Dr Oliver Mathematics
GCSE Mathematics
2018 Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Work out (2)

$$2\tfrac{1}{7} + 1\tfrac{1}{4}.$$

Solution

$$\begin{aligned} 2\tfrac{1}{7} + 1\tfrac{1}{4} &= 3 + \tfrac{4}{28} + \tfrac{7}{28} \\ &= \underline{\underline{3\tfrac{11}{28}}}. \end{aligned}$$

(b) Work out (2)

$$1\tfrac{1}{5} \div \tfrac{3}{4}.$$

Give your answer as a mixed number in its simplest form.

Solution

$$\begin{aligned} 1\tfrac{1}{5} \div \tfrac{3}{4} &= \tfrac{6}{5} \times \tfrac{4}{3} \\ &= \tfrac{2}{5} \times \tfrac{4}{1} \\ &= \tfrac{8}{5} \\ &= \underline{\underline{1\tfrac{3}{5}}}. \end{aligned}$$

2. In a village, the number of houses and the number of flats are in the ratio 7 : 4 and the number of flats and the number of bungalows are in the ratio 8 : 5. (3)
There are 50 bungalows in the village.
How many houses are there in the village?

Solution

The number of houses and the number of flats are in the ratio

$$7 : 4 = 14 : 8$$

which means that the number of houses, the number of flats, and the number of bungalows are in the ratio

$$14 : 8 : 5.$$

Finally, the number of houses is

$$\begin{aligned} \frac{14}{5} \times 50 &= 14 \times 10 \\ &= \underline{\underline{140 \text{ houses}}}. \end{aligned}$$

3. Renee buys 5 kg of sweets to sell. (4)
   She pays £10 for the sweets.
   Renee puts all the sweets into bags.
   She puts 250 g of sweets into each bag.
   She sells each bag of sweets for 65 p.
   Renee sells all the bags of sweets.
   Work out her percentage profit.

Solution

The number of bags of sweets is

$$\frac{5\,000}{250} = 20$$

and the amount raised is

$$20 \times 65 = 1\,300 \text{ p} = £13.$$

Given that she paid £10 for the sweets, her percentage profit is

$$\frac{3}{10} \times 100\% = \underline{\underline{30\%}}.$$

4. A cycle race across America is 3 069.25 miles in length.
   Juan knows his average speed for his previous races is 15.12 miles per hour.
   For the next race across America he will cycle for 8 hours per day.

(a) Estimate how many days Juan will take to complete the race. (3)

Solution
We round the cycle race to 1 sf; average speed to 2 sf; we make him cycle for 10 hours:

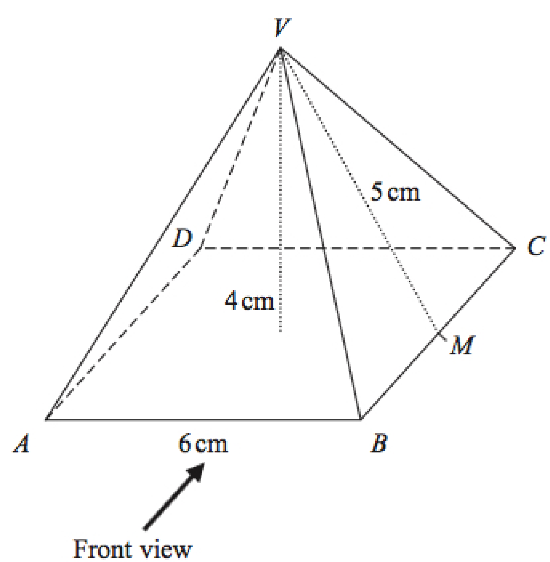
$$\frac{3\,069.25}{15.12 \times 8} \approx \frac{3\,000}{15 \times 10} = \underline{\underline{20 \text{ days}}}.$$

Juan trains for the race.
The average speed he can cycle at increases.
It is now 16.27 miles per hour.

(b) How does this affect your answer to part (a)? (1)

Solution
It will be <u>less</u> than the answer to part (a).

