Dr Oliver Mathematics
AQA Further Maths Level 2
June 2019 Paper 2
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

1. (a) (3)

$$a\begin{pmatrix} 3 \\ 5 \end{pmatrix} = 4\begin{pmatrix} 2a+3 \\ b \end{pmatrix}.$$

Work out the values of a and b.

(b) (2)

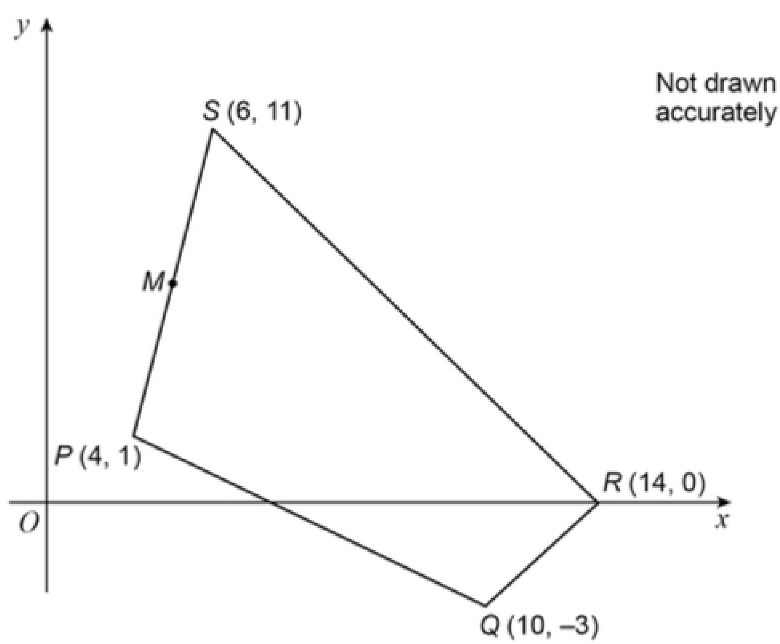
$$\begin{pmatrix} m & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ -2 & -1 \end{pmatrix} = \mathbf{I},$$

where $\mathbf{I}$ is the identity matrix.

Work out the value of m.

2. Here is a sketch of quadrilateral $PQRS$. (3)

M is the midpoint of PS.



Use gradients to show that MR is parallel to PQ.

3. (4)

$$-2 < a < 0 \text{ and } -1 < b < 1.$$

Tick the correct box for each statement.

	Always true	**Sometimes true**	**Never true**
$a^2 < 0$			
$-1 < b^3 < 1$			
$\frac{b}{a} < 0$			
$a - b > 0$			

4. P is a point on a curve. (4)

The curve has gradient function

$$\frac{x^5 - 17}{10}.$$

The tangent to the curve at P is parallel to the line

$$3x - 2y = 9.$$

Work out the x-coordinate of P.

5. (a) Write (2)

$$\sqrt[4]{a \times a^{-9}}$$

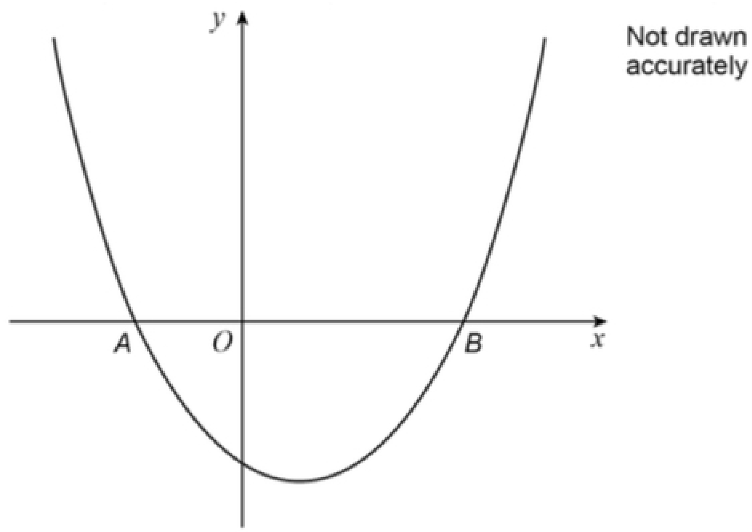
as an integer power of a.

