.01 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = \pi\left[2 \cdot 0.6 \cdot (-0.02) \cdot (2) + (0.6)^2 \cdot (0.01)\right]$$
$$\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = \underline{\underline{-\tfrac{111}{2\,500}\pi \text{ m}^3\,\text{s}^{-1}}}.$$

13. A curve is defined parametrically, for all t, by the equations

$$x = 2t + \tfrac{1}{2}t^2 \text{ and } y = \tfrac{1}{3}t^3 - 3t.$$

(a) Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ as functions of t. (5)

Solution

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2 + t \text{ and } \frac{\mathrm{d}y}{\mathrm{d}t} = t^2 - 3.$$

Now,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} \\ &= \underline{\underline{\frac{t^2-3}{2+t}}}\end{aligned}$$

and

$$\begin{aligned}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \frac{\mathrm{d}t}{\mathrm{d}x} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{t^2-3}{2+t}\right) \cdot \frac{1}{2+t} \\ &= \frac{(2+t)(2t) - (t^2-3)(1)}{(2+t)^2} \cdot \frac{1}{2+t} \\ &= \frac{(4t+2t^2) - (t^2-3)}{(2+t)^3} \\ &= \underline{\underline{\frac{3+4t+t^2}{(2+t)^3}}}.\end{aligned}$$

(b) Find the values of t at which the curve has stationary points and determine their nature. (3)

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{t^2 - 3}{2 + t} = 0$$
$$\Rightarrow t^2 - 3 = 0$$
$$\Rightarrow t = \pm\sqrt{3}.$$

Next,

$$t = \sqrt{3} \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\sqrt{3}$$
$$t = -\sqrt{3} \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2\sqrt{3}.$$

Hence, $(\frac{3-4\sqrt{2}}{2}, 2\sqrt{3})$ is a minimum turning point and $(\frac{3+4\sqrt{2}}{2}, -2\sqrt{3})$ is a maximum turning point

(c) Show that the curve has exactly two points of inflexion. (2)

Solution

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0 \Rightarrow \frac{3 + 4t + t^2}{(2 + t)^3} = 0$$
$$\Rightarrow 3 + 4t + t^2 = 0$$
$$\Rightarrow (3 + t)(t + 1) = 0$$
$$\Rightarrow t = -3 \text{ or } t = -1;$$

hence, the curve has exactly two points of inflexion, as required.

14. (a) Use Gaussian elimination to obtain the solution of the following system of equations in terms of the parameter λ. (5)

$$4x + 6z = 1$$
$$2x - 2y + 4z = -1$$
$$-x + y + \lambda z = 2.$$

Solution

$$\left(\begin{array}{ccc|c} 4 & 0 & 6 & 1 \\ 2 & -2 & 4 & -1 \\ -1 & 1 & \lambda & 2 \end{array}\right)$$

Do $R_2 - \frac{1}{2} \times R_1$ and $R_3 + \frac{1}{4}R_1$:

$$\left(\begin{array}{ccc|c} 4 & 0 & 6 & 1 \\ 0 & -2 & 1 & -\frac{3}{2} \\ 0 & 1 & \lambda + \frac{3}{2} & \frac{9}{4} \end{array}\right)$$

Do $R_3 + \frac{1}{2}R_2$:

$$\left(\begin{array}{ccc|c} 4 & 0 & 6 & 1 \\ 0 & -2 & 1 & -\frac{3}{2} \\ 0 & 0 & \lambda + 2 & \frac{3}{2} \end{array}\right)$$

Hence,

$$
\begin{aligned}
(\lambda+2)z = \tfrac{3}{2} &\Rightarrow z = \frac{\frac{3}{2}}{\lambda+2} \\
&\Rightarrow \underline{\underline{z = \frac{3}{2(\lambda+2)}}} \\
&\Rightarrow -2y + \frac{3}{2(\lambda+2)} = -\tfrac{3}{2} \\
&\Rightarrow 2y = \frac{3}{2(\lambda+2)} + \tfrac{3}{2} \\
&\Rightarrow 2y = \frac{3+3(\lambda+2)}{2(\lambda+2)} \\
&\Rightarrow 2y = \frac{3\lambda+9}{2(\lambda+2)} \\
&\Rightarrow \underline{\underline{y = \frac{3\lambda+9}{4(\lambda+2)}}} \\
&\Rightarrow 4x + \frac{18}{2(\lambda+2)} = 1 \\
&\Rightarrow 4x = 1 - \frac{18}{2(\lambda+2)} \\
&\Rightarrow 4x = \frac{2(\lambda+2)-18}{2(\lambda+2)} \\
&\Rightarrow 4x = \frac{2\lambda-14}{2(\lambda+2)} \\
&\Rightarrow \underline{\underline{x = \frac{\lambda-7}{4(\lambda+2)}}}.
\end{aligned}
$$

(b) Describe what happens when $\lambda = -2$. (1)

Solution

When $\lambda = -2$, $0 = \frac{3}{2}$ and so there are $\underline{\underline{\text{no solutions}}}$.

When $\lambda = -1.9$, the solution is

$$x = -22.25,\ y = 8.25,\ z = 15.$$

(c) Find the solution when $\lambda = -2.1$. (2)

Solution

For $\lambda = -2.1$,

$$\underline{\underline{x = 22\tfrac{3}{4},\ y = -6\tfrac{3}{4},\ z = -15.}}$$

(d) Comment on these solutions. (1)

Solution

Although the values of λ are close, the values of x, y, and z are radically different. Hence, the system is $\underline{\underline{\text{ill-conditioned}}}$ near $\lambda = -2$.

15. (a) Express (4)

$$\frac{1}{(x-1)(x+2)^2}$$

in partial fractions.