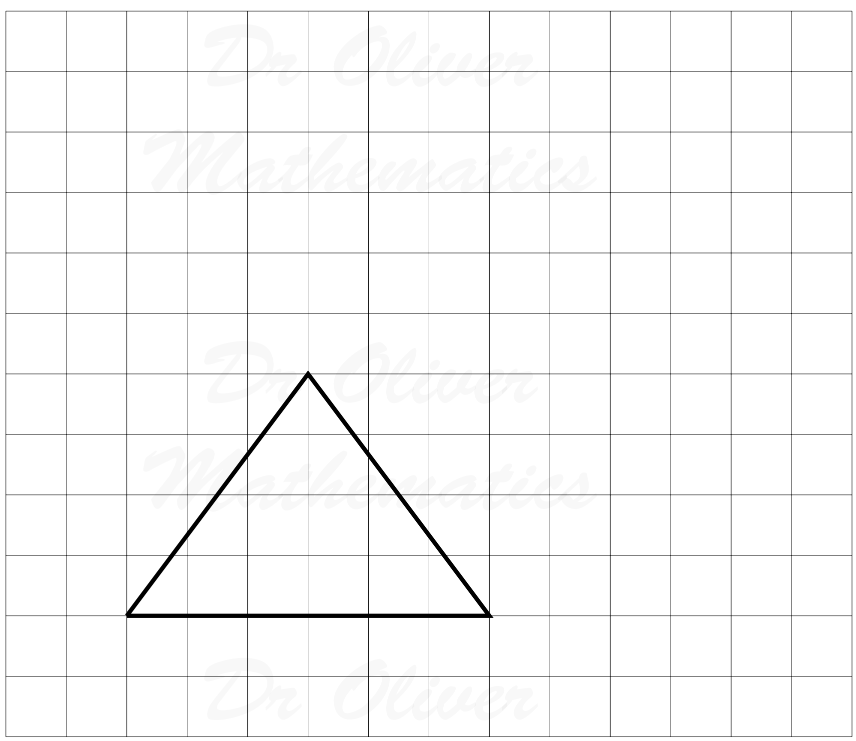
5. Here is a solid square-based pyramid, $VABCD$.



The base of the pyramid is a square of side 6 cm.
The height of the pyramid is 4 cm.
M is the midpoint of BC and $VM = 5$ cm.

(a) Draw an accurate front elevation of the pyramid from the direction of the arrow. (2)

Solution

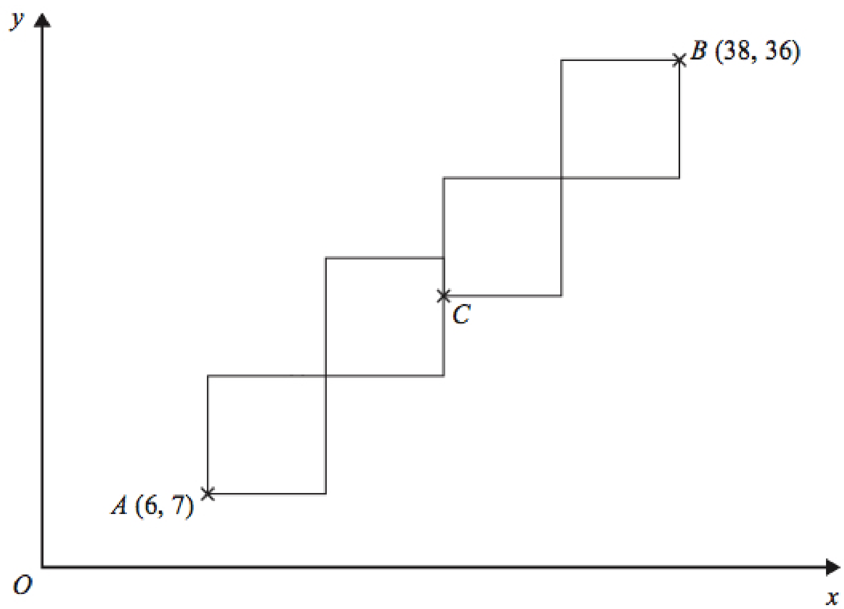


(b) Work out the total surface area of the pyramid. (4)

Solution

$$\begin{aligned} \text{Area} &= 4 \text{ triangular faces} + \text{base} \\ &= (4 \times \tfrac{1}{2} \times 6 \times 5) + (6 \times 6) \\ &= 60 + 36 \\ &= \underline{\underline{96 \text{ cm}^2}}. \end{aligned}$$

6. A pattern is made from four identical squares. (5)
The sides of the squares are parallel to the axes.



Point A has coordinates $(6, 7)$.
Point B has coordinates $(38, 36)$.
Point C is marked on the diagram.

Work out the coordinates of C.