(b) Simplify fully (3)

$$\frac{(4cd^2)^3}{2cd^4}.$$

6. Here is a sketch of the curve

$$y = (2x + 3)(x - 2).$$



The curve intersects the x-axis at A and B.

(a) Complete the coordinates of A and B. (2)

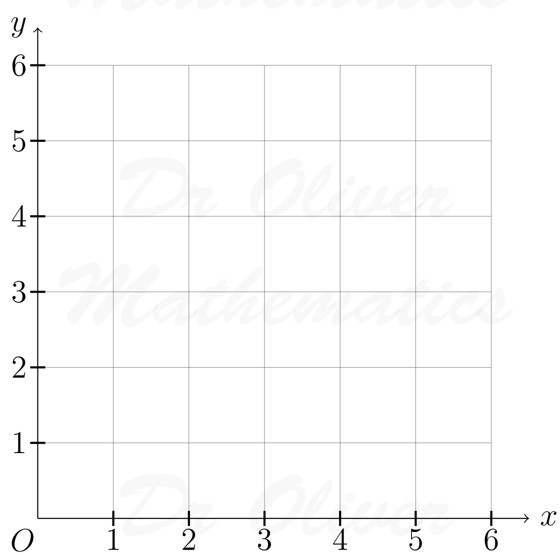
$$A(\qquad, 0) \qquad B(\qquad, 0)$$

(b) Write down the range of values for x for which (1)

$$(2x + 3)(x - 2) < 0.$$

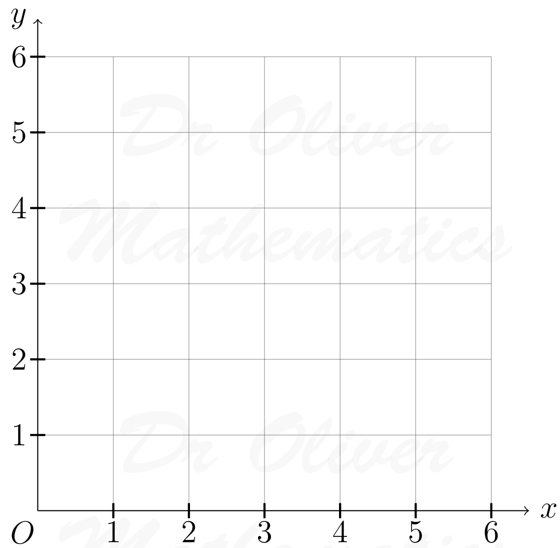
7. (a) On the grid, sketch a graph for which (1)

the rate of change of y with respect to x is always zero.



(b) On the grid, sketch a graph for which (1)

the rate of change of y with respect to x is always a positive constant.



8. (a) A linear sequence has first term (2)

$$7 + 12\sqrt{5}.$$

The term-to-term rule is

$$\text{add } 9 - 2\sqrt{5}.$$

One term of the sequence is an integer.

Work out the value of this integer.

(b) The nth term of a different sequence is (2)

$$\frac{3n^2 - 1}{n^2 + 1}.$$

Work out the sum of the first three terms.

(c) The first four terms of a quadratic sequence are (3)

$$-3 \qquad 3 \qquad 13 \qquad 27.$$

Work out an expression for the nth term.

9. Factorise fully (2)

$$(p + 6)^{11} - (p + 6)^{10}.$$

10. (a) (2)

$$f(x) = x^3 - 2.$$

The domain of $f(x)$ is $x \leqslant 3$.

Work out the range of $f(x)$.

