Dr Oliver Mathematics
Mathematics: Higher
2012 Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

Section A

1. A sequence is defined by the recurrence relation (2)

$$u_{n+1} = 3u_n + 4, \text{ with } u_0 = 1.$$

Find the value of u_2.

A. 7

B. 10

C. 25

D. 35

2. What is the gradient of the tangent to the curve with equation (2)

$$y = x^3 - 6x + 1$$

at the point where $x = -2$?

A. -24

B. 3

C. 5

D. 6

3. If (2)

$$x^2 - 6x + 14$$

is written in the form

$$(x - p)^2 + q,$$

what is the value of q?

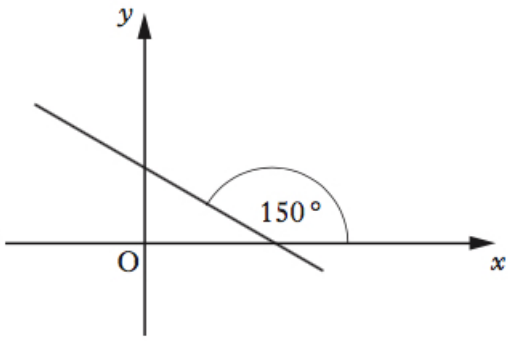
A. -22

B. 5

C. 14

D. 50

4. What is the gradient of the line shown in the diagram? (2)



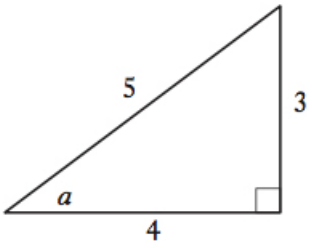
A. $-\sqrt{3}$

B. $-\dfrac{1}{\sqrt{3}}$

C. $-\frac{1}{2}$

D. $-\dfrac{\sqrt{3}}{2}$

5. The diagram shows a right-angled triangle with sides and angles as marked. (2)



What is the value of $\cos 2a$?

A. $\frac{7}{25}$

B. $\frac{3}{5}$

C. $\frac{24}{25}$

D. $\frac{6}{5}$

6. If (2)

$$y = 3x^{-2} + 2x^{\frac{3}{2}},\ x > 0,$$

determine $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

A. $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$

B. $-3x^{-1} + 3x^{\frac{1}{2}}$

C. $-6x^{-3} + 3x^{\frac{1}{2}}$

D. $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

7. If (2)

$$\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$$

are perpendicular, what is the value of t?

A. -3

B. -2

C. $\frac{2}{3}$

D. 1

8. The volume of a sphere is given by the formula (2)

$$V = \tfrac{4}{3}\pi r^3.$$

What is the rate of change of V with respect to r, at $r = 2$?

A. $\frac{16}{3}\pi$

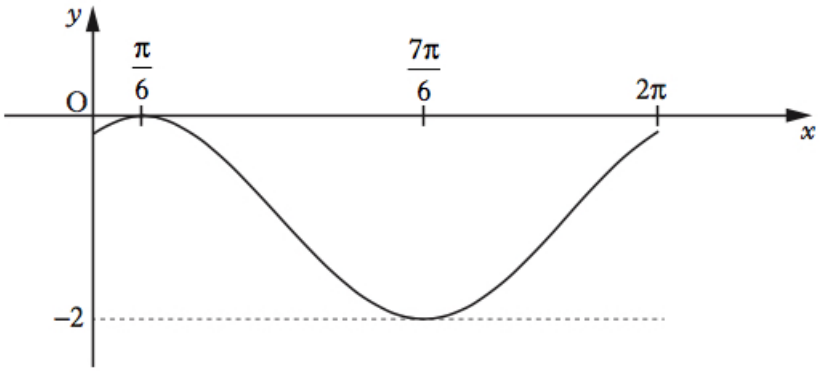
B. $\frac{32}{3}\pi$

C. 16π

D. 32π

9. The diagram shows the curve with equation of the form (2)

$$y = \cos(x + a) + b, \text{ for } 0 \leqslant x \leqslant 2\pi.$$



What is the equation of this curve?

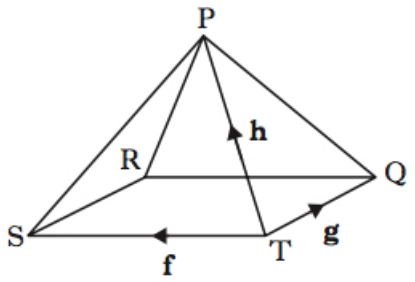
A. $y = \cos(x - \frac{1}{6}\pi) - 1$

B. $y = \cos(x - \frac{1}{6}\pi) + 1$

C. $y = \cos(x + \frac{1}{6}\pi) - 1$

D. $y = \cos(x + \frac{1}{6}\pi) + 1$

10. The diagram shows a square-based pyramid $PQRST$. (2)



$\overrightarrow{TS}$, $\overrightarrow{TQ}$, and $\overrightarrow{TP}$ represent **f**, **g**, and **h** respectively.
Express $\overrightarrow{RP}$ in terms of **f**, **g**, and **h**.

A. $-\mathbf{f} + \mathbf{g} - \mathbf{h}$

B. $-\mathbf{f} - \mathbf{g} + \mathbf{h}$

C. $\mathbf{f} - \mathbf{g} - \mathbf{h}$

D. $\mathbf{f} + \mathbf{g} + \mathbf{h}$

11. Find (2)

$$\int \frac{1}{6x^2}\,\mathrm{d}x,\ x \neq 0.$$

A. $-12x^{-3} + c$

B. $-6x^{-1} + c$

C. $-\frac{1}{3}x^{-3} + c$

D. $-\frac{1}{6}x^{-1} + c$

12. Find the maximum value of (2)

$$2 - 3\sin(x - \tfrac{1}{3}\pi)$$

and the value of x where this occurs in the interval $0 \leqslant x \leqslant 2\pi$.