Solution

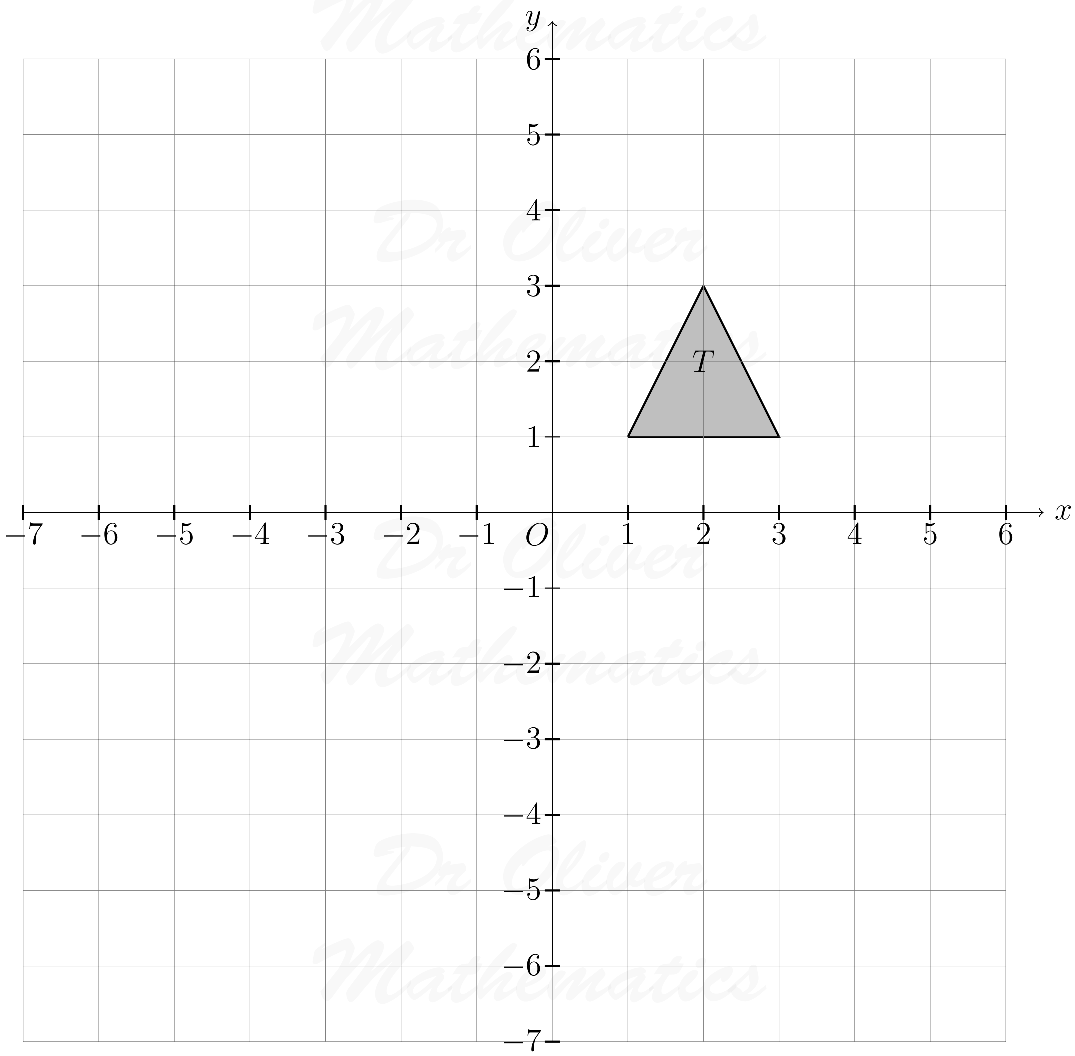
The horizontal distance between A and B is

$$38 - 6 = 32 \text{ cm}$$

and this means that the four squares are each 8 cm. Hence, the coordinates of C are