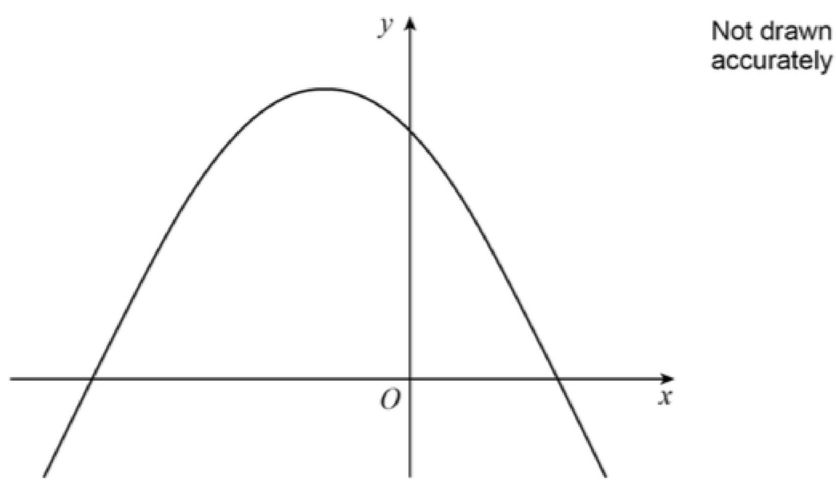
(b) (2)

$$g(x) = 5 - x^2.$$

The domain of $g(x)$ is $-2 \leqslant x \leqslant 1$.

Work out the range of $g(x)$.

11. Here is a sketch of a quadratic curve which has a maximum point at $(-2, 5)$. (1)

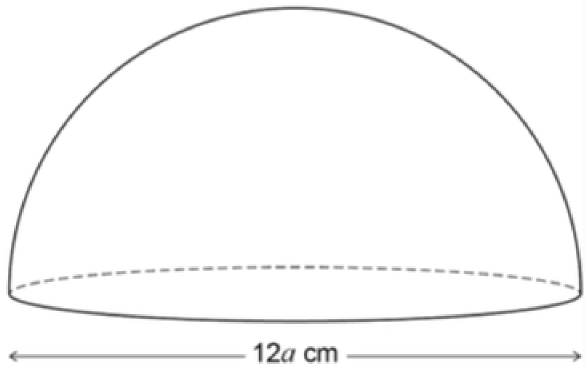


What is the equation of the normal to the curve at the maximum point?
Circle your answer.

$x = -2$ $\quad$ $y = 5$ $\quad$ $x = 5$ $\quad$ $y = -2$

12. The diagram shows a solid hemisphere. (3)

- The diameter is $12a$ cm.
- The volume is 486π cm^3.



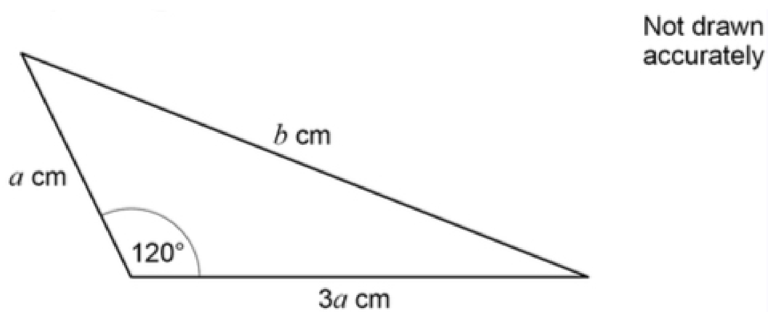
Work out the value of a.

13. Simplify fully (4)

$$\frac{x - x^3}{2x + 2x^2}.$$

You **must** show your working.

14. Here is a triangle. (3)



Use the cosine rule to work out the ratio

$$b^2 : a^2.$$

15. Rearrange (4)

$$m = \frac{2p + 1}{p} + \frac{p + 5}{3p}$$

to make p the subject.

16. The curve (3)

$$y = 2\sqrt{x - a} + 5$$

passes through the point $(1, 8)$

Work out the value of a.