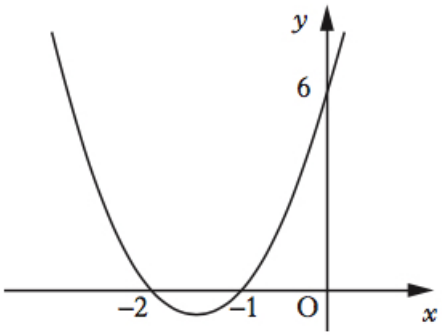
A. Maximum value is -1 when $x = \frac{11}{6}\pi$

B. Maximum value is 5 when $x = \frac{11}{6}\pi$

C. Maximum value is -1 when $x = \frac{5}{6}\pi$

D. Maximum value is 5 when $x = \frac{5}{6}\pi$

13. A parabola intersects the axes at $x = -2$, $x = -1$, and $y = 6$, as shown in the diagram. (2)



What is the equation of the parabola?

A. $y = 6(x-1)(x-2)$

B. $y = 6(x+1)(x+2)$

C. $y = 3(x-1)(x-2)$

D. $y = 3(x+1)(x+2)$

14. Find (2)

$$\int (2x-1)^{\frac{1}{2}}\,\mathrm{d}x$$

where $x > \frac{1}{2}$.

A. $\frac{1}{3}(2x-1)^{\frac{3}{2}} + c$

B. $\frac{1}{2}(2x-1)^{-\frac{1}{2}} + c$

C. $\frac{1}{2}(2x-1)^{\frac{3}{2}} + c$

D. $\frac{1}{3}(2x-1)^{-\frac{1}{2}} + c$

15. If (2)

$$\mathbf{u} = k\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix},$$

where $k > 0$ and $\mathbf{u}$ is a unit vector, determine the value of k.

A. $\frac{1}{2}$

B. $\frac{1}{8}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{10}}$

16. If $y = 3\cos^4 x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (2)

A. $12\cos^3 x \sin x$

B. $12\cos^3 x$

C. $-12\cos^3 x \sin x$

D. $-12\sin^3 x$

17. Given that (2)

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \text{ and } \mathbf{a}.(\mathbf{a}+\mathbf{b}) = 7,$$

what is the value of $\mathbf{a}.\mathbf{b}$?

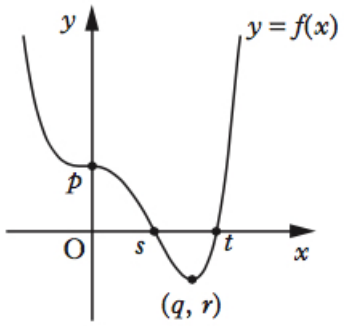
A. $\frac{7}{25}$

B. $-\frac{18}{5}$

C. -6

D. -18

18. The graph of $y = \mathrm{f}(x)$ shown has stationary points at $(0, p)$ and (q, r). (2)



Here are two statements about $f(x)$:
(1) $\mathrm{f}(x) < 0$ for $s < x < t$;
(2) $\mathrm{f}'(x) < 0$ for $x < q$.
Which of the following is true?

A. Neither statement is correct.

B. Only statement (1) is correct.

C. Only statement (2) is correct.

D. Both statements are correct.

19. Solve (2)

$$6 - x - x^2 < 0.$$

A. $-3 < x < 2$

B. $x < -3$ or $x > 2$

C. $-2 < x < 3$

D. $x < -2$ or $x > 3$

20. Simplify (2)

$$\frac{\log_b 9a^2}{\log_b 3a},$$

where $a > 0$ and $b > 0$.

A. 2

B. $3a$

C. $\log_b 3a$

D. $\log_b(9a^2 - 3a)$.

Section B

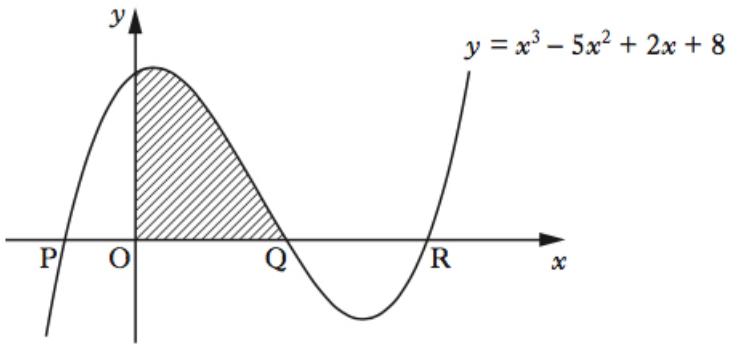
21. (a) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$. (6)

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

The diagram shows the curve with equation

$$y = x^3 - 5x^2 + 2x + 8.$$



The curve crosses the x-axis at P, Q, and R.

(b) Determine the shaded area. (6)

22. (a) The expression (4)

$$\cos x - \sqrt{3}\sin x$$

can be written in the form

$$k\cos(x + a),$$

where $k > 0$ and $0 \leqslant a < 2\pi$.

Calculate the values of k and a.

(b) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x- and y-axes, in the interval $0 \leqslant x < 2\pi$. (3)

23. (a) Find the equation of l_1, the perpendicular bisector of the line joining $P(3, -3)$ to $Q(-1, 9)$. (4)

(b) Find the equation of l_2 which is parallel to PQ and passes through $R(1, -2)$. (2)

(c) Find the point of intersection of l_1 and l_2. (3)

(d) Hence find the shortest distance between PQ and l_2. (2)