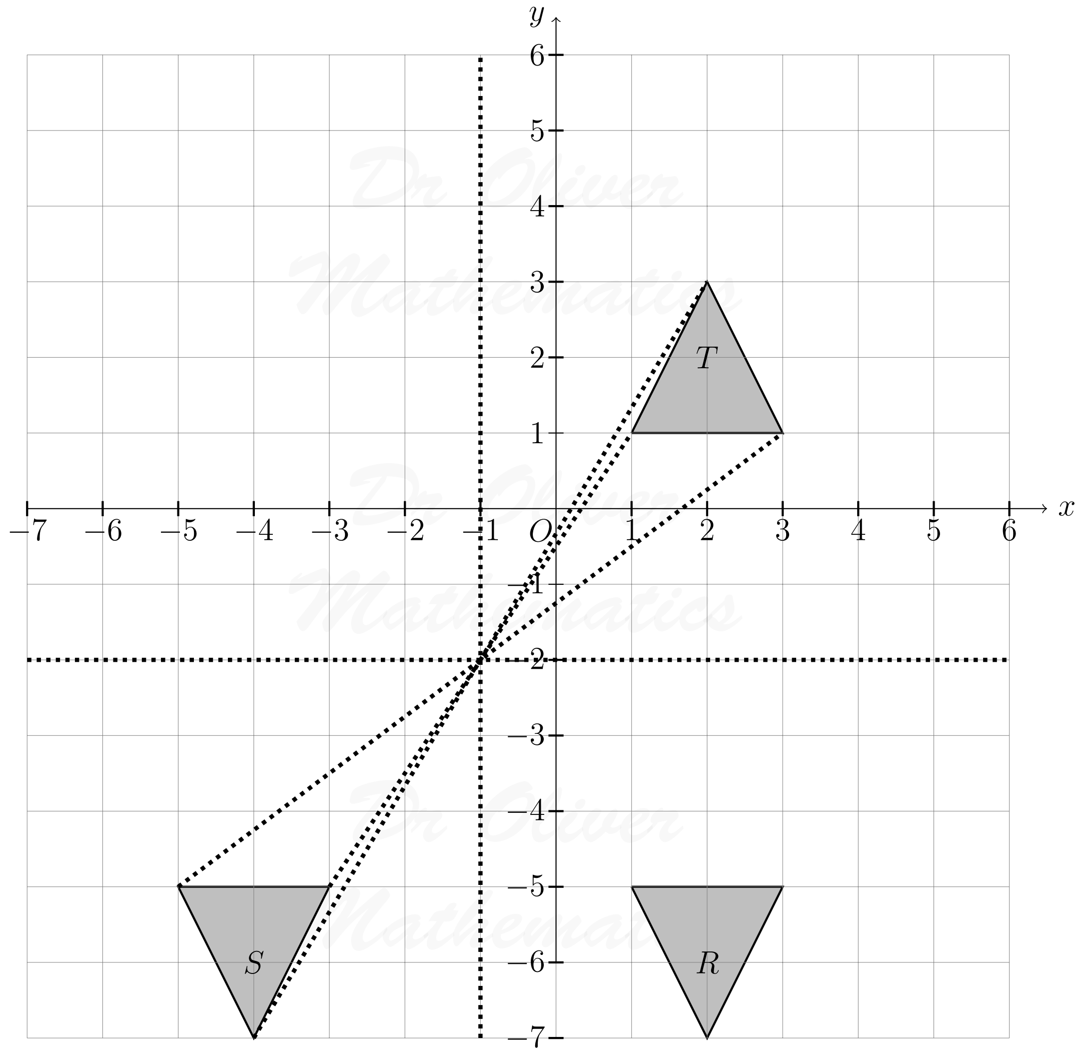
$$(38 - 2 \times 8, 36 - 2 \times 8) = \underline{\underline{(22, 20)}}.$$

7. Shape **T** is reflected in the line $x = -1$ to give shape **R**. (2)
Shape **R** is reflected in the line $y = -2$ to give shape **S**.



Describe the **single** transformation that will map shape **T** to shape **S**.

Solution



Rotation, about 180°, about (−1, −2).

8. The perimeter of a right-angled triangle is 72 cm. (4)
The lengths of its sides are in the ratio 3 : 4 : 5.

Work out the area of the triangle.