17. Show that (5)

$$(x+1)(x+3)(x+4) - x(x^2+7x+11)$$

can be written in the form

$$(x+a)(x+b),$$

where a and b are positive integers.

18. Solve (3)

$$4(x-5)^2 = k^2,$$

where k is a constant.

Give your answers in their simplest form in terms of k.

19.
- ABC is a right-angled triangle.
- ACD is an isosceles triangle.
- All dimensions are in centimetres.

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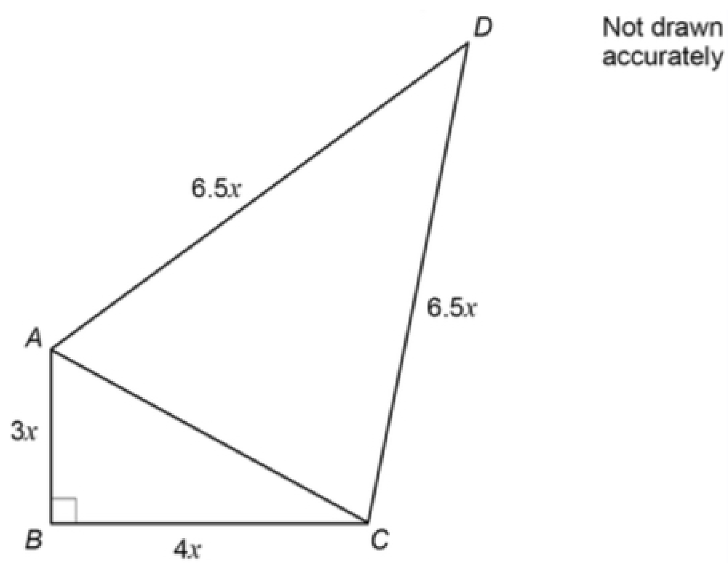


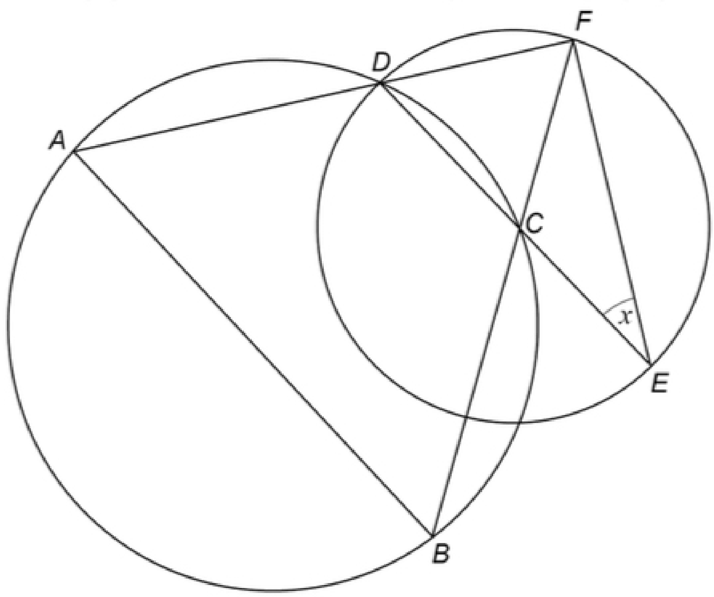
(a) Show that $AC = 5x$. (1)

(b) Work out an expression, in cm^2, for the area of quadrilateral $ABCD$. (5)

Give your answer in the form px^2, where p is an integer.

20.
- A, B, C, and D are points on a circle.
- D, E, and F are points on a different circle, centre C.

- DCE, ADF, and BCF are straight lines.
- Angle $DEF = x$.