Solution

$$\begin{aligned}\frac{1}{(x-1)(x+2)^2} &\equiv \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \\ &\equiv \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x-1)(x+2)^2}\end{aligned}$$

and hence

$$1 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1).$$

$\underline{x = 1}$: $1 = 9A \Rightarrow A = \frac{1}{9}$.
$\underline{x = -2}$: $1 = -3C \Rightarrow C = -\frac{1}{3}$.
$\underline{x = 0}$: $1 = 4A - 2B - C \Rightarrow B = -\frac{1}{9}$.
Hence,

$$\frac{1}{(x-1)(x+2)^2} \equiv \underline{\underline{\frac{\frac{1}{9}}{x-1} - \frac{\frac{1}{9}}{x+2} - \frac{\frac{1}{3}}{(x+2)^2}}}.$$

(b) Obtain the general solution of the differential equation (7)

$$(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} - y = \frac{x-1}{(x+2)^2},$$

expressing your answer in the form $y = \mathrm{f}(x)$.

Solution

$$(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} - y = \frac{x-1}{(x+2)^2} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} + \left(-\frac{1}{x-1}\right) y = \frac{1}{(x+2)^2}$$

$$\begin{aligned} \text{IF} &= \mathrm{e}^{\int -\frac{1}{x-1}\,\mathrm{d}x} \\ &= \mathrm{e}^{-\ln(x-1)} \\ &= \mathrm{e}^{\ln\frac{1}{x-1}} \\ &= \frac{1}{x-1} \end{aligned}$$

$$\begin{aligned} &\Rightarrow \left(\frac{1}{x-1}\right)\frac{\mathrm{d}y}{\mathrm{d}x} + \left(-\frac{1}{(x-1)^2}\right) y = \frac{1}{(x-1)(x+2)^2} \\ &\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{x-1}\right) = \frac{1}{(x-1)(x+2)^2} \\ &\Rightarrow \frac{y}{x-1} = \int \left(\frac{\frac{1}{9}}{x-1} - \frac{\frac{1}{9}}{x+2} - \frac{\frac{1}{3}}{(x+2)^2}\right) \mathrm{d}x \\ &\Rightarrow \frac{y}{x-1} = \tfrac{1}{9}\ln|x-1| - \tfrac{1}{9}\ln|x+2| + \tfrac{1}{3}(x+2)^{-1} + c \end{aligned}$$

and, hence,

$$\underline{\underline{y = (x-1)\left[\tfrac{1}{9}\ln|x-1| - \tfrac{1}{9}\ln|x+2| + \tfrac{1}{3}(x+2)^{-1} + c\right]}}.$$

16. (a) Prove by induction that (6)

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta,$$

for all integers $n \geqslant 1$.

Solution

$\underline{n=1}$: $(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta$ and so the solution is true for $n = 1$.

Suppose the solution is true for $n = k$, i.e.,

$$(\cos\theta + \mathrm{i}\sin\theta)^k \equiv \cos k\theta + \mathrm{i}\sin k\theta.$$

Then,

$$
\begin{aligned}
&(\cos\theta + \mathrm{i}\sin\theta)^{k+1} \\
=&(\cos\theta + \mathrm{i}\sin\theta)(\cos\theta + \mathrm{i}\sin\theta)^{k} \\
=&(\cos\theta + \mathrm{i}\sin\theta)(\cos k\theta + \mathrm{i}\sin k\theta) \\
=&(\cos k\theta\cos\theta - \sin k\theta\sin\theta) + \mathrm{i}(\cos k\theta\sin\theta + \cos\theta\sin k\theta) \\
=&\cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta
\end{aligned}
$$

and so the result is true for $n = k + 1$.

Hence, by mathematical induction, the expression is true for all $n \in \mathbb{N}$, as required.

(b) Show that the real part of (4)

$$\frac{\left(\cos\frac{1}{18}\pi + \mathrm{i}\sin\frac{1}{18}\pi\right)^{11}}{\left(\cos\frac{1}{36}\pi + \mathrm{i}\sin\frac{1}{36}\pi\right)^{4}}$$

is zero.

Solution

$$
\begin{aligned}
\frac{\left(\cos\frac{1}{18}\pi + \mathrm{i}\sin\frac{1}{18}\pi\right)^{11}}{\left(\cos\frac{1}{36}\pi + \mathrm{i}\sin\frac{1}{36}\pi\right)^{4}} &= \frac{\left(\cos\frac{11}{18}\pi + \mathrm{i}\sin\frac{11}{18}\pi\right)}{\left(\cos\frac{1}{9}\pi + \mathrm{i}\sin\frac{1}{9}\pi\right)} \\
&= \frac{\left(\cos\frac{11}{18}\pi + \mathrm{i}\sin\frac{11}{18}\pi\right)}{\left(\cos\frac{1}{9}\pi + \mathrm{i}\sin\frac{1}{9}\pi\right)} \times \frac{\left(\cos\frac{1}{9}\pi - \mathrm{i}\sin\frac{1}{9}\pi\right)}{\left(\cos\frac{1}{9}\pi - \mathrm{i}\sin\frac{1}{9}\pi\right)};
\end{aligned}
$$

Hence, the real part is

$$
\begin{aligned}
\frac{\cos\frac{11}{18}\pi\cos\frac{1}{9}\pi - \sin\frac{11}{18}\pi\sin\frac{1}{9}\pi}{\left(\cos^2\frac{1}{9}\pi + \sin^2\frac{1}{9}\pi\right)} &= \cos(\tfrac{11}{18}\pi - \tfrac{1}{9}\pi) \\
&= \cos(\tfrac{1}{2}\pi) \\
&= \underline{\underline{0}},
\end{aligned}
$$

as required.