Solution

Well,

$$3 + 4 + 5 = 12$$

and the shortest side, the middle side, and the longest side are

$$\tfrac{3}{12} \times 72 = \tfrac{1}{4} \times 72 = 18 \text{ cm},$$

$$\tfrac{4}{12} \times 72 = \tfrac{1}{3} \times 72 = 24 \text{ cm},$$

and

$$\tfrac{5}{12} \times 72 = 5 \times 6 = 30 \text{ cm}.$$

Hence, the area of the triangle is

$$\tfrac{1}{2} \times 18 \times 24 = 9 \times 24 = \underline{\underline{216 \text{ cm}^2}}.$$

9. (a) Write down the value of $36^{\frac{1}{2}}$. (1)

Solution

$$36^{\frac{1}{2}} = \sqrt{36} = \underline{\underline{6}}.$$

(b) Write down the value of 23^0. (1)

Solution

$$23^0 = \underline{\underline{1}}.$$

(c) Work out the value of $27^{-\frac{2}{3}}$. (2)

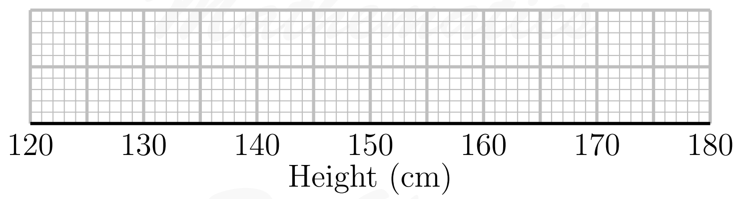
Solution

$$\begin{aligned} 27^{-\frac{2}{3}} &= \frac{1}{27^{\frac{2}{3}}} \\ &= \frac{1}{(27^{\frac{1}{3}})^2} \\ &= \frac{1}{3^2} \\ &= \underline{\underline{\tfrac{1}{9}}}. \end{aligned}$$

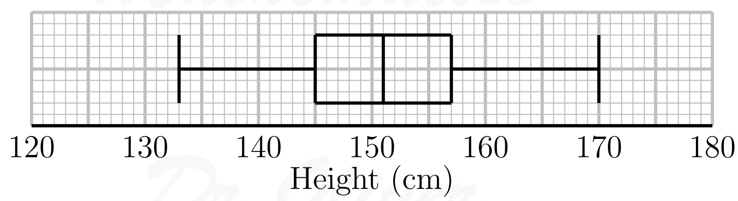
10. The table gives some information about the heights of 80 girls.

Least height	133 cm
Greatest height	170 cm
Lower quartile	145 cm
Upper quartile	157 cm
Median	151 cm

(a) Draw a box plot to represent this information. (3)



Solution



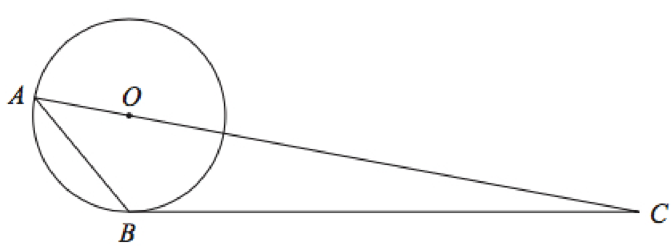
(b) Work out an estimate for the number of these girls with a height between 133 cm and 157 cm. (2)

Solution

An estimate is

$$\tfrac{3}{4} \times 80 = \underline{\underline{60 \text{ girls}}}.$$

11. A and B are points on a circle, centre O. (5)



BC is a tangent to the circle.
AOC is a straight line.
Angle $ABO = x^\circ$.
Find the size of angle ACB, in terms of x.
Give your answer in its simplest form.
Give reasons for each stage of your working.

Solution

Let D be the point on the circle that is opposite A, i.e., that AD is a diameter of the circle.
$\angle BAO = x^\circ$ (base angles of an isosceles triangle)
$\angle AOB = 180 - x - x = (180 - 2x)^\circ$ (completing the triangle)
$\angle BAD = \angle DBC$ (alternating segment theorem)
$\angle OBD = \angle ODB = \frac{1}{2}(180 - 2x) = (90 - x)^\circ$ (completing $\triangle OBD$)
Finally,

$$\begin{aligned}\angle ACB &= 180 - x - \angle ABC \\ &= 180 - x - [x + (90 - x) + x] \\ &= 180 - x - (90 + x) \\ &= 180 - x - 90 - x \\ &= \underline{\underline{(90 - 2x)^\circ}}.\end{aligned}$$

12. Prove that the square of an odd number is always 1 more than a multiple of 4 (4)

Solution

Let $2n + 1$ be the odd number. Then

$$\begin{aligned}(2n + 1)^2 &= 4n^2 + 4n + 1 \\ &= 4(n^2 + n) + 1 \\ &= \underline{\underline{4 \times \text{some integer} + 1}};\end{aligned}$$

hence, the claim that the square of an odd number is always 1 more than a multiple of 4 is true.