Not drawn accurately

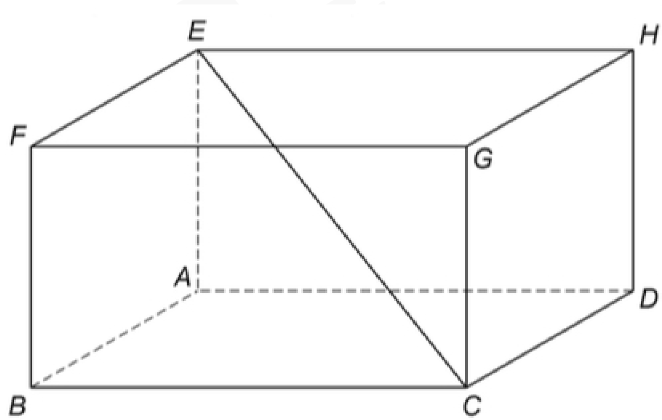
(a) Prove that (3)

$$\text{angle } BAD = 2x.$$

(b) In the case when AB is parallel to DE, work out the size of angle x. (2)

21. $ABCDEFGH$ is a cuboid. (4)

- $BC = 15$ cm.
- $CD = 12$ cm.
- $DH = 8$ cm.



Work out the size of the angle between the line CE and the plane $CDHG$.

22. (a) Show that (3)

$$\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x}$$

is equivalent to $\tan x$.

(b) Hence solve (2)

$$\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = -1,$$

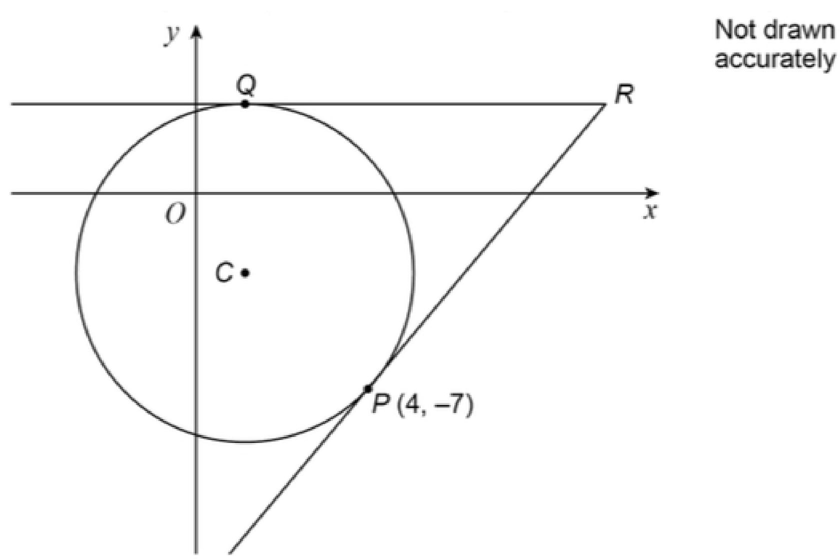
for $0^\circ \leqslant x \leqslant 360^\circ$.

23. A circle has centre C and equation

$$(x-1)^2 + (y+3)^2 = 25.$$

$P(4, -7)$ and Q are points on the circle.

- The tangent at Q is parallel to the x-axis.
- The tangents at P and Q intersect at point R.



(a) Write down the coordinates of C. (1)

(b) Show that the equation of the tangent at Q is (1)

$$y = 2.$$

(c) Work out the x-coordinate of R. (4)

24. Show that the curve (4)

$$y = \tfrac{3}{5}x^5 + x^4$$

has **exactly two** stationary points.

25. (3)

$$\mathrm{f}(x) = x^3 - 10x - c, \text{ where } c \text{ is a positive integer.}$$

$(x + c)$ is a factor of $\mathrm{f}(x)$.

Use the factor theorem to work out the value of c.

26. $\mathrm{f}(x)$ is a function with domain all values of x. (4)

$$\mathrm{f}(x) = x^2 + 6x - a, \text{ where } a \text{ is a constant.}$$

Work out the possible values of a.

Give your answer as an inequality.

27. The curve $y = \mathrm{f}(x)$ has (3)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x+2)^6 + (x+2)^4.$$

The curve has exactly one stationary point at P where $x = -2$.

Use the expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to show that P is a point of inflection.