13. (3)

$$\sqrt{5}(\sqrt{8} + \sqrt{18})$$

can be written in the form $a\sqrt{10}$ where a is an integer.

Find the value of a.

Solution

$$\begin{aligned}\sqrt{5}(\sqrt{8}+\sqrt{18}) &= \sqrt{5}(\sqrt{4\times 2}+\sqrt{9\times 2}) \\ &= \sqrt{5}(\sqrt{4}\times\sqrt{2}+\sqrt{9}\times\sqrt{2}) \\ &= \sqrt{5}(2\sqrt{2}+3\sqrt{2}) \\ &= \sqrt{5}\times 5\sqrt{2} \\ &= 5\sqrt{5\times 2} \\ &= \underline{\underline{5\sqrt{10}}};\end{aligned}$$

hence, $\underline{\underline{a=5}}$.

14. y is inversely proportional to d^2. (5)
When $d = 10$, $y = 4$.

d is directly proportional to x^2.
When $x = 2$, $d = 24$.

Find a formula for y in terms of x.
Give your answer in its simplest form.

Solution

$$y \propto \frac{1}{d^2} \Rightarrow y = \frac{k}{d^2}$$

for some constant, k. Now,

$$4 = \frac{k}{10^2} \Rightarrow k = 400$$

which means

$$y = \frac{400}{d^2}.$$

Next,

$$d \propto x^2 \Rightarrow d = lx^2$$

for some constant, l. Now,

$$24 = l \times 2^2 \Rightarrow l = 6$$

which means

$$d = 6x^2.$$

Finally,

$$\begin{aligned} y &= \frac{400}{d^2} \\ &= \frac{400}{(6x^2)^2} \\ &= \frac{400}{36x^4} \\ &= \underline{\underline{\frac{100}{9x^4}}}. \end{aligned}$$

15. (a) Factorise (1)

$$a^2 - b^2.$$

Solution

$$a^2 - b^2 = \underline{\underline{(a+b)(a-b)}}.$$

(b) Hence, or otherwise, simplify fully (3)

$$(x^2+4)^2 - (x^2-2)^2.$$

Solution

$$\begin{aligned} (x^2+4)^2 - (x^2-2)^2 &= [(x^2+4) + (x^2-2)] \cdot [(x^2+4) - (x^2-2)] \\ &= (2x^2+2) \cdot 6 \\ &= \underline{\underline{12x^2 + 12}}. \end{aligned}$$

16. There are only red counters, blue counters and purple counters in a bag. (3)
The ratio of the number of red counters to the number of blue counters is 3 : 17.

Sam takes at random a counter from the bag.

The probability that the counter is purple is 0.2.

Work out the probability that Sam takes a red counter.

Solution

Well,

$$\text{P(red or blue)} = 1 - 0.2 = 0.8$$

which means that

$$\text{P(red)} = \tfrac{3}{20} \times 0.8 = \underline{\underline{0.12}}.$$

17. Simplify fully (3)

$$\frac{3x^2 - 8x - 3}{2x^2 - 6x}.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & \quad\quad\quad\quad -8 \\ \text{multiply to:} & (+3) \times (-3) = -9 \end{array}\right\} -9,\ +1$$

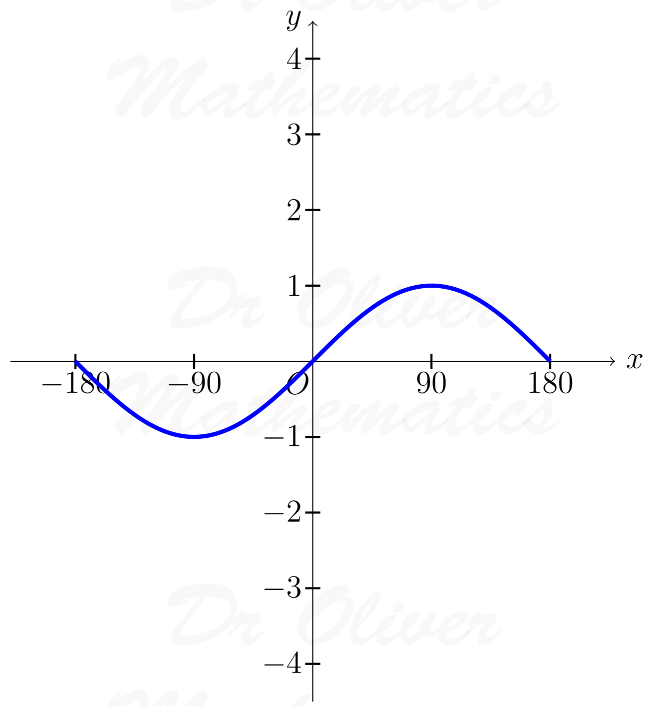
E.g.,

$$\begin{aligned} 3x^2 - 8x - 3 &= 3x^2 - 9x + x - 3 \\ &= 3x(x-3) + (x-3) \\ &= (3x+1)(x-3). \end{aligned}$$

Finally,

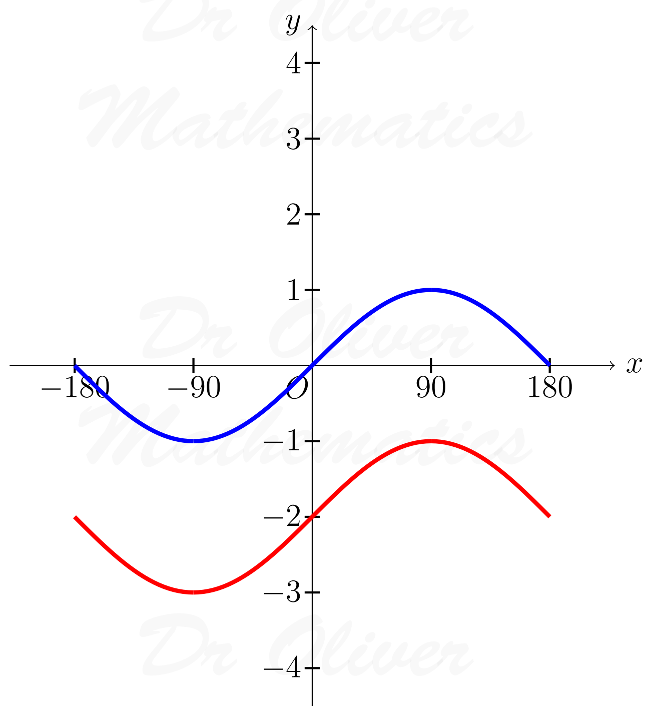
$$\begin{aligned} \frac{3x^2 - 8x - 3}{2x^2 - 6x} &= \frac{(3x+1)(x-3)}{2x(x-3)} \\ &= \underline{\underline{\frac{3x+1}{2x}}}. \end{aligned}$$

18. Here is the graph of $y = \sin x^\circ$ for $-180 \leqslant x \leqslant 180$. (2)



On the grid, sketch the graph of $y = \sin x^\circ - 2$ for $-180 \leqslant x \leqslant 180$.

Solution



19. The point P has coordinates $(3, 4)$. (5)
The point Q has coordinates (a, b).

A line perpendicular to PQ is given by the equation $3x + 2y = 7$.

Find an expression for b in terms of a.

Solution

$$3x + 2y = 7 \Rightarrow 2y = -3x + 7$$
$$\Rightarrow y = -\tfrac{3}{2}x + \tfrac{7}{2};$$

hence, the line through P and Q has gradient $\frac{2}{3}$.
Now, the equation of line through P and Q is

$$y - 4 = \tfrac{2}{3}(x - 3) \Rightarrow y - 4 = \tfrac{2}{3}x - 2$$
$$\Rightarrow y = \tfrac{2}{3}x + 2;$$

hence, $\underline{\underline{b = \frac{2}{3}a + 2}}$.

20. n is an integer such that (5)

$$3n + 2 \leqslant 14 \text{ and } \frac{6n}{n^2 + 5} > 1.$$

Find all the possible values of n.

Solution

First inequality:

$$3n + 2 \leqslant 14 \Rightarrow 3n \leqslant 12$$
$$\Rightarrow n \leqslant 4.$$

Second inequality:

$$\frac{6n}{n^2 + 5} > 1 \Rightarrow 6n > n^2 + 5$$
$$\Rightarrow n^2 - 6n + 5 < 0$$

$$\left.\begin{array}{ll}\text{add to:} & -6 \\ \text{multiply to:} & +5\end{array}\right\} -5,\ -1$$

$$\Rightarrow (n-1)(n-5) < 0$$
$$\Rightarrow 1 < n < 5.$$

So

$$1 < n \leqslant 4$$

and this means, since n is an integer, $\underline{\underline{2, 3, \text{ and } 4}}$ will work.